1

Magnetic field and its sources

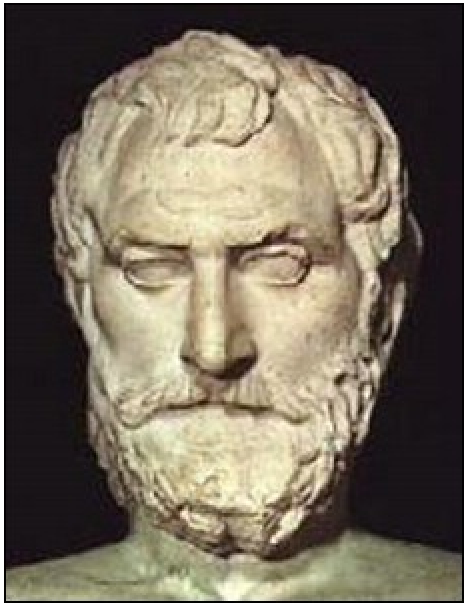
Magnetization reversal in thin films and some relevant experimental methods

Maciej Urbaniak

Today's plan

- The beginnings of the science of magnetism
- The field of the currents - Biot-Savart law
- The field of magnetic dipoles
- Magnetic fields in the universe
- Basic magnetic field measurements methods

Thales of Miletus (about 585BCE)- the first mention of the influence of loadstone on iron[1]



Aristotle: ‘Thales, too, to judge from what is recorded of his views, seems to suppose that the soul is in a sense the cause of movement, since he says that a stone [magnet, or lodestone] has a soul because it causes **movement to iron**’ (On the soul (Perì Psūchês), 405 a20-22)

Probably the first practical application of magnetism

Sushruta Samhita (Indian book from IV century CE giving supposedly teachings of surgeon Sushruta acting about 600 BCE):
A loose, unbarbed arrow, lodged in a wound with a broad mouth and lying in an Anuloma direction, **should be withdrawn by applying a magnet** to its end.

Lucretius (98-55 BCE)- the first recorded theory of magnetic interactions (following the view of Epicurus and Democritus [1]. De rerum natura (O naturze wszechrzeczy, translation in polish E. Szymański):

Teraz powiem, na mocy jakiego natury prawa
Może żelazo przyciągać **ten kamień, który Greki**
Magnesem zwą od ziemi Magnetów — w tym bo dalekim
Kraju kamień ten cenny rodzi się i przebywa.
Ludzi uczonych od dawna nie darmo on zadziwia:
…
Teraz cel osiągniemy dokładniej już i prędzej.
Bo skoro wszystkie dane sprawdzone i gotowe,
Z ich pomocą prawdziwie poznamy siły owe,
Dzięki którym kamień żelazo do siebie przyzywa.
Naprzód musi z kamienia dużo ziaren wypływać.
Istny prąd, co roztrąca swem mocnem uderzeniem
Warstwę powietrza między żelazem i kamieniem.
Gdy się opróżni przestrzeń i w środku miejsca sporo,
Zaraz ziarna żelaza wyskoczą, wnet się zbiorą
Próżnię wypełnić, zaczem zbliża się i ogniwo,
Całem swem ciałem dążąc ku kamieniowi co żywo.

"In other word, tiny particles emanating from the loadstone sweep away the air and the consequent suction draws in the iron" -Fowler [1]

Lucretius:

•the gold is to heavy to be attracted by magnets

•the wood is so light...

And of course there were Chinese. They new magnetic needle from ca. 400 BCE. But the first Chinese mention of the use of magnetic needle for navigation refers to the period 1086-99 and concerns the used by "*Muslim sailors between Canton and Sumatra*" [5].

The South-Pointing Fish

source: shanghai.cultural-china.com

William Gilbert (1544-1603) – royal physician to Queen Elizabeth I

-”De Magnete” (1600) the first scientific investigation of magnetism [1]:

• **the earth is a giant magnet** (previously there was a belief that there was a magnetic island or star *Polaris* that attracted compass needles)

• magnetic (and electric) attraction depends on **the distance between bodies**

Working iron in a smithy

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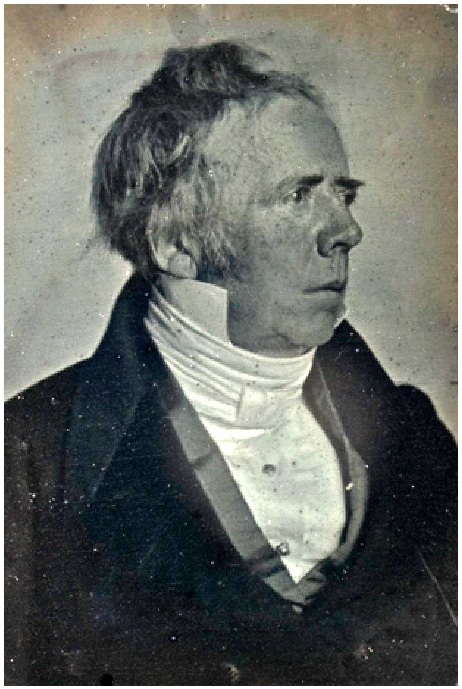
inducing magnetic anisotropy by metalworking

Working iron in a smithy

Hans Christian Ørsted (1777–1851)



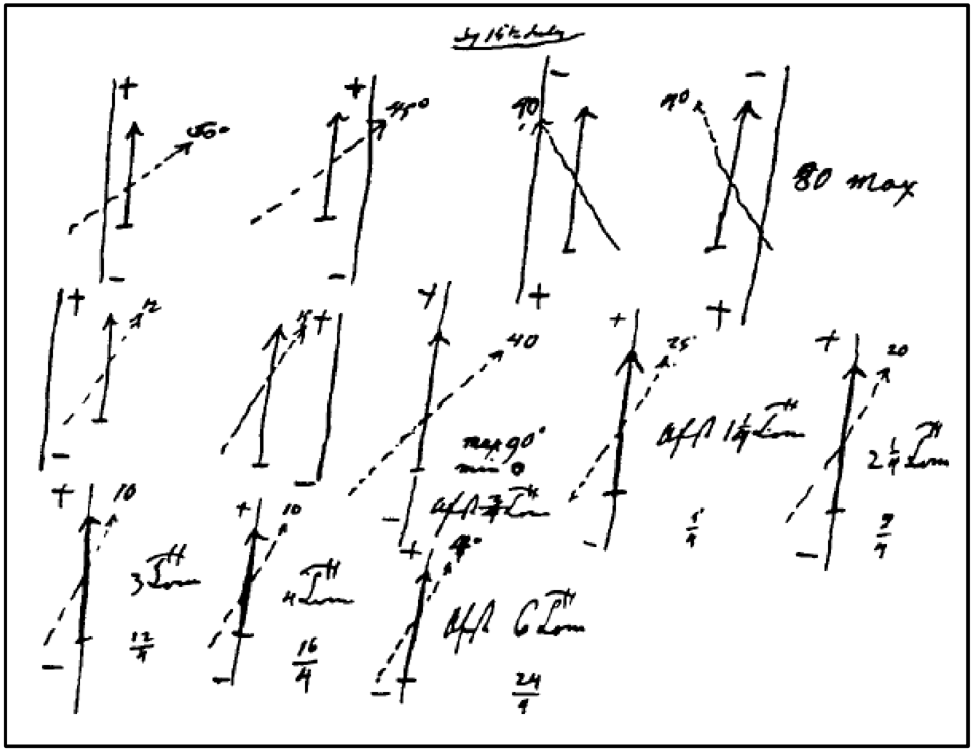
• Around 1750 Benjamin Franklin magnetized sewing needles by the electrical discharge of a Leyden jar [6] but the effect was due to Joule heating in the Earth's magnetic field.

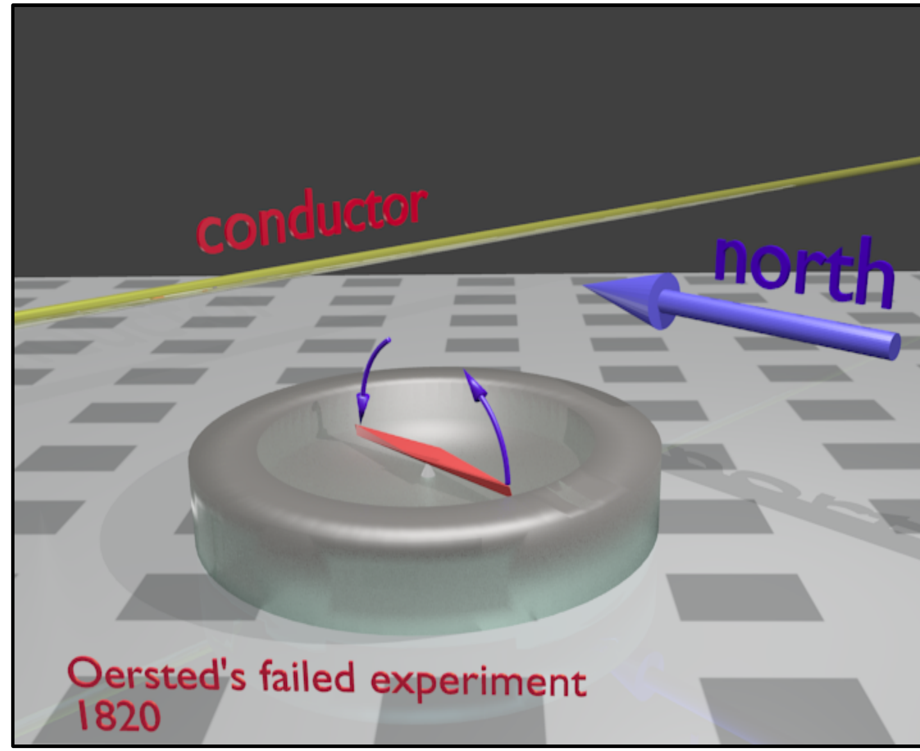
• In 1795 Coulomb established that magnetic forces obey the inverse square law [6].

• In 1805 Hachette and Désormes unsuccessfully attempted to build a electric compass [6].

• In 1820 Ørsted discovers that electric **current deflects magnetic needle** – the begin of electromagnetism.

Hans Christian Ørsted (1777–1851)

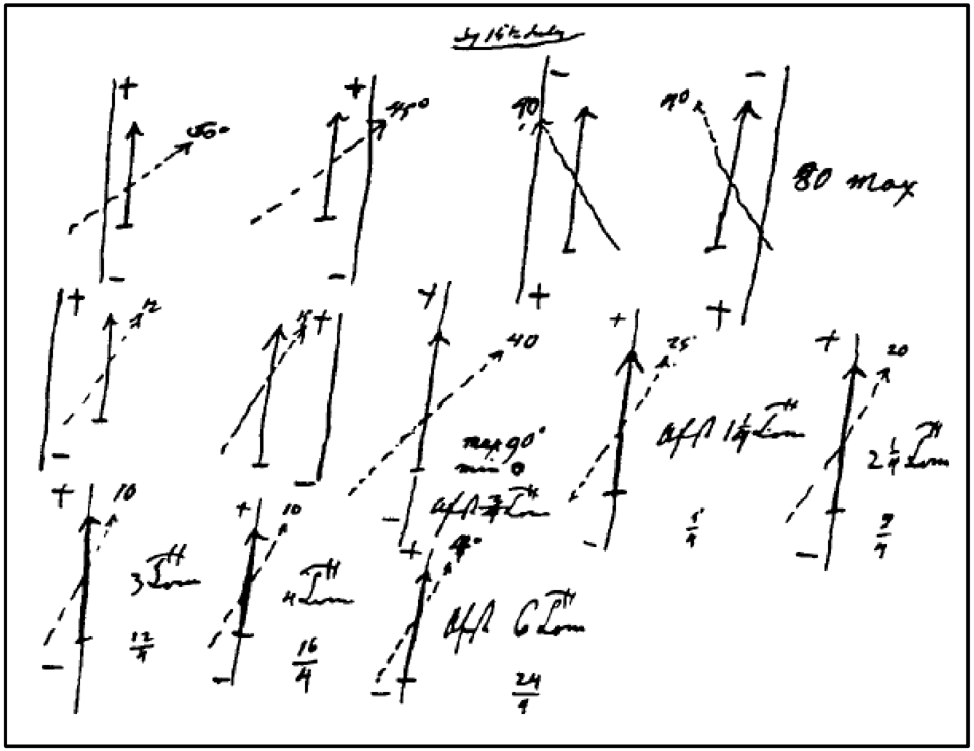


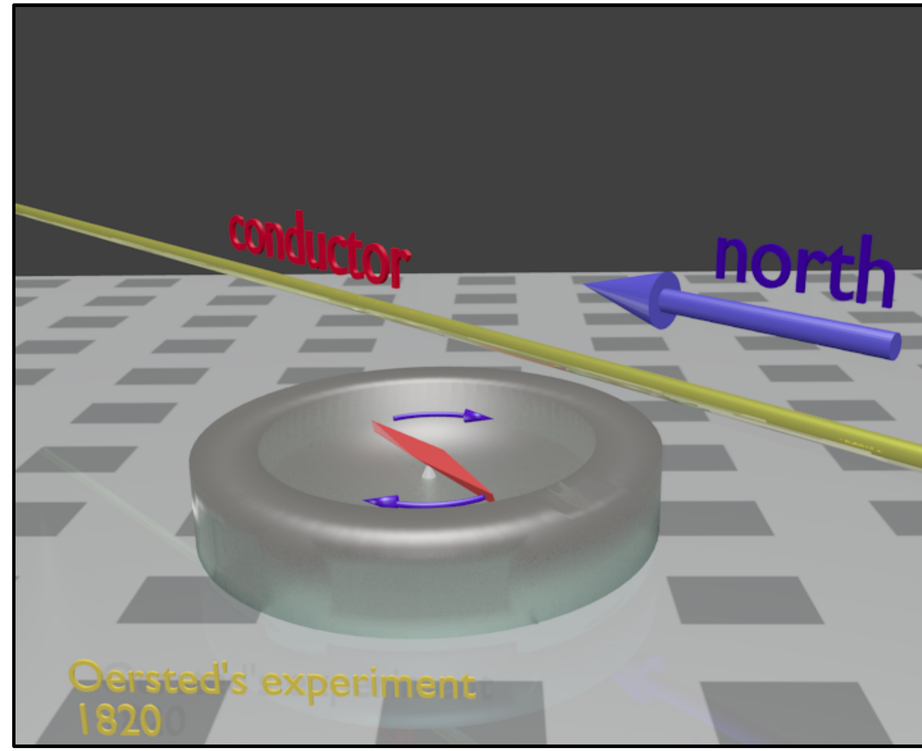


Ørsted's laboratory notes from 1820.07.15

- Before 1820 Ørsted's first hypothesis was that the magnetic effect should be parallel to the wire [6] – it lead to the misplacement of the wire relative to the south-north direction: a force couple would act to turn the needle in a vertical plane, and the suspension of the needle would prevent this kind of motion. So, if Ørsted attempted such experiments, he could observe no effect [6].

Hans Christian Ørsted (1777–1851)





Ørsted's laboratory notes from 1820.07.15

- Before 1820 Ørsted's first hypothesis was that the magnetic effect should be parallel to the wire [6] – it lead to the misplacement of the wire relative to the south-north direction.
- According to Ørsted's final view, the **magnetic effect of an electric current rotates around the conducting wire**

Biot-Savart Law – 1820

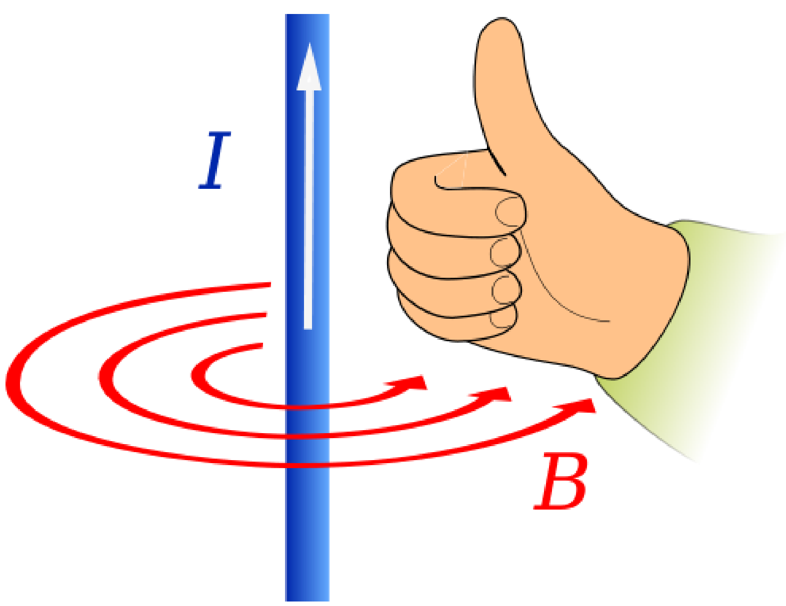
• Jean-Baptiste Biot (1774-1862), Félix Savart (1791-1841)

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\hat{d}l \times \vec{r}}{|\vec{r}|^3}$$

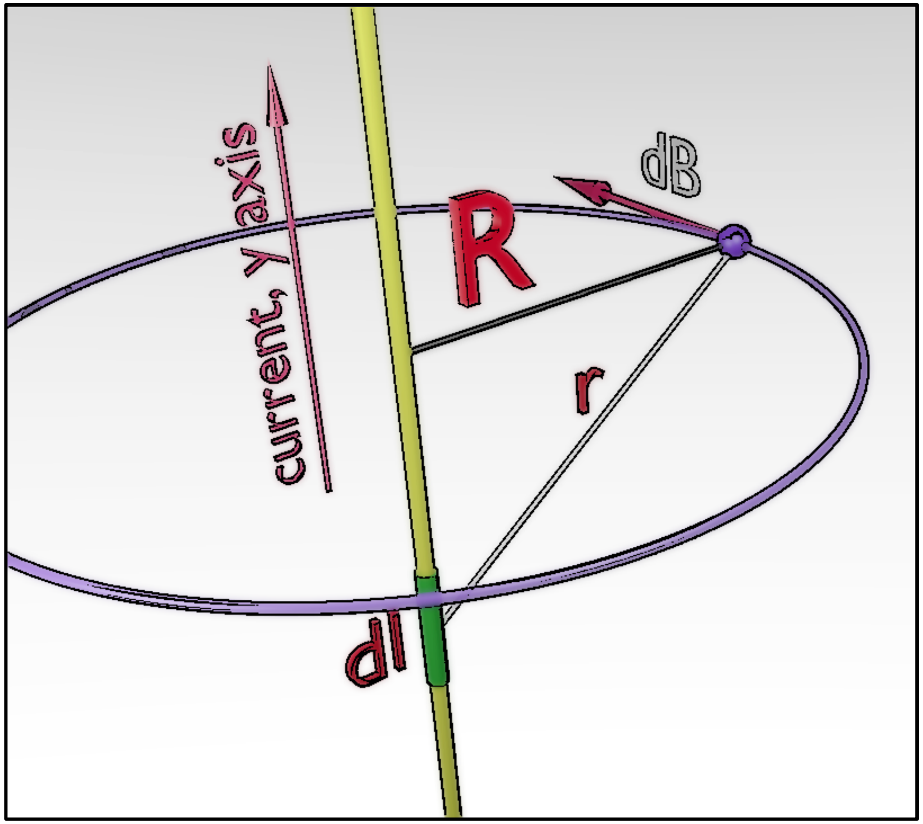
$\mu_0 = 4\pi 10^{-7}\, Hm^{-1}$ -vacuum permeability

•magnetic field is created by the electric current

•meaningful only for closed circuits



author: Jfmelero; from Wikimedia Commons



Biot-Savart Law – 1820

• Field of an infinite conductor

$$d\vec{B}=\frac{\mu_0 I}{4\pi}\frac{\hat{dl}\times\vec{r}}{|\vec{r}|^3}=\frac{\mu_0 I}{4\pi}\frac{dy|\vec{r}|\sin(\theta)}{|\vec{r}|^3}=\frac{\mu_0 I}{4\pi}\frac{dy|\vec{r}|\frac{R}{|\vec{r}|}}{|\vec{r}|^3}=\frac{\mu_0 I}{4\pi}\frac{R\,dy}{|\vec{r}|^3}=\frac{\mu_0 I}{4\pi}\frac{R\,dy}{\left(\sqrt{R^2+y^2}\right)^3}$$

The problem has a circular symmetry so the magnitude of ***B*** depends only on R.

$$\vec{B}(R)=\frac{\mu_0 I R}{4\pi}\int_{-\infty}^{\infty}\frac{dy}{\left(\sqrt{R^2+y^2}\right)^3}=\frac{\mu_0 I R}{4\pi}\left[\frac{y}{R^2\left(\sqrt{R^2+y^2}\right)}\right]_{-\infty}^{+\infty}=\frac{\mu_0 I}{4\pi R}\left[\frac{\infty}{\infty}-\left(\frac{-\infty}{+\infty}\right)\right]=\frac{\mu_0 I}{2\pi R}$$

$$\boxed{\vec{B}(R)=\frac{\mu_0 I}{2\pi R}}$$

•An infinite straight conductor carrying a current of 1 A creates a magnetic field which is weaker than earth's magnetic (~10^{-5} T) field at a distance greater than **4 millimeters** from the wire.

• Passing a current through a straight wire is not an effective way of generating magnetic field [11].

Basic properties of static magnetic field

It follows from Biot-Savart law that [7,8]:

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\times\frac{r-r'}{|r-r'|^3}d^3r'$$

$\vec{J}(\vec{r}\,')$ - current density

Using the identity:

$$\nabla_{\vec{r}}\left(\frac{1}{|r-r'|}\right) = \nabla_{\vec{r}}\left(\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\right) = -\frac{\vec{r}-\vec{r}\,'}{|r-r'|^3}$$

We obtain:

$$\vec{B}(\vec{r})=\frac{-\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\times\nabla\left(\frac{1}{|r-r'|}\right)d^3r'$$

Using the identity $\nabla\times(\beta\vec{a})=\beta\nabla\times\vec{a}-\vec{a}\times\nabla\beta$ with $\vec{J}\rightarrow\vec{a}$ and $\beta\rightarrow 1/|r-r'|$ we get:

$$\vec{J}(\vec{r}\,')\times\nabla\left(\frac{1}{|r-r'|}\right)=\frac{1}{|r-r'|}\nabla\times\vec{J}(\vec{r}\,')-\nabla\times\frac{\vec{J}(\vec{r}\,')}{|r-r'|}$$

, but ***J*** does not depend on ***r***, so...

Basic properties of static magnetic field

so...

$$\vec{J}(\vec{r}\,')\times\nabla\left(\frac{1}{|r-r'|}\right)=-\nabla\times\frac{J(\vec{r}\,')}{|r-r'|}$$

, and thus

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\int\nabla\times\frac{J(\vec{r}\,')}{|r-r'|}d^3r'$$

, and since rotation operator does not act on primed coordinates we can rewrite (nabla moves outside the integral):

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\nabla\times\int\frac{J(\vec{r}\,')}{|r-r'|}d^3r' \qquad (1)$$

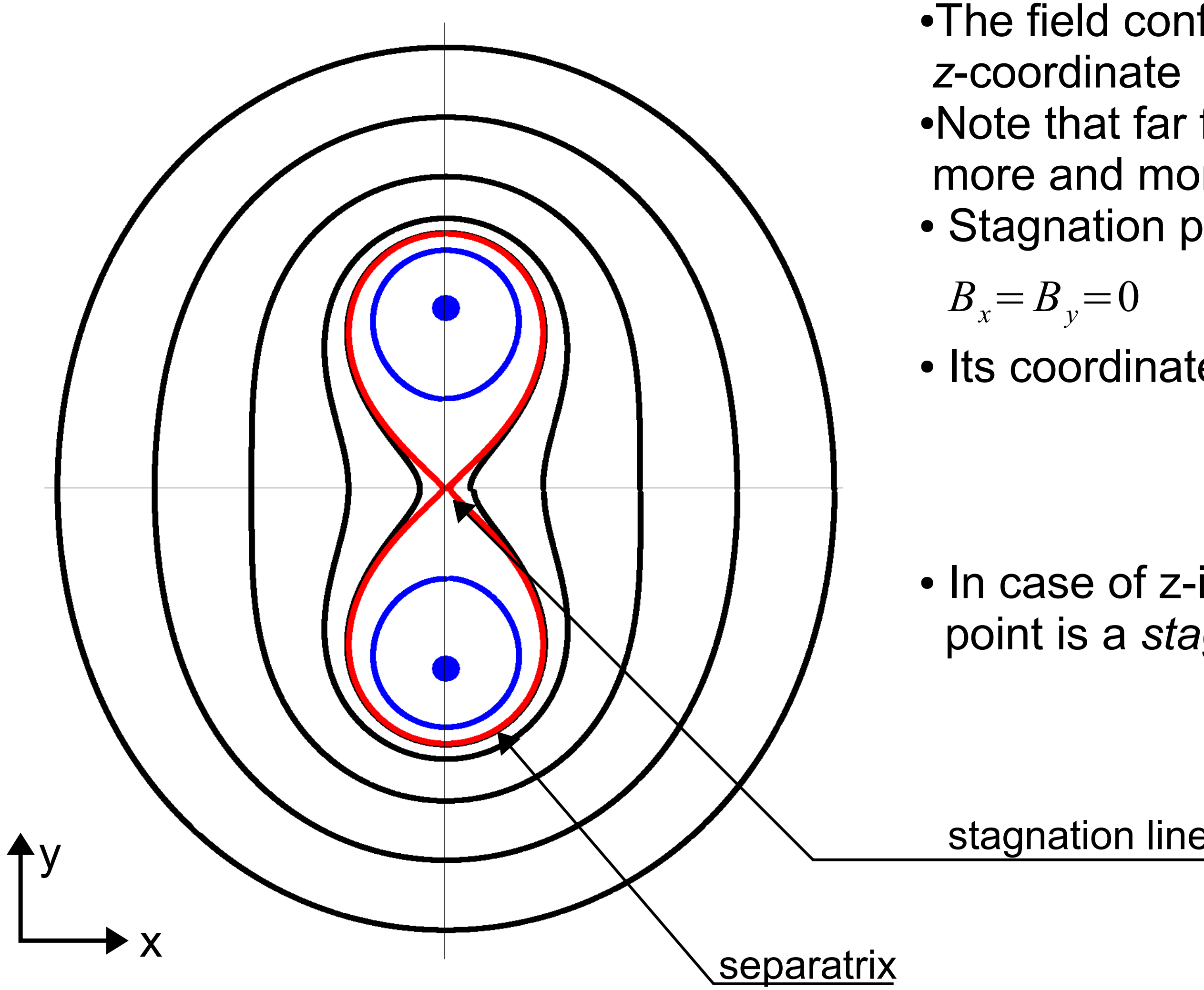
Using vector identity $\nabla\cdot(\nabla\times\vec{a})=0$ we get the first differential equation of magnetostatics:

$$\boxed{\nabla\cdot\vec{B}=0}$$

- there are no sources or sinks of magnetic induction vector (there are no magnetic charges emanating magnetic induction)
- ***B*** is a solenoidal field

Separatrix and stagnation points

- Directions of magnetic field of two parallel, infinite currents lines:



two currents of the same direction and magnitude

- The field configuration does not depend on *z*-coordinate
- Note that far from currents the field lines are more and more circle-like
- Stagnation point is defined by [19]:

$$B_x = B_y = 0$$

- Its coordinates are:

$$x_s = 0 \qquad y_s = \frac{d}{2}\frac{I_1 - I_2}{I_1 + I_2}$$

- In case of z-independent field the stagnation point is a *stagnation line*

Basic properties of static magnetic field

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\nabla\times\int\frac{J(\vec{r}\,')}{|r-r\,'|}d^3r\,'=\nabla\times\boxed{\frac{\mu_0}{4\pi}\int\frac{J(\vec{r}\,')}{|r-r\,'|}d^3r\,'}$$

This is called **magnetic vector potential**

$$\vec{B}(\vec{r})=\nabla\times\vec{A}(\vec{r}) \quad {}^*$$

$$\nabla\cdot(\nabla\times\vec{a})=0$$

For an arbitrary ***A*** the magnetic induction ***B*** is divergenceless

*Because $\nabla\times\nabla\phi=0$ one can add gradient of scalar function to ***A*** without changing ***B***.

Basic properties of static magnetic field

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\nabla\times\int\frac{J(\vec{r}\,')}{|r-r'|}d^3r' \qquad (1)$$

From (1), using the identity $\nabla\times(\nabla\times\vec{a})=\nabla(\nabla\cdot\vec{a})-\nabla^2\vec{a}$, the rotation of magnetic induction is:

$$\nabla\times\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\nabla\left(\nabla\cdot\int\frac{J(\vec{r}\,')}{|r-r'|}d^3r'\right)-\frac{\mu_0}{4\pi}\nabla^2\left(\int\frac{J(\vec{r}\,')}{|r-r'|}d^3r'\right)$$

In the first term we use the identities $\nabla\cdot(\beta\vec{a})=\vec{a}\cdot\nabla\beta+\beta\nabla\cdot\vec{a}$ and $\nabla\left(\frac{1}{|r-r'|}\right)=-\nabla'\left(\frac{1}{|r-r'|}\right)$ to get:

$$\nabla\cdot\int\frac{J(\vec{r}\,')}{|r-r'|}d^3r'=\int\nabla\cdot\frac{J(\vec{r}\,')}{|r-r'|}d^3r'=-\int\nabla'\cdot\frac{J(\vec{r}\,')}{|r-r'|}d^3r'=$$

$$\boxed{\vec{a}=J(\vec{r}\,') \qquad \beta=\frac{1}{|r-r'|}}$$

$$-\int\left(J(\vec{r}\,')\cdot\nabla'\frac{1}{|r-r'|}+\frac{1}{|r-r'|}\nabla'\cdot J(\vec{r}\,')\right)d^3r'$$

In manetostatics we assume $\nabla\cdot J(\vec{r})=0$ (no charge accumulation) so we get, remembering that nabla acts here on unprimed coordinates (for the second integral) [7]:

$$\nabla\times\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\nabla\left(\int J(\vec{r}\,')\cdot\nabla'\frac{1}{|r-r'|}d^3r'\right)-\frac{\mu_0}{4\pi}\int J(\vec{r}\,')\nabla^2\frac{1}{|r-r'|}d^3r' \qquad (2)$$

Basic properties of static magnetic field

$$\nabla\times\vec{B}(\vec{r})=\ldots-\frac{\mu_0}{4\pi}\int J(\vec{r}\,')\nabla^2\frac{1}{|r-r'|}d^3r' \qquad (2)$$

From (2), using the identity:

$$\nabla^2\left(\frac{1}{|r-r'|}\right)=-4\pi\delta(r-r')$$ (Dirac's delta)

we get:

$$\nabla\times\vec{B}(\vec{r})=-\frac{\mu_0}{4\pi}\nabla\left(\int J(\vec{r}\,')\cdot\nabla'\frac{1}{|r-r'|}d^3r'\right)+\mu_0 J(\vec{r})$$

We integrate the remaining integral using integration by parts:

$$\left(\frac{J(\vec{r}\,')}{|r-r'|}\right)'=J(\vec{r}\,')\left(\frac{1}{|r-r'|}\right)'+J'(\vec{r}\,')\left(\frac{1}{|r-r'|}\right)$$

$$\int J(\vec{r}\,')\cdot\nabla'\frac{1}{|r-r'|}d^3r'=-\int\frac{\nabla'\cdot J(\vec{r}\,')}{|r-r'|}d^3r'+\int\nabla'\cdot\left(\frac{J(\vec{r}\,')}{|r-r'|}\right)d^3r'$$

The first integral vanishes by the divergence of current ($\nabla\cdot J(\vec{r})=0$). The second integral can be changed into a surface integral [8,9] by applying:

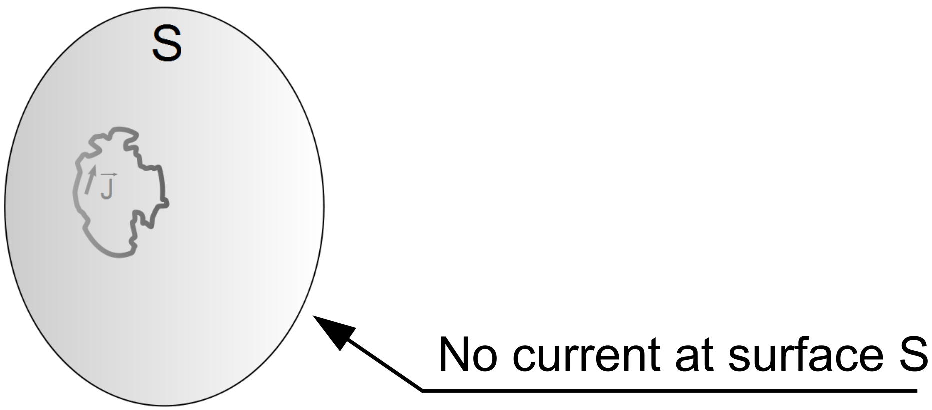
Gauss's theorem $\int_S\vec{A}\cdot dS=\int_V\nabla\cdot\vec{A}\,dV$

Basic properties of static magnetic field

Gauss's theorem $\int_S \vec{A}\cdot dS = \int_V \nabla\cdot\vec{A}\, dV$

$$\int \vec{J}(\vec{r}\,')\cdot\nabla'\frac{1}{|r-r'|}d^3r' = \int \nabla'\cdot\left(\frac{\vec{J}(\vec{r}\,')}{|r-r'|}\right)d^3r' = \int_S \frac{\vec{J}(\vec{r}\,')\cdot\vec{n}}{|r-r'|}dS$$

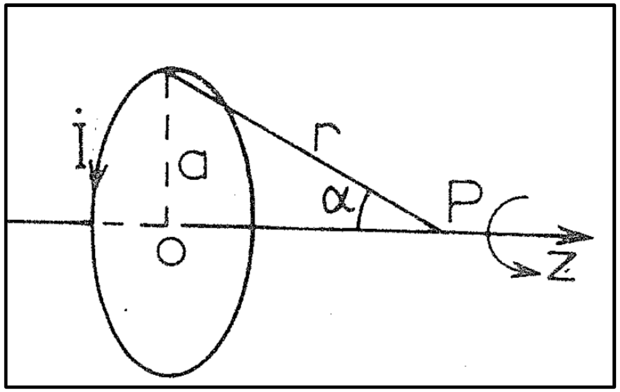
The integral vanishes as the volume enclosing currents is limited but the surface S can be placed **far away** from the currents. Finally we get:



$$\nabla\times\vec{B} = \mu_0\vec{J}(\vec{r})$$

- ***B*** is a solenoidal field

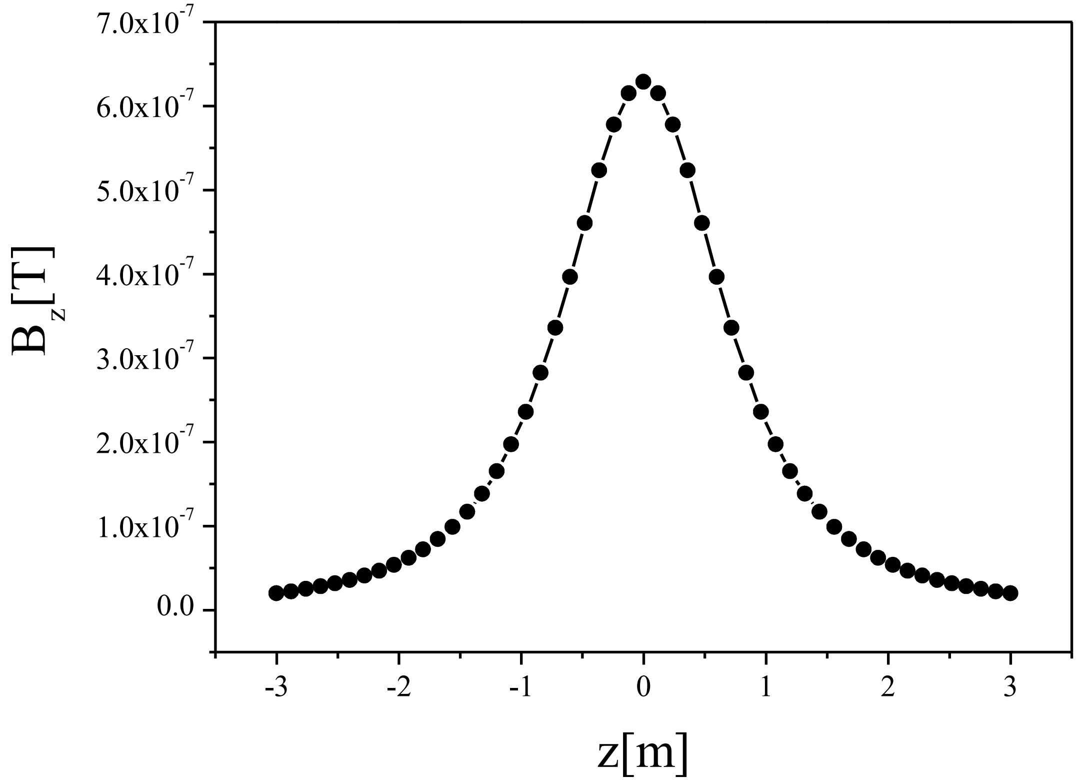
Magnetic field of circular currents loops



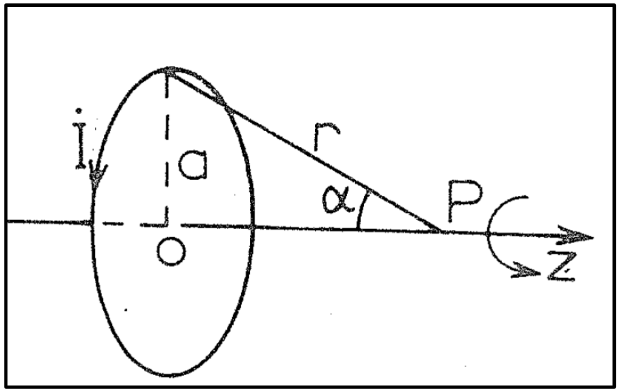
Source E. Durand [12]

- We are interested in the field produced by a current loop
- The exact formulas are quite difficult to derive [see 7,12]
- Here we do a numerical integration from Biot-Savart law (loop radius-1m, current 1A)

Field on symmetry axis



Magnetic field of circular currents loops



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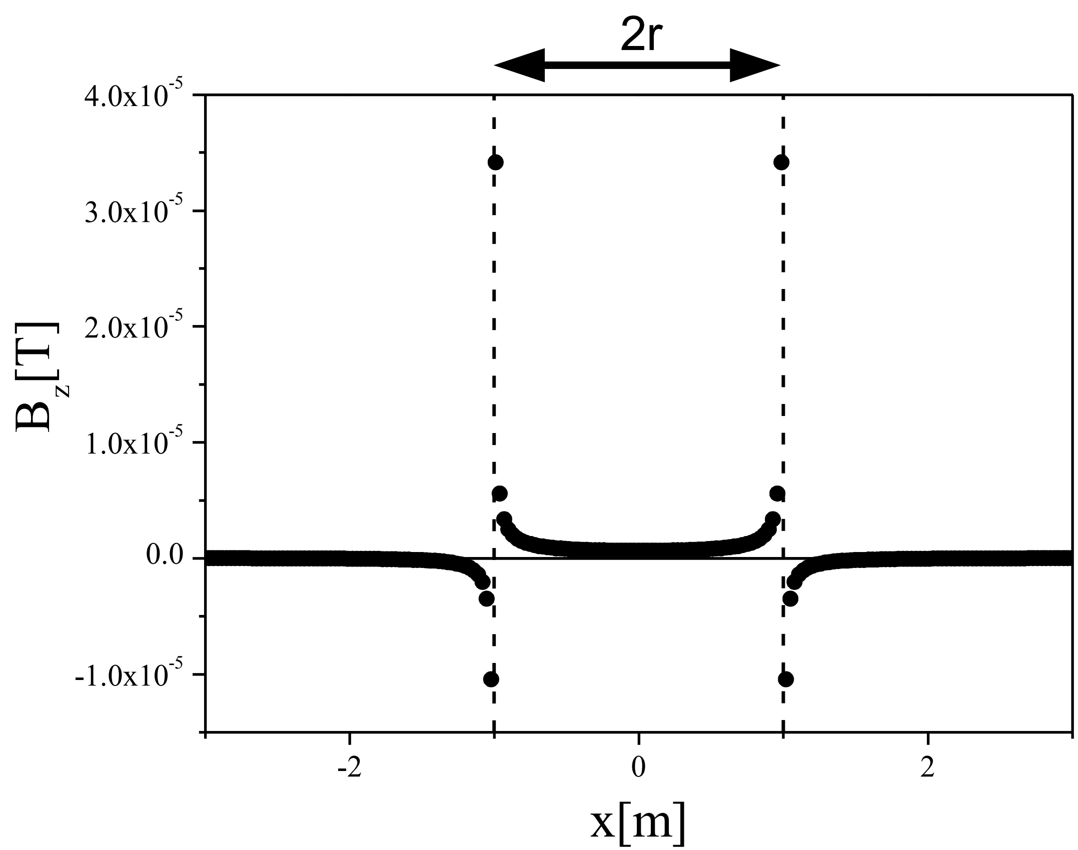
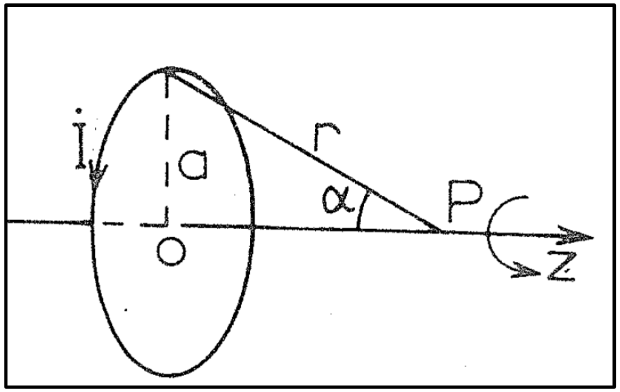
Field in plane of the loop (z=0)



Magnetic field of circular currents loops



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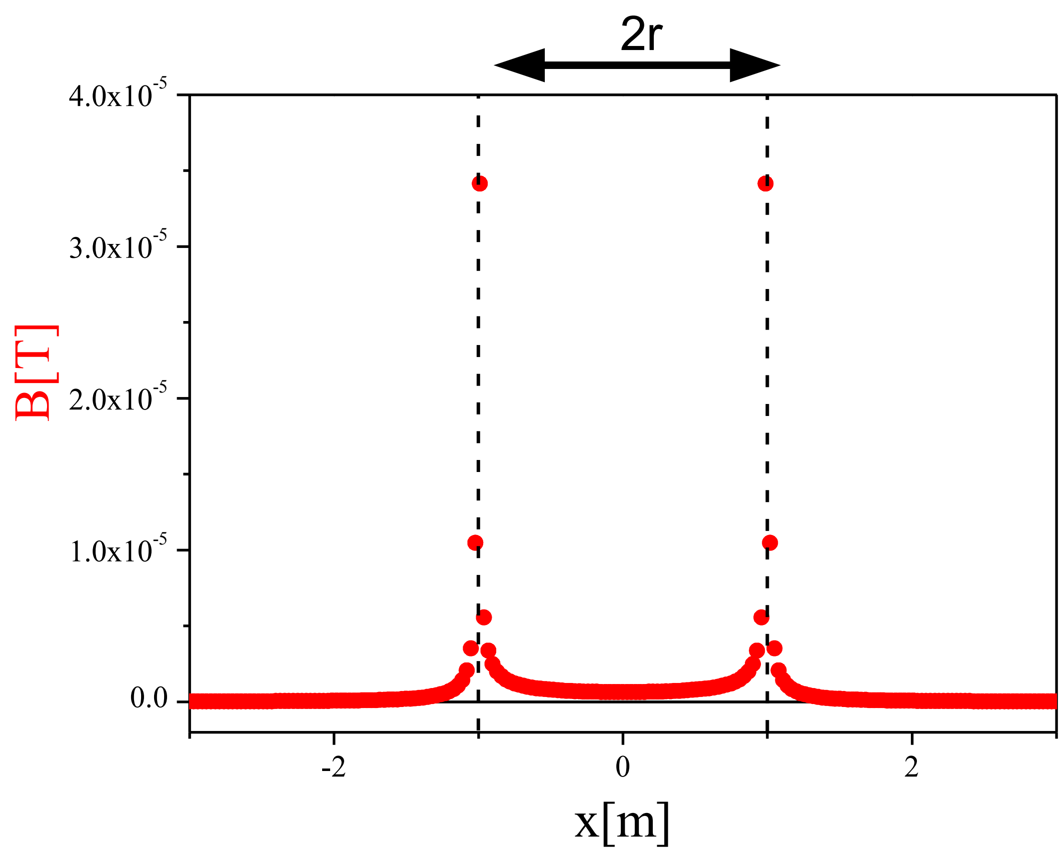
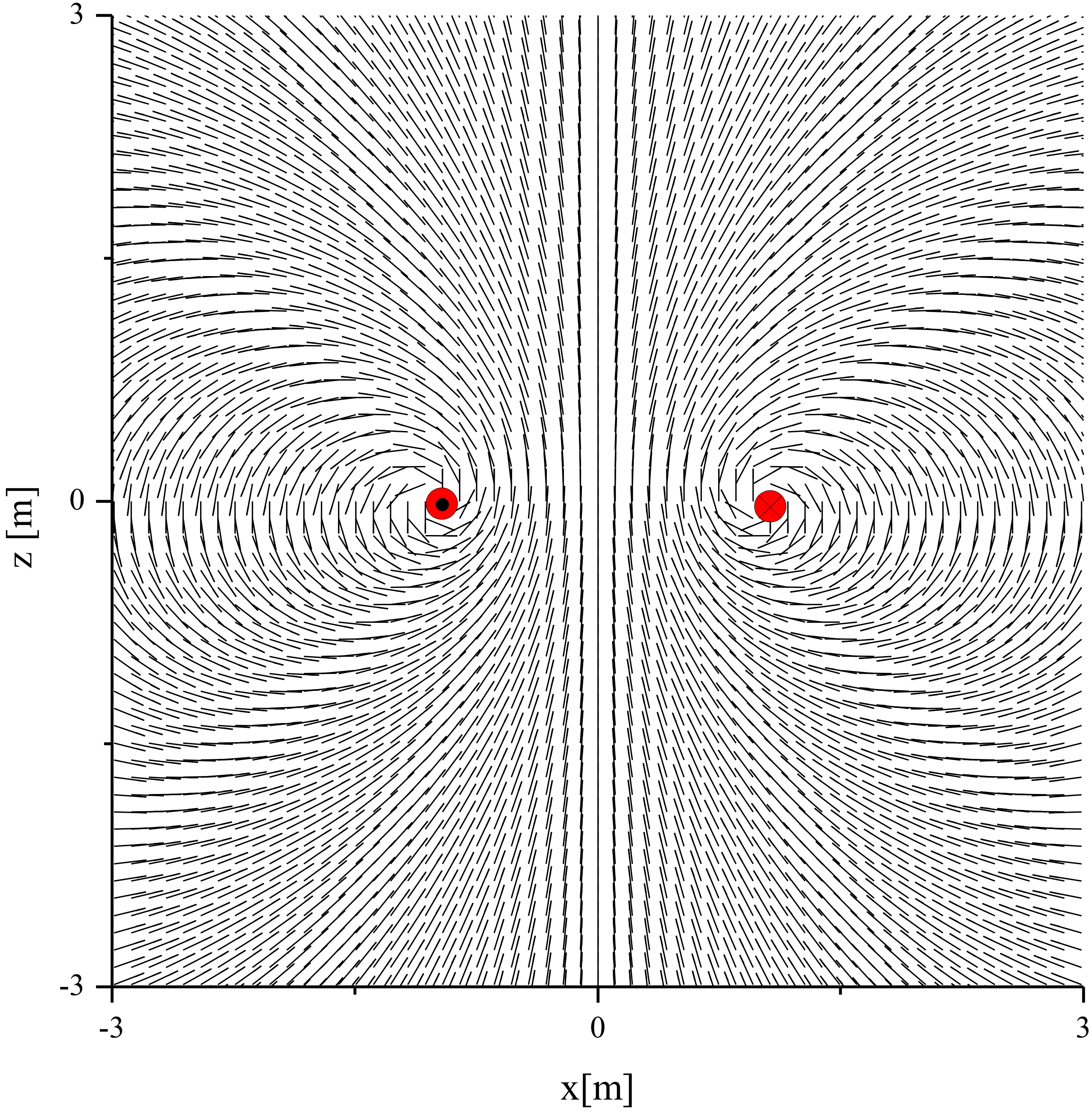
Field in plane of the loop (z=0)

As in the the case of straight wire field **B** is stronger only in the direct vicinity of the current.



Magnetic field of circular currents loops



For numerical integration loop divided into 50 parts

Magnetic field of circular currents loops

It is usual to display magnetic fields as streamlines:

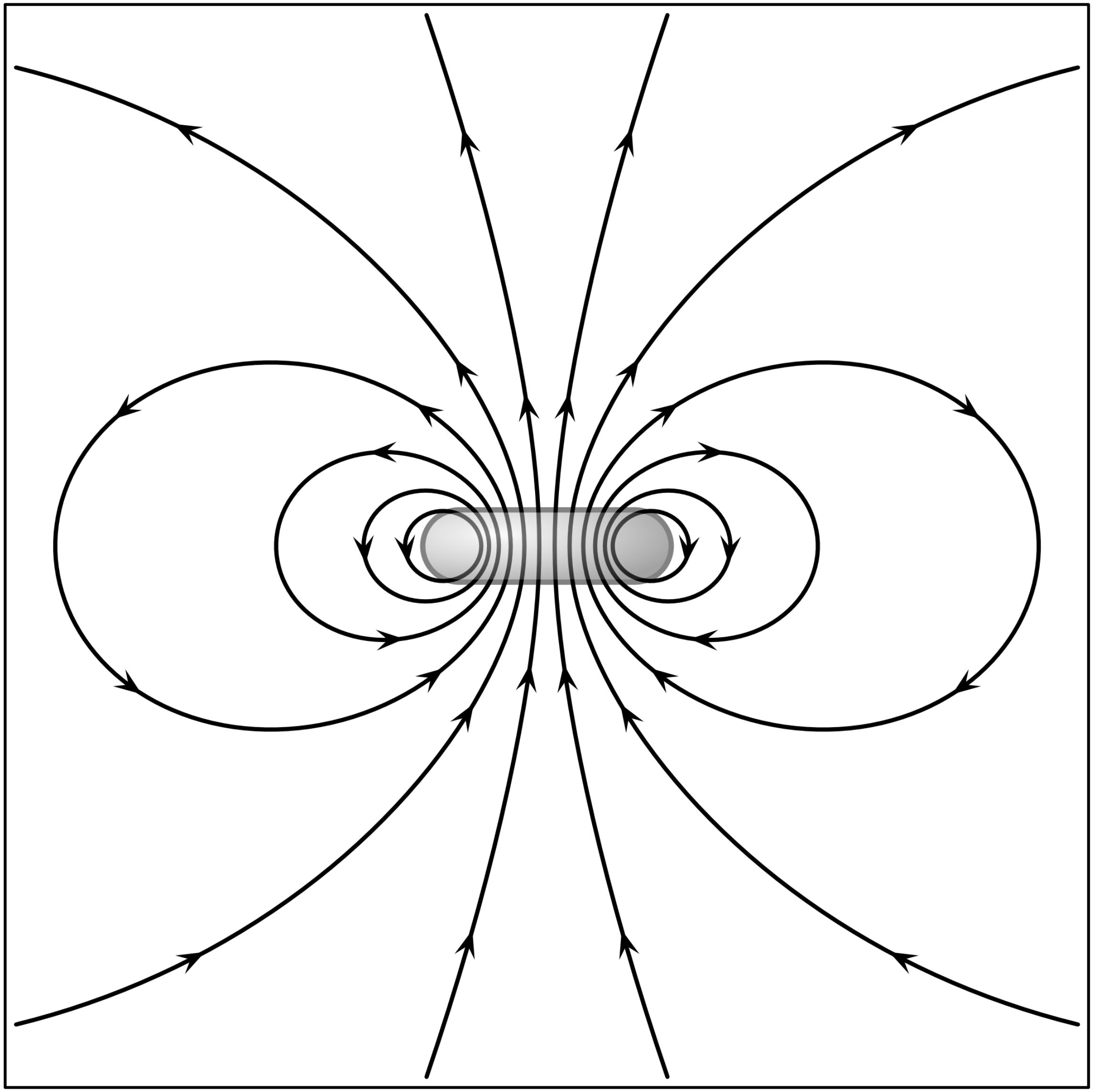


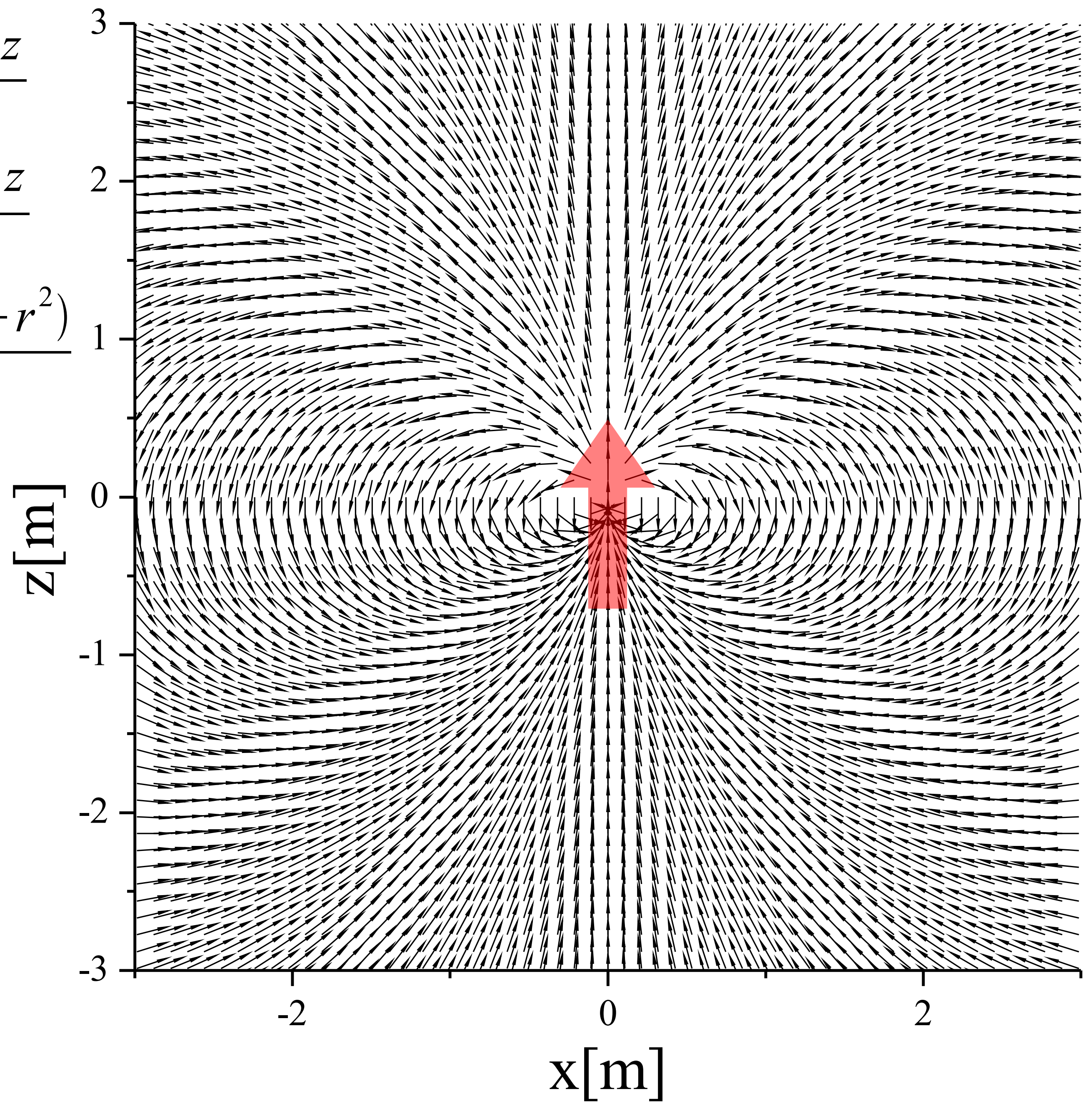
image source: Wikimedia Commons; author: Geek3 (modified by MU)

Magnetic field of a dipole

$$B_x = \frac{3m_z x z}{r^5}$$

$$B_y = \frac{3m_z y z}{r^5}$$

$$B_z = \frac{m_z(3z^2 - r^2)}{r^5}$$



y=0

E. M. Purcell, Elektryczność i Magnetyzm PWN, Warszawa 1971, str. 422

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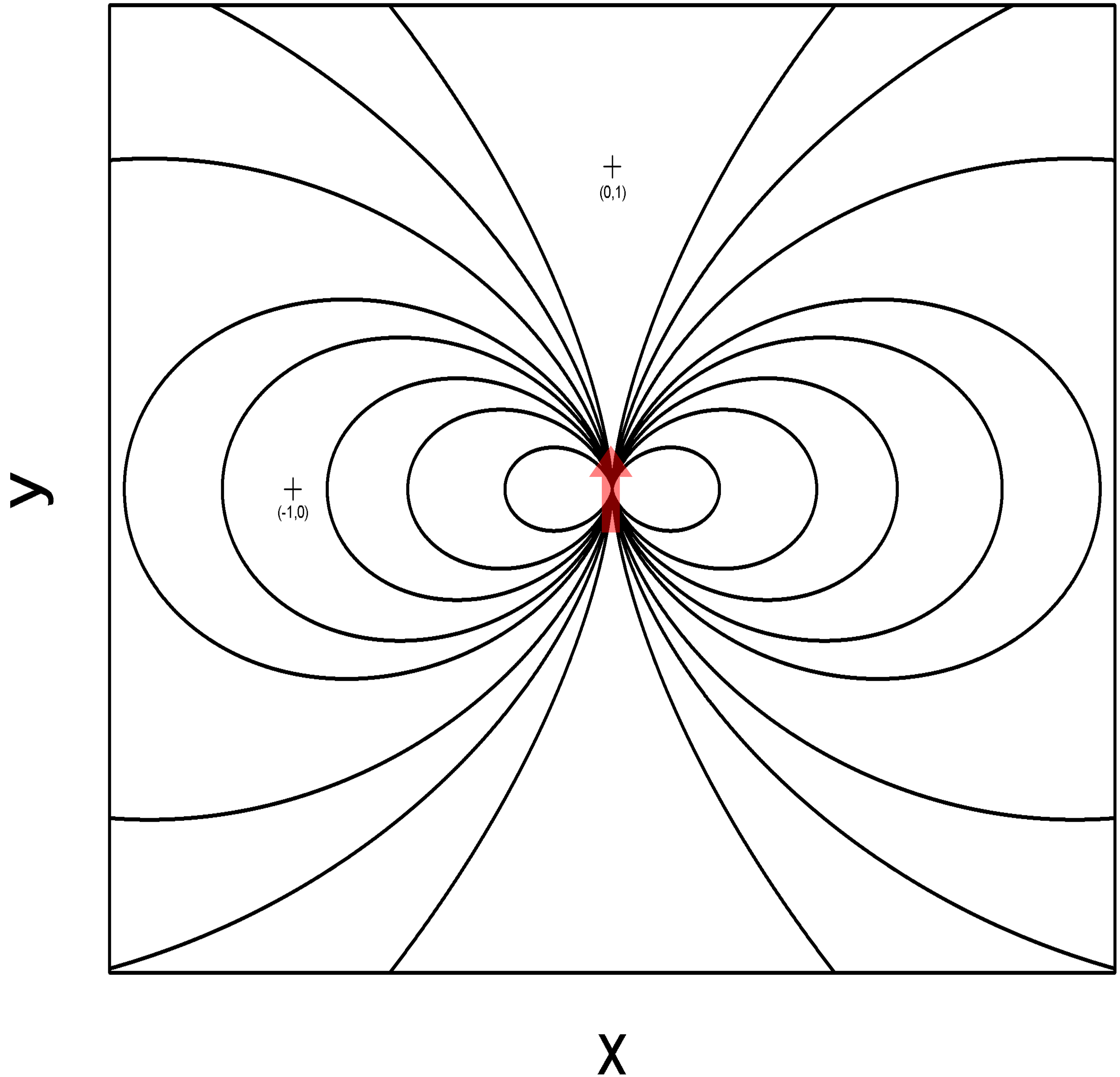
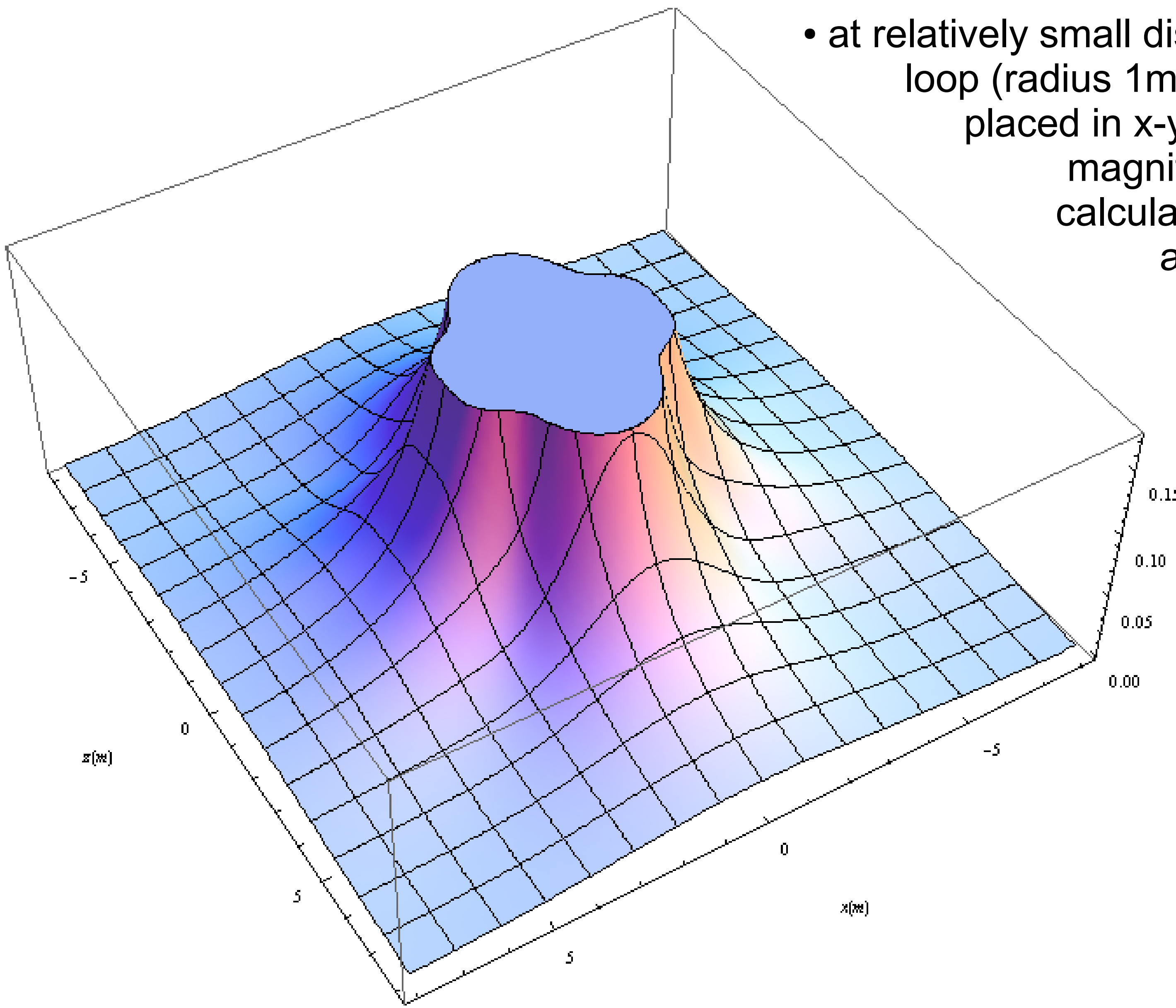


Magnetic field of a dipole – difference between dipol and current fields



- at relatively small distances from the current loop (radius 1m, centered at (0,0,0) and placed in x-y plane) the difference of magnitudes between the fields calculated from Biot-Savart law and dipole approximation are well below 5%.

At large distances from the current distribution the field can be approximated by the dipol field.

Magnetic field of circular currents loops

Far from the current loop the induction is given by the approximate expressions [7]:

$$B_r = \left(\frac{\mu_0 I r^2}{2} \right) \frac{\cos(\phi)}{r^3} \qquad B_r = \left(\frac{\mu_0 I r^2}{4} \right) \frac{\sin(\phi)}{r^3}$$

Comparing the above expressions with the dipole field:

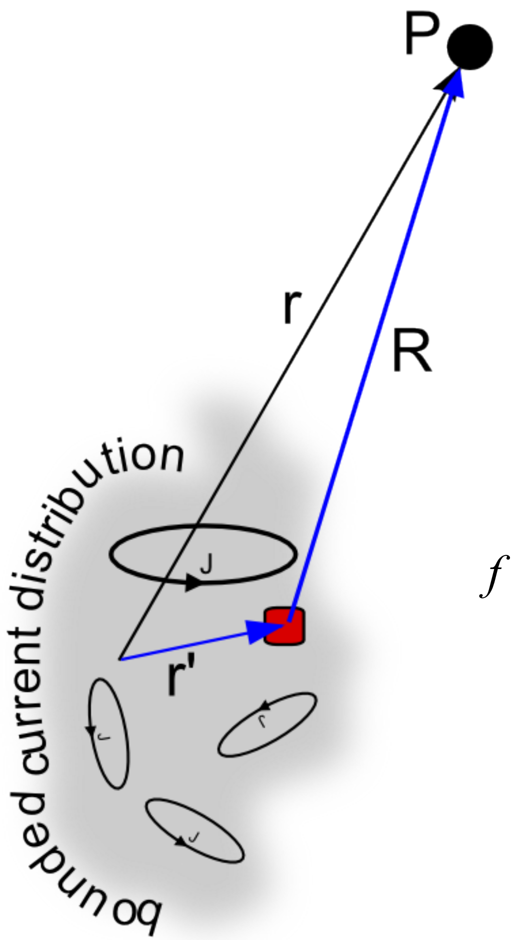
$$B_r = \frac{2 p \cos(\phi)}{r^3} \qquad B_r = \frac{p \sin(\phi)}{r^3}$$

We conclude:

Seen from distances large compared to the circular loop radius its magnetic induction ***B*** has a dipolar character.

Multipole expansion of magnetic fields

$$\vec{B}(\vec{r})=\nabla\times\frac{\mu_0}{4\pi}\int\frac{J(\vec{r}\,')}{|r-r'|}d^3r'$$



- We assume that the currents density is null outside some bounded volume
- The magnetic vector potential of the distribution is given by:

$$\vec{A}=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d^3r' \qquad (3)$$

- We express the denominator of the integrand in a Taylor series expansion* [9]:

$$f(\vec{r}-\vec{r}\,')=f(\vec{r})-\left[x'\frac{\partial}{\partial x}+y'\frac{\partial}{\partial y}+z'\frac{\partial}{\partial z}\right]f(\vec{r})+\frac{1}{2}\sum_{i,j}x'_i x'_j\frac{\partial^2 f(\vec{r})}{\partial x'_i\partial x'_j}+\ldots$$

- We have:

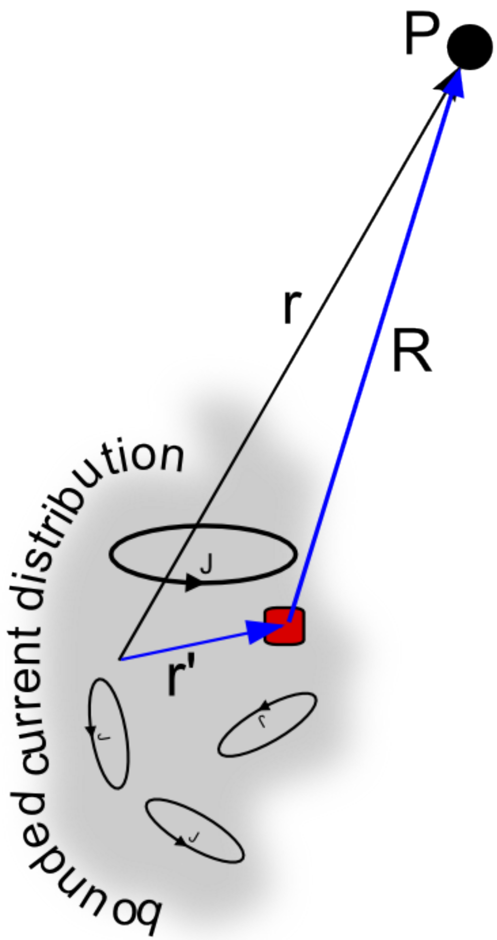
$$\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\right)=\frac{-(x-x')}{((x-x')^2+(y-y')^2+(z-z')^2)^{3/2}}$$

- It follows, taking derivatives at ***r'***=0, that:

$$\vec{r}\,'\cdot\nabla\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)=-\vec{r}\,'\cdot\frac{\vec{r}}{|\vec{r}|^3}$$

* usually one uses expansion into spherical harmonics

Multipole expansion of magnetic fields



- At the moment we are interested in the first two terms of the expansion:

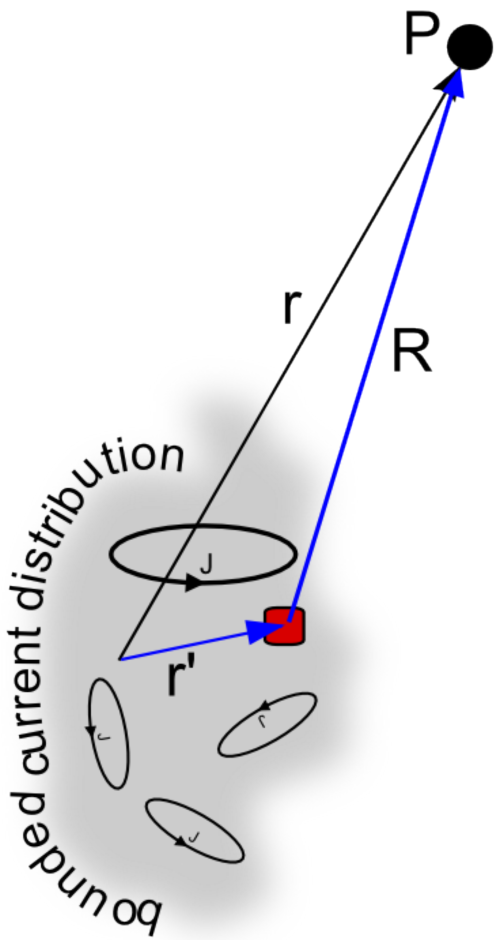
$$\frac{1}{|\vec{r}-\vec{r}\,'|}=\frac{1}{|\vec{r}|}+\frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3}+\ldots$$

- Combining this wit the expression (3) for vector potential we get:

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d^3r'=\frac{\mu_0}{4\pi}\int\vec{J}(\vec{r}\,')\left(\frac{1}{|\vec{r}|}+\frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3}+\ldots\right)d^3r'=$$
$$\frac{\mu_0}{4\pi}\int\vec{J}(\vec{r}\,')\left(\frac{1}{|\vec{r}|}\right)d^3r'+\frac{\mu_0}{4\pi}\int\vec{J}(\vec{r}\,')\left(\frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3}\right)d^3r'+\ldots$$

Multipole expansion of magnetic fields

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int\vec{J}(\vec{r}\,')\left(\frac{1}{|\vec{r}|}\right)d^3r'+\frac{\mu_0}{4\pi}\int\vec{J}(\vec{r}\,')\left(\frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3}\right)d^3r'+\ldots$$



- We take on the first integral [9]:
  - the current distribution is a divergenceless
  - we can consider any time-independent current distribution as a sum of circulating currents
  - through each current tube there passes a current $I=\vec{J}\cdot\Delta S$

$$\frac{\mu_0}{4\pi}\int\vec{J}(\vec{r}\,')\left(\frac{1}{|\vec{r}|}\right)d^3r'=\frac{\mu_0}{4\pi}\left(\frac{1}{|\vec{r}|}\right)\int\vec{J}(\vec{r}\,')d^3r'$$

- For each current circuit we have:

$$\int\vec{J}(\vec{r}\,')dV'=\int\vec{J}(\vec{r}\,')\underbrace{\Delta S'\cdot ds'}_{\text{volume element of the circuit}}=I\oint ds'$$

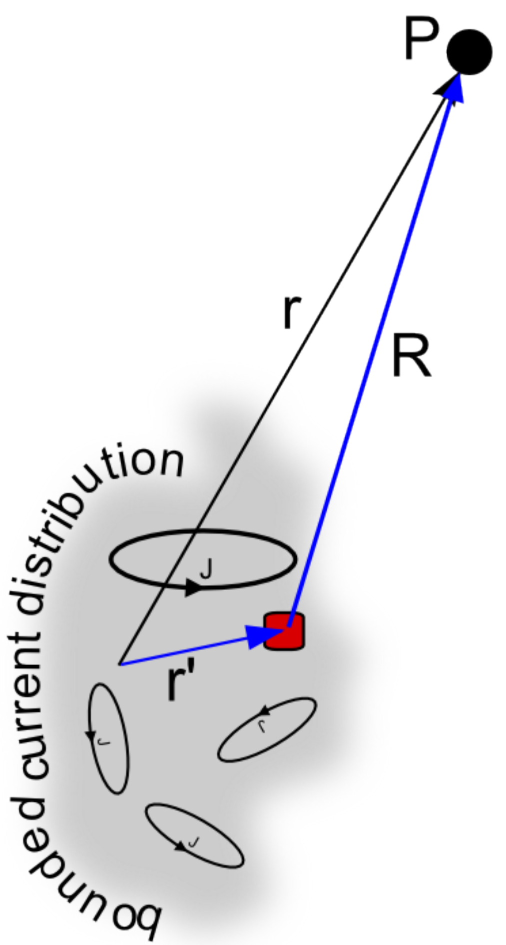
closed circuit

- Since path integral of *ds* along closed path is zero we conclude that the first term of the multipole expansion of the field of the current vanishes.

There are no magnetic monopoles *

*We have already seen that with $\nabla\cdot\vec{B}=0$.

Multipole expansion of magnetic fields



- Alternatively [14] the first integral* can be rewritten by the use of the vector identity:

$$\nabla\cdot(f\vec{A}) = f\,\nabla\cdot\vec{A} + \vec{A}\cdot\nabla f$$

- It follows (as divergence of the current vanishes):

$$\nabla\cdot(x\vec{J}) = x\underbrace{\nabla\cdot\vec{J}}_{0} + \vec{J}\cdot\nabla x = \vec{J}\cdot\nabla x = \vec{J}\,\hat{x} = J_x$$

- We have then:

$$\int J_x(\vec{r})\,d^3r = \int \nabla\cdot(x\vec{J})\,d^3r = \oint\!\!\oint x\vec{J}\,dS = 0$$

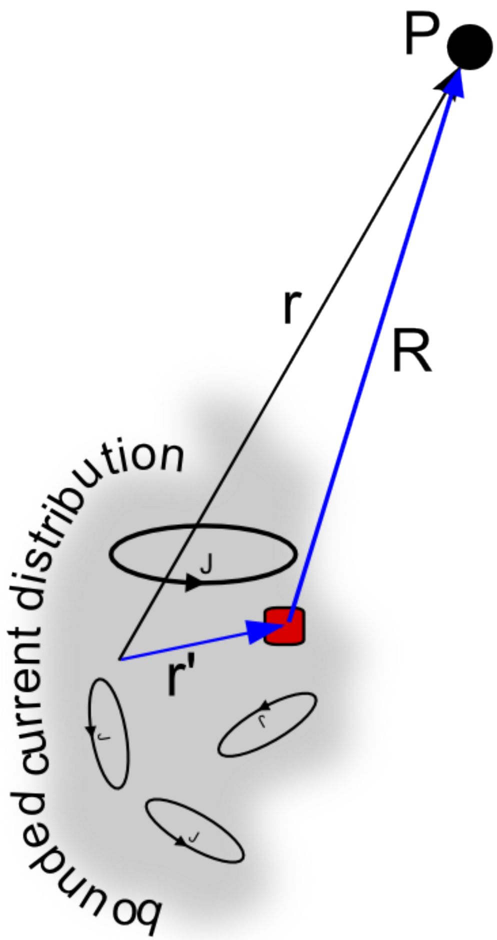
as the current density ***J*** vanishes at the outer boundary.

- Similar consideration holds for other Cartesian components of ***J***, so finally we have:

$$\frac{\mu_0}{4\pi}\left(\frac{1}{|\vec{r}|}\right)\int \vec{J}(\vec{r}\,')\,d^3r' = 0$$

* $$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\left(\frac{1}{|\vec{r}|}\right)d^3r' + \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\left(\frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3}\right)d^3r' + \ldots$$

Multipole expansion of magnetic fields



- We rewrite now the second integral of Taylor expansion:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\left(\frac{1}{|\vec{r}|}\right)d^3r' + \boxed{\frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\left(\frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3}\right)d^3r'} + \ldots$$

- We have, for arbitrary scalar functions *f* and *g*:

$$\frac{\partial}{\partial x}(g f \vec{J}) = f g \frac{\partial}{\partial x}\vec{J} + f \vec{J}\frac{\partial}{\partial x} g + g \vec{J}\frac{\partial}{\partial x} f$$

$$g \vec{J}\frac{\partial}{\partial x} f = \frac{\partial}{\partial x}(g f \vec{J}) - f g \frac{\partial}{\partial x}\vec{J} - f \vec{J}\frac{\partial}{\partial x} g$$

- Going now 3D [15]:

$$\int g \vec{J}\cdot\nabla' f\, d^3x' = \sum_i \int g J_i \partial'_i f\, d^3x' =$$

$$\sum_i \int \left[\partial'_i (J_i f g) - f J_i \partial'_i g - f g \partial'_i J_i\right] d^3x' =$$

$$\int \left[\nabla'\cdot(\vec{J} f g) - f \vec{J}\cdot\nabla' g - f g \nabla'\vec{J}\right] d^3x' =$$

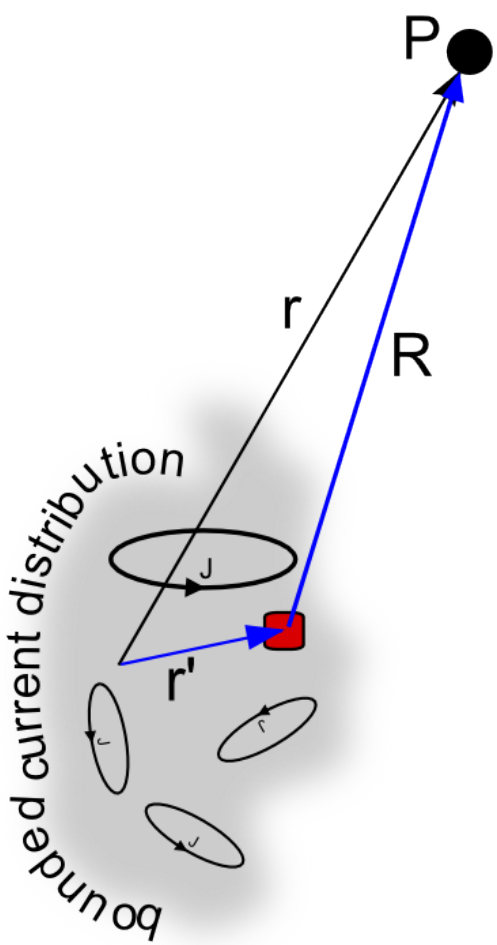
$$\oiint f g \vec{J}\cdot dS - \int \left[f \vec{J}\cdot\nabla' g + f g \nabla'\vec{J}\right] d^3x'$$

= 0 since current density vanishes on the outer boundary

- Since ***J*** is divergenceless we have:

$$\int \left(g \vec{J}\cdot\nabla' f + f \vec{J}\cdot\nabla' g + f g \nabla'\vec{J}\right) d^3x' = 0 \qquad (4)$$

Multipole expansion of magnetic fields



- We rewrite (4) using the substitutions $f=x'_i$ $\quad g=x'_j$

$$\int\left(g\vec{J}\cdot\nabla' f+f\vec{J}\cdot\nabla' g+f g\underbrace{\nabla'\vec{J}}_{0}\right)d^3x'=0 \qquad (4)$$

$$\int\left(x'_j\vec{J}\cdot\nabla' x'_i+x'_i\vec{J}\cdot\nabla' x'_j\right)d^3x'=0$$

$$\int\left(x'_j\vec{J}\cdot\hat{x}_i+x'_i\vec{J}\cdot\hat{x}_j\right)d^3x'=0$$

$$\int\left(x'_jJ_i+x'_iJ_j\right)d^3x'=0$$

- We note that:

$$\int\left(x'_jJ_i\right)d^3x'=\int\left(-x'_iJ_j\right)d^3x'$$

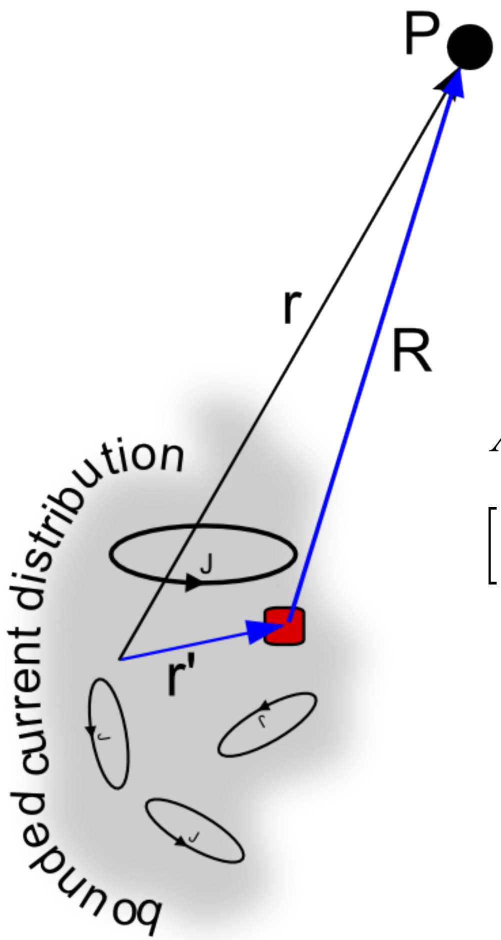
$$\int 2x'_jJ_i\,d^3x'=\int\left(x'_jJ_i+x'_jJ_i\right)d^3x'=\int\left(x'_jJ_i-x'_iJ_j\right)d^3x'$$

$$\int\left(x'_jJ_i\right)d^3x'=-\frac{1}{2}\int\left(x'_iJ_j-x'_jJ_i\right)d^3x' \qquad (5)$$

- We calculate now *i-th* **component** of the second term of the expansion of vector potential ***A***:

$$A_i(\vec{r})=\frac{\mu_0}{4\pi}\frac{\vec{r}}{|\vec{r}|^3}\cdot\underbrace{\int J_i(\vec{r}\,')\vec{r}\,'d^3r'}_{\text{scalar}}$$

Multipole expansion of magnetic fields



- We rewrite the expression for the component *i* of potential ***A***:

$$A_i(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{r}}{|\vec{r}|^3}\cdot\int J_i(\vec{r}\,')\vec{r}\,' d^3r' = \frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\sum_i r_j \int r'_j J_i(\vec{r}\,') d^3r' \overset{\text{using (5)}}{=}$$

$$-\frac{1}{2}\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\sum_j r_j \int \left(r'_i J_j - r'_j J_i\right) d^3r'$$

$r_i \equiv x_i$

- The *x* component of ***A*** is then:

$$A_x(\vec{r}) = -\frac{1}{2}\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\sum_j r_j \int \left(r'_x J_j - r'_j J_x\right) d^3r' = -\frac{1}{2}\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\times$$

$$\left[\cancelto{0}{r_x \int \left(r'_x J_x - r'_x J_x\right) d^3r'} + \boxed{r_y \int \left(r'_x J_y\right.} \left. - r'_y J_x\right) d^3r' + r_z \int \left(r'_x J_z - r'_z J_x\right) d^3r'\right]$$

- Note that: $\left[\vec{r}\times(\vec{r}\,'\times\vec{J})\right]_x = r_y r'_x J_y - r_y r'_y J_x - r_z r'_z J_x + r_z r'_x J_z$

and consequently:

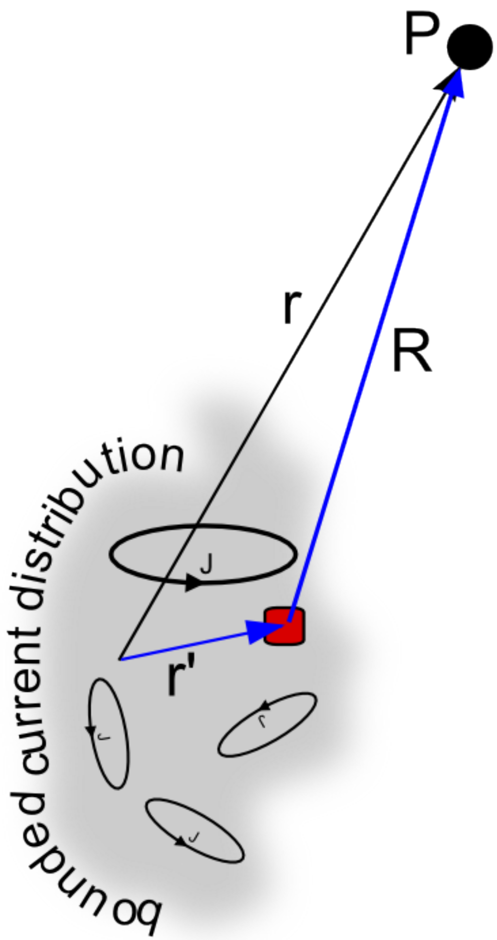
$$A_x(\vec{r}) = -\frac{1}{2}\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\left[\vec{r}\times\int \left(\vec{r}\,'\times\vec{J}(\vec{r}\,')\right) d^3r'\right]_x$$

$$\boxed{\vec{A}(\vec{r}) = -\frac{1}{2}\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\vec{r}\times\int \left(\vec{r}\,'\times\vec{J}(\vec{r}\,')\right) d^3r'}$$

Vector potential from the first two terms of the expansion:

$$\frac{1}{|\vec{r}-\vec{r}\,'|} = \frac{1}{|\vec{r}|} + \frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3} + \ldots$$

Multipole expansion of magnetic fields



$$\vec{A}(\vec{r}) = -\frac{1}{2}\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\vec{r}\times\boxed{\int(\vec{r}\,'\times\vec{J}(\vec{r}\,'))\,d^3r\,'}$$

- We define **magnetic dipole moment** of current distribution [7]:

$$\vec{m} = \frac{1}{2}\int(\vec{r}\,'\times\vec{J}(\vec{r}\,'))\,d^3r\,' \qquad \left[m\cdot\frac{A}{m^2}\cdot m^3 = A\cdot m^2\right]$$

- The integrand of the above expression is called **magnetization**

$$\vec{M}(\vec{r}) = \frac{1}{2}\vec{r}\,'\times\vec{J}(\vec{r}\,')$$

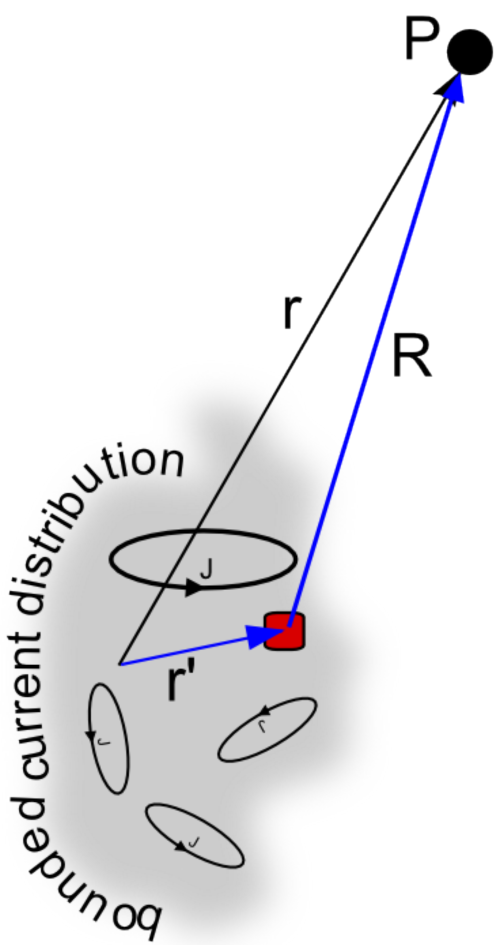
$M_{Fe} \approx 1.7\times10^6$ A/m

$M_{Co} \approx 1.4\times10^6$ A/m

$M_{Ni} \approx 0.5\times10^6$ A/m

at RT

Multipole expansion of magnetic fields



- From the expression for the potential ***A*** and the definition of ***m*** we have [14]:

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\vec{m}\times\vec{r}$$

- We have from the definition of $\vec{A}$: $\vec{B}(\vec{r})=\nabla\times\vec{A}(\vec{r})$

- Using $\nabla\times(\vec{a}\,f)=f\,\nabla\times\vec{a}-\vec{a}\times(\nabla f)$ we obtain:

$$\vec{B}(\vec{r})=\nabla\times\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\left[\frac{1}{|\vec{r}|^3}\nabla\times(\vec{m}\times\vec{r})-(\vec{m}\times\vec{r})\times\nabla\frac{1}{|\vec{r}|^3}\right]$$

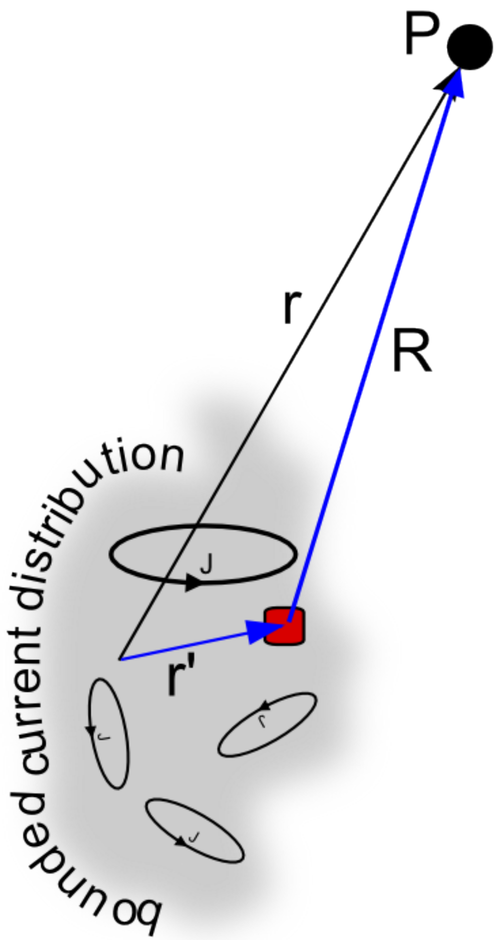
- Using $\nabla\times(\vec{a}\times\vec{b})=(\vec{b}\cdot\nabla)\vec{a}-(\vec{a}\cdot\nabla)\vec{b}+\vec{a}(\nabla\cdot\vec{b})-\vec{b}(\nabla\cdot\vec{a})$ we obtain:

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\left\{\frac{1}{|\vec{r}|^3}[(\vec{r}\cdot\nabla)\vec{m}-(\vec{m}\cdot\nabla)\vec{r}+\vec{m}(\nabla\cdot\vec{r})-\vec{r}(\nabla\cdot\vec{m})]-(\vec{m}\times\vec{r})\times\nabla\frac{1}{|\vec{r}|^3}\right\}$$

0 as ***m*** does not depend on ***r***

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\left\{\frac{1}{|\vec{r}|^3}[-(\vec{m}\cdot\nabla)\vec{r}+\vec{m}(\nabla\cdot\vec{r})]-(\vec{m}\times\vec{r})\times\nabla\frac{1}{|\vec{r}|^3}\right\}$$

Multipole expansion of magnetic fields



- We have, from the definition of nabla:

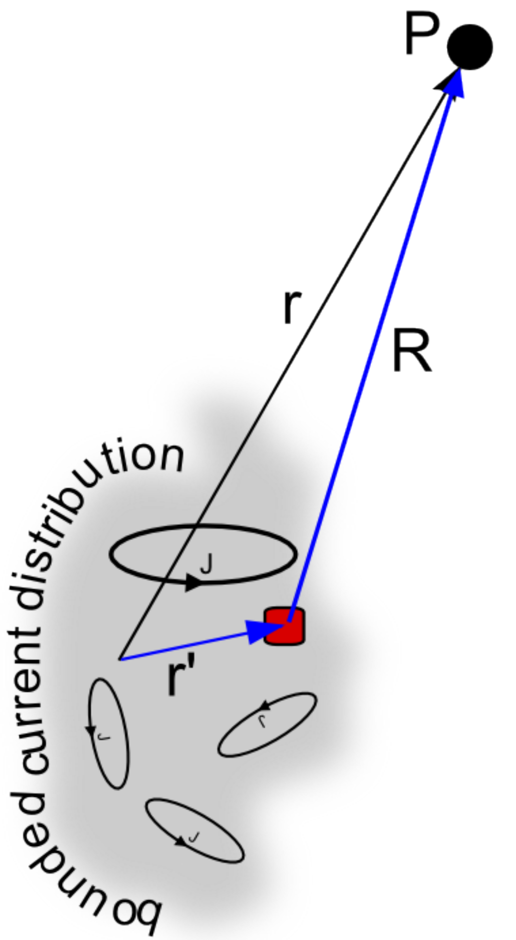
$$(\vec{m}\cdot\nabla)\vec{r}=\vec{m}$$

$$\nabla\cdot\vec{r}=3$$

$$\nabla\left(\frac{1}{|\vec{r}|^3}\right)=-\frac{3\,\vec{r}}{|\vec{r}|^5}$$

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\left\{\frac{1}{|\vec{r}|^3}\left[-(\vec{m}\cdot\nabla)\vec{r}+\vec{m}(\nabla\cdot\vec{r})\right]-(\vec{m}\times\vec{r})\times\nabla\frac{1}{|\vec{r}|^3}\right\}$$

Multipole expansion of magnetic fields



- We have, from the definition of nabla:

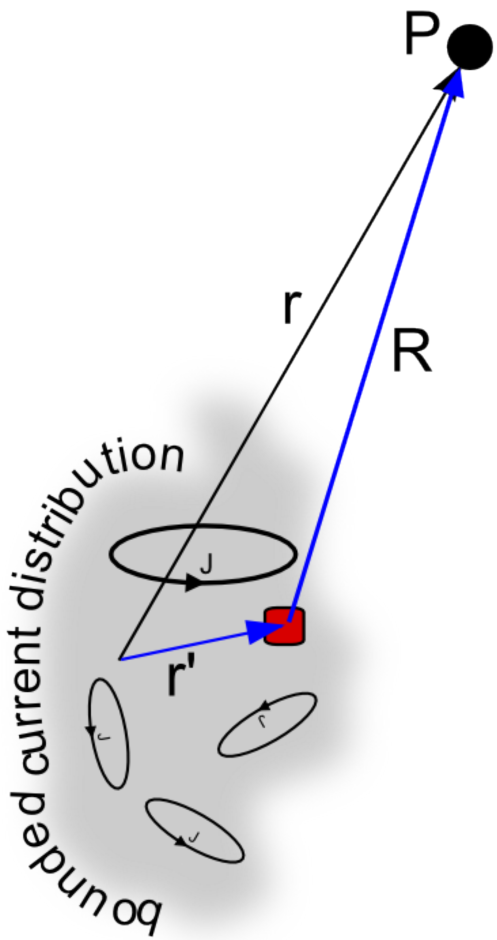
$$(\vec{m}\cdot\nabla)\vec{r}=\vec{m}$$

$$\nabla\cdot\vec{r}=3$$

$$\nabla\left(\frac{1}{|\vec{r}|^3}\right)=-\frac{3\,\vec{r}}{|\vec{r}|^5}$$

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\left\{\frac{1}{|\vec{r}|^3}\left[-\vec{m}+3\,\vec{m}\right]+\frac{3}{|\vec{r}|^5}(\vec{m}\times\vec{r})\times\vec{r}\right\}$$

Multipole expansion of magnetic fields



- To transform the third term we use the identity:

$$(\vec{b}\times\vec{c})\times\vec{a}=\vec{c}(\vec{a}\cdot\vec{b})-\vec{b}(\vec{a}\cdot\vec{c})$$

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\left\{\frac{1}{|\vec{r}|^3}[-\vec{m}+3\vec{m}]+\frac{3}{|\vec{r}|^5}(\vec{m}\times\vec{r})\times\vec{r}\right\}=$$

$$\frac{\mu_0}{4\pi}\left\{\frac{1}{|\vec{r}|^3}2\vec{m}+\frac{3}{|\vec{r}|^5}(\vec{r}(\vec{r}\cdot\vec{m})-\vec{m}(\vec{r}\cdot\vec{r}))\right\}=$$

$$\frac{\mu_0}{4\pi}\left\{\frac{1}{|\vec{r}|^3}\underline{2\vec{m}}+\frac{3}{|\vec{r}|^5}(\vec{r}(\vec{m}\cdot\vec{r})\underline{-\vec{m}}|\vec{r}|^2)\right\}=\frac{\mu_0}{4\pi}\left\{\frac{-\vec{m}}{|\vec{r}|^3}+\frac{3}{|\vec{r}|^5}(\vec{r}(\vec{m}\cdot\vec{r}))\right\}$$

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\frac{3\frac{\vec{r}}{|\vec{r}|}(\vec{m}\cdot\frac{\vec{r}}{|\vec{r}|})-\vec{m}}{|\vec{r}|^3}=\boxed{\frac{\mu_0}{4\pi}\frac{3\hat{r}(\vec{m}\cdot\hat{r})-\vec{m}}{|\vec{r}|^3}}\;*$$

- We should compare it with the expression for the field of **electric dipole** [7, 14]:

$$\vec{E}(\vec{r})=\frac{1}{4\pi\varepsilon_0}\frac{3\hat{r}(\vec{p}\cdot\hat{r})-\vec{p}}{|\vec{r}|^3}$$

*magnetic induction from first two terms of the expansion $\frac{1}{|\vec{r}-\vec{r}\,'|}=\frac{1}{|\vec{r}|}+\frac{\vec{r}\cdot\vec{r}\,'}{|\vec{r}|^3}+\dots$

Multipole expansion – dependence on origin

- The values of the components of the successive terms of the multipole expansion of the field depend in general on the *origin of the coordinate system*.
- The dipole moment of the current distribution does not depend on the origin.
- It can be shown that quadrupole moments of the current distribution do not depend on origin provided that the dipole moment is zero.

Introducing magnetic field strength H*

- We are looking for the field produced by the distribution of spatially limited closed current loops. Remembering the definition of magnetization:

$$\vec{m}=\frac{1}{2}\int(\vec{r}\,'\times\vec{J})\,d^3r'=\int\vec{M}(\vec{r}\,')\,d^3r'$$

- We obtain potential at ***r*** from magnetic moments localized at ***r'***-s:

$$\vec{A}_{magn.dipol}(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{\vec{M}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,d^3r' \quad \Longleftarrow \quad \vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|^3}\vec{m}\times\vec{r}$$

- Further, using first $\nabla'\left(\frac{1}{|r-r'|}\right)=\frac{\vec{r}-\vec{r}\,'}{|(r-r')|^3}$ and then $\nabla\times(f\vec{a})=f\nabla\times\vec{a}+\nabla f\times\vec{a}$ we rewrite the integrand:

$$\vec{M}(\vec{r}\,')\times\nabla'\left(\frac{1}{|r-r'|}\right)=\frac{1}{|r-r'|}\nabla'\times\vec{M}(\vec{r}\,')-\nabla'\times\frac{\vec{M}(\vec{r}\,')}{|r-r'|}$$

- We assume that moments associated with current loops occupy a finite volume (i.e. **magnetization vanishes at infinity**) and integrate first the second term of integrand.

*this section is taken from K.J. Ebeling and J. MJ. Mähnß [14]

Introducing magnetic field strength H*

- We assume that moments associated with current loops occupy a finite volume (i.e. **magnetization vanishes at infinity**) and integrate first the second term of the integrand.

$$\int \nabla' \times \frac{\vec{M}(\vec{r}\,')}{|r-r'|} d^3 r' \Rightarrow \int_{-\infty}^{+\infty} \left[\frac{\partial}{\partial y'} \frac{\vec{M}_z(\vec{r}\,')}{|r-r'|} - \frac{\partial}{\partial z'} \frac{\vec{M}_y(\vec{r}\,')}{|r-r'|} \right] dx'\, dy'\, dz' =$$

x-component of the curl

$$\iint_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \frac{\partial}{\partial y'} \frac{\vec{M}_z(\vec{r}\,')}{|r-r'|} dy' \right] dx'\, dz' - \iint_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \frac{\partial}{\partial z'} \frac{\vec{M}_y(\vec{r}\,')}{|r-r'|} dz' \right] dx'\, dy' =$$

$$\iint_{-\infty}^{+\infty} \underbrace{\left[\frac{\vec{M}_z(\vec{r}\,')}{|r-r'|} \right]_{-\infty}^{+\infty}}_{0} dx'\, dz' - \iint_{-\infty}^{+\infty} \underbrace{\left[\frac{\vec{M}_y(\vec{r}\,')}{|r-r'|} \right]_{-\infty}^{+\infty}}_{0} dx'\, dy' = 0 \Rightarrow \int \nabla' \times \frac{\vec{M}(\vec{r}\,')}{|r-r'|} d^3 r' = 0$$

- Finally for a contribution of the magnetization to the magnetic potential ***A*** we have:

$$\boxed{\vec{A}_{magn.dipol}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\nabla' \times \vec{M}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} d^3 r'}$$

*this section is taken from K.J. Ebeling and J. Mähnß [14]

Introducing magnetic field strength H*

- The overall potential ***A*** can be written as**:

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{\vec{j}_{free}(\vec{r}\,')+\nabla'\times\vec{M}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d^3r'$$

The effect of magnetic moment distribution on magnetic field is the same as that of current distribution given by:

$$\vec{j}_{bound}(\vec{r})=\nabla\times\vec{M}(\vec{r})$$

We distinguish two types of currents contributing to magnetic field:

- the free currents – flowing in lossy circuits (coils, electromagnets) or superconducting coils; in general one can influence (switch on/off) and measure free currents
- the bound currents – due to intratomic or intramolecular currents and to magnetic moments of elementary particles with spin [13]

**A was defined previously (p. 16) as:

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{j(\vec{r}\,')}{|r-r'|}d^3r'$$

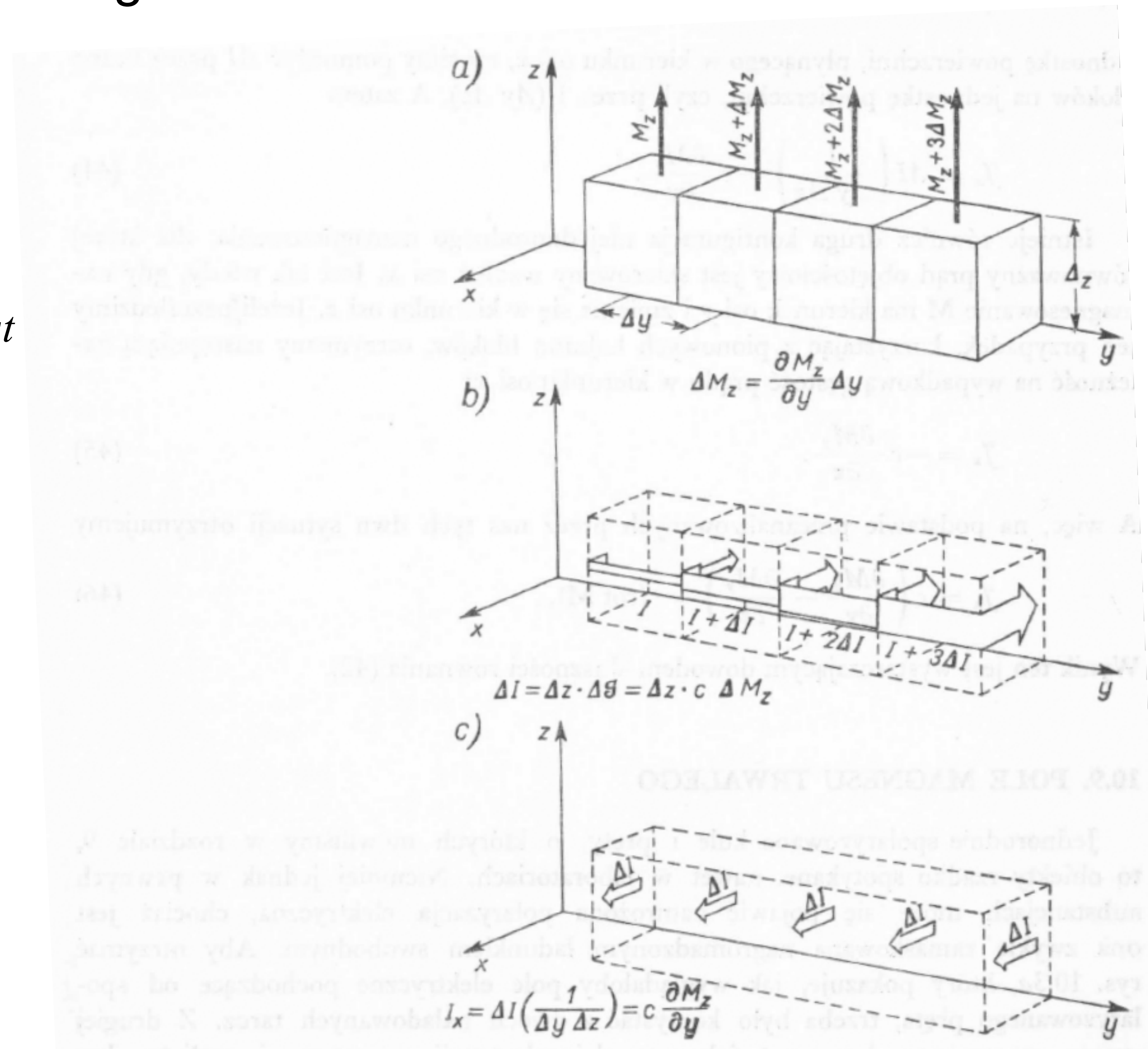
*this section is taken from K.J. Ebeling and J. Mähnß [14]

Introducing magnetic field strength H

- An illustration of the fact that:

$$\vec{j}_{bound}(\vec{r}) = \nabla \times \vec{M}(\vec{r})$$

$$\nabla \times \vec{M}(\vec{r}) \neq 0 \rightarrow \vec{M}(\vec{r}) \neq const$$



Rys. 10.20. Namagnesowanie niejednorodne jest równoważne gęstości prądu objętościowego.

image source E. M. Purcell, Elektryczność i Magnetyzm, PWN Warszawa 1971

Introducing magnetic field strength H*

- From Biot-Savart law we have:

$$\nabla\times\vec{B}=\mu_0\vec{j}(\vec{r})=\mu_0\vec{j}_{free}+\mu_0\vec{j}_{bound}=\mu_0\vec{j}_{free}+\mu_0\nabla\times\vec{M} \quad (6)$$

- We introduce a vector:

$$\boxed{\vec{H}=\frac{1}{\mu_0}\vec{B}-\vec{M}}$$

In cgs system:

$$\boxed{\vec{H}=\vec{B}-4\pi\vec{M}}$$

- From (6) we have:

$$\nabla\times\vec{B}-\mu_0\nabla\times\vec{M}=\mu_0\nabla\times\left(\frac{1}{\mu_0}\vec{B}-\vec{M}\right)=\mu_0\vec{j}_{free}$$

- It follows that the rotation of field strength **H is determined solely by the free currents**:

$$\boxed{\nabla\times\vec{H}=\vec{j}_{free}}$$

- In general $\boxed{\nabla\cdot\vec{H}\neq 0}$ i.e. magnetic field strength is not source-free.

*this section is taken from K.J. Ebeling and J. Mähnß [14]

Magnetic moment of an electron

- Spin magnetic moment (Bohr magneton): $\mu_B = \frac{e h}{4 \pi m_e} = 9.27400968(20) \times 10^{-24} A m^2$ *

- Magnetic moment of electron originates from spin i.e. angular momentum of electron which is equal to $\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\frac{h}{2\pi}$ and its component along arbitrary direction can take on values $\pm\frac{1}{2}\frac{h}{2\pi}$.
- the magnitude of magnetic moment of electron is **constant**; only its orientation can be changed.
- **Giromagnetic ratio**: $\gamma = \frac{\vec{m}}{\vec{L}}$, $\vec{L}$ - angular momentum
- Giromagnetic ratio for a classical rigid body (with mass density proportional to charge density) equals $\gamma = \frac{q}{2m}$
- Giromagnetic ratio for spin magnetic moment is **twice** that of classic circular movement of a charge (like for examlpe electron circulating nucleus).

$$\gamma_e = 1.760859708(39) \times 10^{11} s^{-1} T^{-1}$$

* it is customary to express magnetic moment in J T^{-1} which is equivalent [1T=1 kg $s^{-2}A^{-1}$].

Magnetic moment of an electron

- Spin g_e factor:

$$\vec{m}_e = -g_e \frac{e}{2m} \vec{S} \qquad \vec{m} = \gamma \vec{L}$$

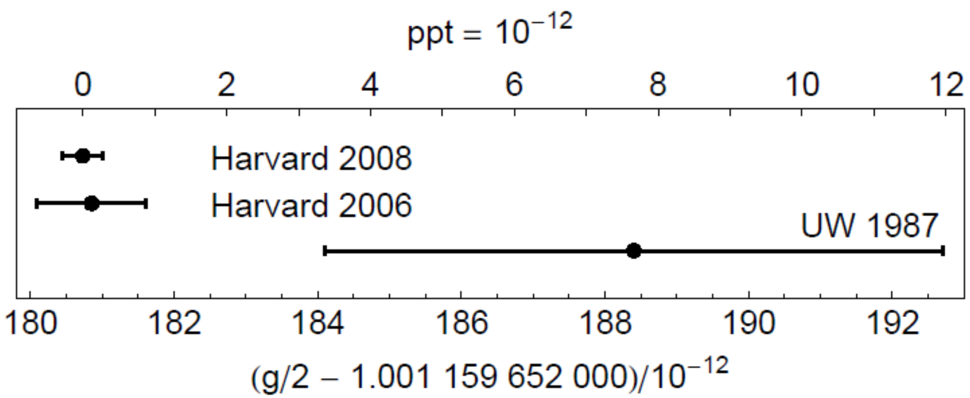


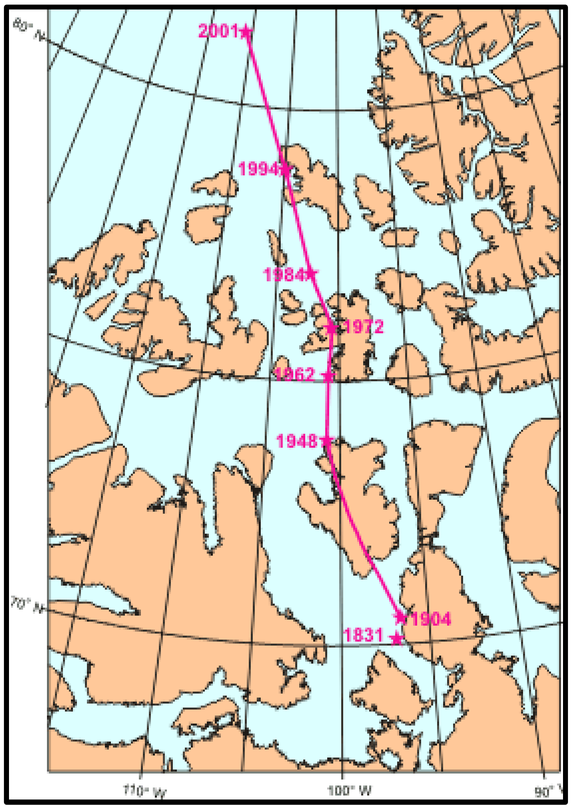
Fig. 6.1. Most accurate measurements of the electron $g/2$.

G. Gabrielse, Measurements of the Electron Magnetic Moment

- At large distances electron magnetic field has a dipolar character
- The external field exerts on electron the torque which is equal to the one exerted on the current loop with equal magnetic moment
- Within the electron $\nabla \vec{B} = 0$ as in classical sources of magnetic field [13].

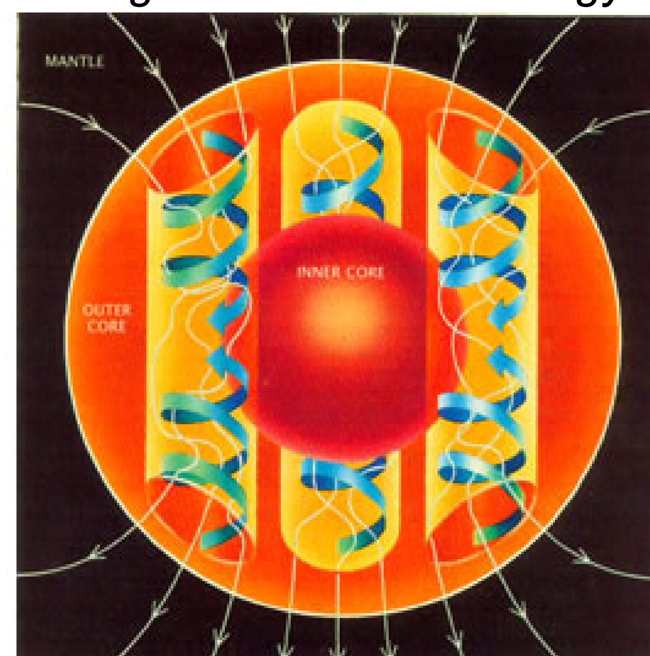
Magnetic fields in the universe - Earth

- Earth can be approximated by the magnetic dipole moment of 1.5×10^{16} Am^2 [15] (it corresponds to the iron cube of 4.5 km sides)
- On its surface the earth's magnetic field is of the order of 0.025-0.065 mT



from Wikimedia Commons, author NASA, USA

- Position of earth magnetic pole may change by several kilometers a year
- USGS: “*Basically, the motion of the electrically conducting iron in the presence of the Earth's magnetic field induces electric currents. Those electric currents generate their own magnetic field, and, as the result of this internal feedback, the process is self-sustaining, so long as there is an energy source sufficient to maintain convection.*” (near the surface the magnetic ores can contribute to local magnetic field)



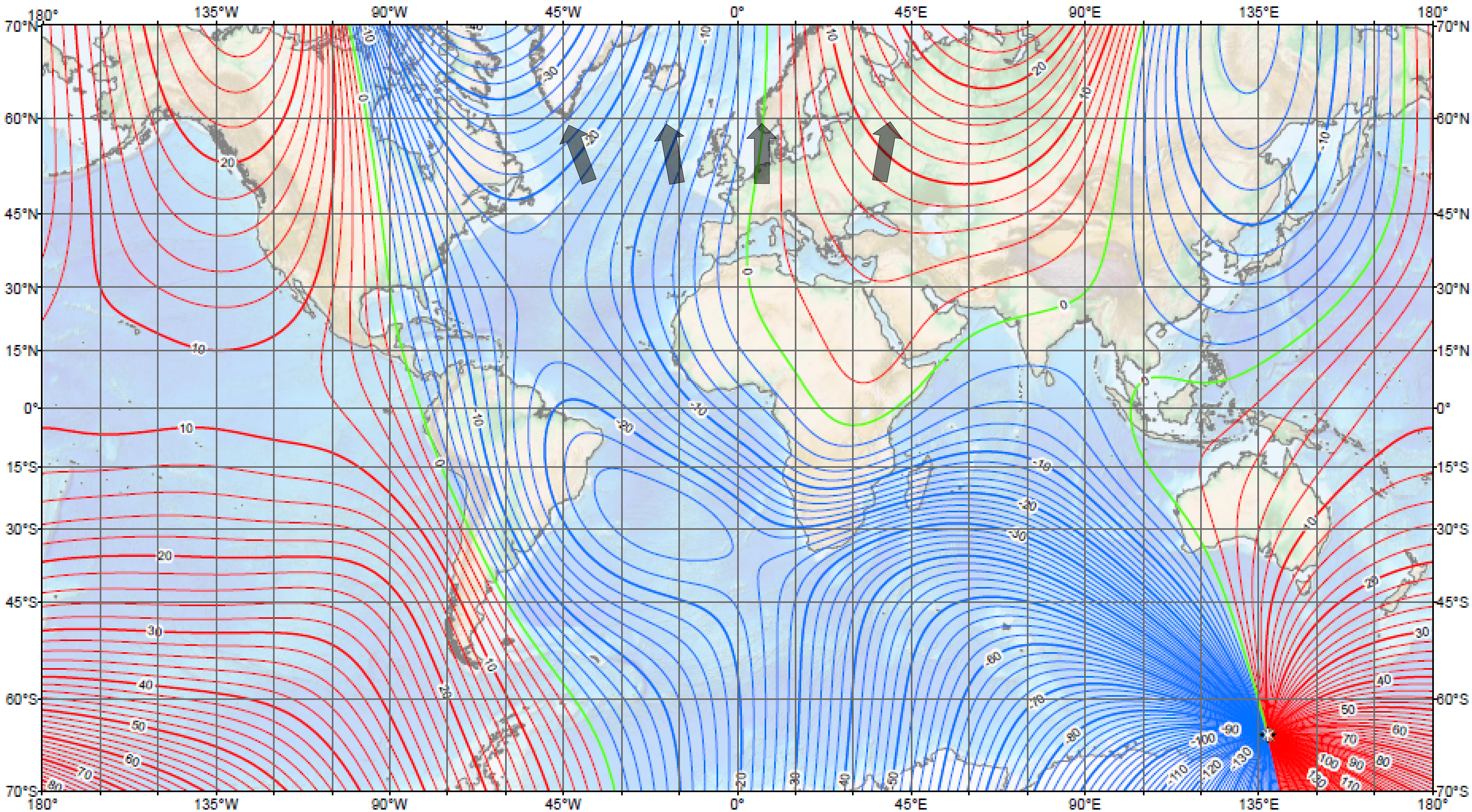
from Wikimedia Commons, author USGS, USA (geomag.usgs.gov)

Magnetic fields in the universe - Earth

Contour interval: 2 degrees,
red contours positive (**east**)
blue negative (**west**)
green (agonic) zero line.

- Magnetic field model used in navigation:

US/UK World Magnetic Model -- Epoch 2010.0
Main Field Declination (D)



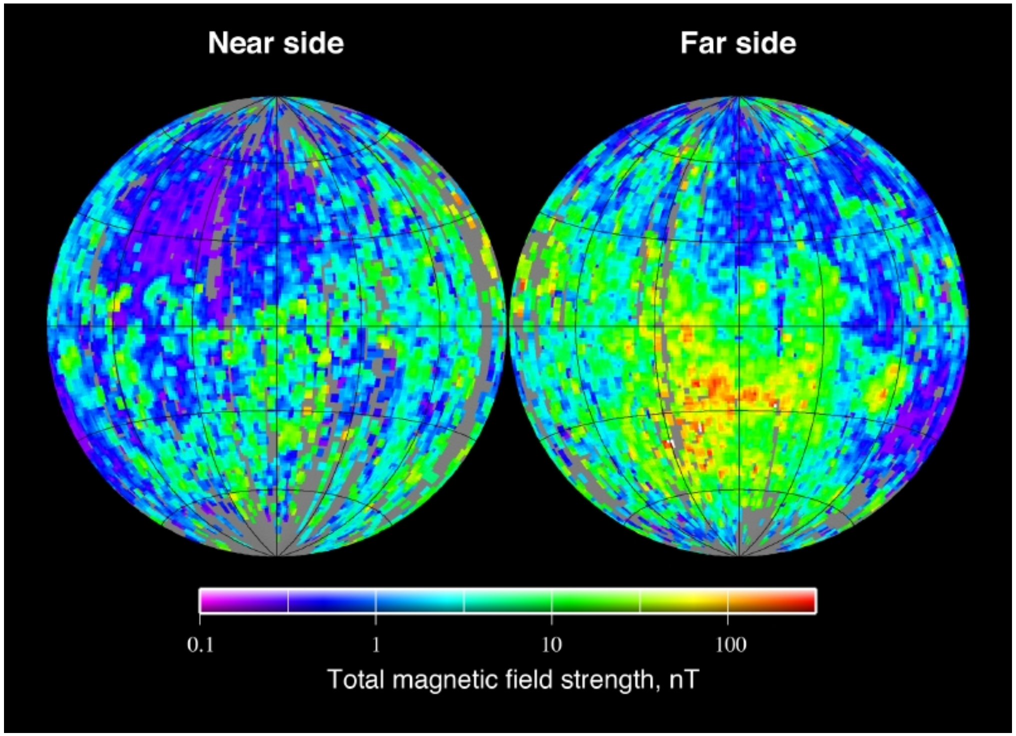
Main field declination (D)
Contour interval: 2 degrees, red contours positive (east); blue negative (west); green (agonic) zero line.
Mercator Projection.
: Position of dip poles

Map developed by NOAA/NGDC & CIRES
http://ngdc.noaa.gov/geomag/WMM/
Map reviewed by NGA/BGS
Published January 2010

image source: http://ngdc.noaa.gov/geomag/WMM/

Magnetic fields in the universe – the solar system and outer space

• Lunar surface magnetic field is of the order of 10^{-9} T [15] – the moon has no (dipolar) magnetosphere.



from Wikimedia Commons, author Mark A. Wieczorek

Total magnetic field strength at the surface of the Moon as derived from the Lunar Prospector electron reflectometer experiment.

• Magnetic fields of planets may be very weak (Venus) or much higher than earth's field (Jupiter – on equator 0.42 mT [about 10× earth's field], magnetic dipole moment about 10^5 times higher than that of earth) [16].

• Stellar magnetic fields may reach 0.4 T [16] on our Sun (although field not simply dipolar) and 100-**1000 MT** in neutron stars (the strongest fields in the universe) [16].

Magnetic fields in the universe – large scale Galactic structure

• Magnetic field in the disc of our galactic possesses large scale structure:
-magnetic fields in all inner spiral arms are going counterclockwise when viewed from the North Galactic pole
-fields in all interarm regions are oriented clockwise

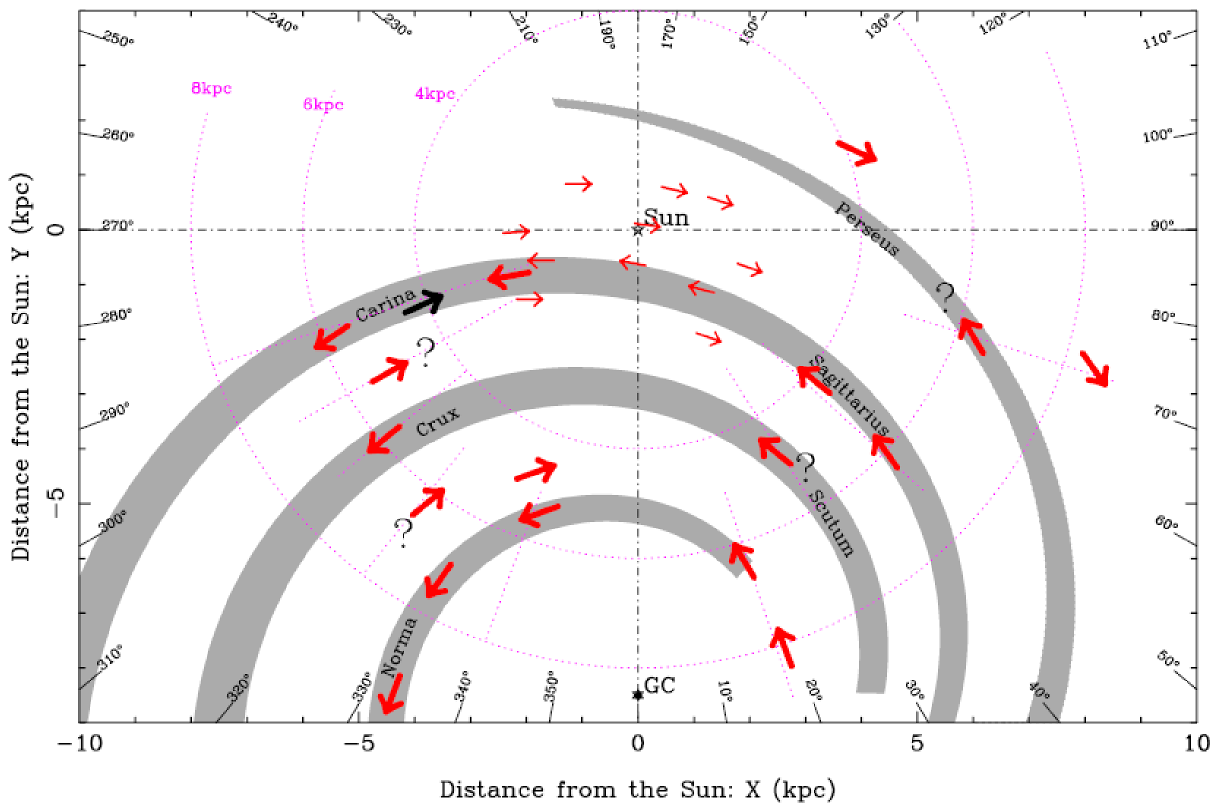


Figure 1. The observed large-scale structure of magnetic fields in the Galactic disk. See Han et al. (2005) for details.

JinLin Han, Journal of Physics: Conference Series **47**, 120 (2006)

Magnetoreception

- "The magnetic field of the Earth provides a pervasive and reliable source of directional information that certain animals can use as an orientation cue while migrating, homing, or moving around their habitat."- Sönke Johnsen and Kenneth J. Lohmann
- Diverse animal species (bees, salamandres, turtles, birds etc.) possess magnetoreceptory senses
- Humans do not seem to have the ability to sense either direction or the intensity of magnetic field.

Figure 2. Bacterial magnetoreceptors. Transmission electron micrograph of the bacterium *Magnetospirillum magnetotacticum* showing the chain of magnetosomes inside the cell. The magnetite crystal incorporated in each magnetosome is about 42 nm long. (Image courtesy of Dennis Bazylinski.)

- Magnetobacterias possess magnetosomes containing magnetic materials (single domain size range; for example magnetite Fe_3O_4)
- The torque on chain of magnetosomes is strong enough to turn the entire bacteria along magnetic field direction (field inclination). They use this to sense what direction is "down" - they prefere deeper, less oxygenated mud.
- In higher animals the mechanical sensors in cells are supposed to detect the rotation of magnetosomes.

S. Johnsen, K.J. Lohmann, Physics Today March 2008

Magnetic field measurement

- Magnetic sensor can be divided according to different criteria:

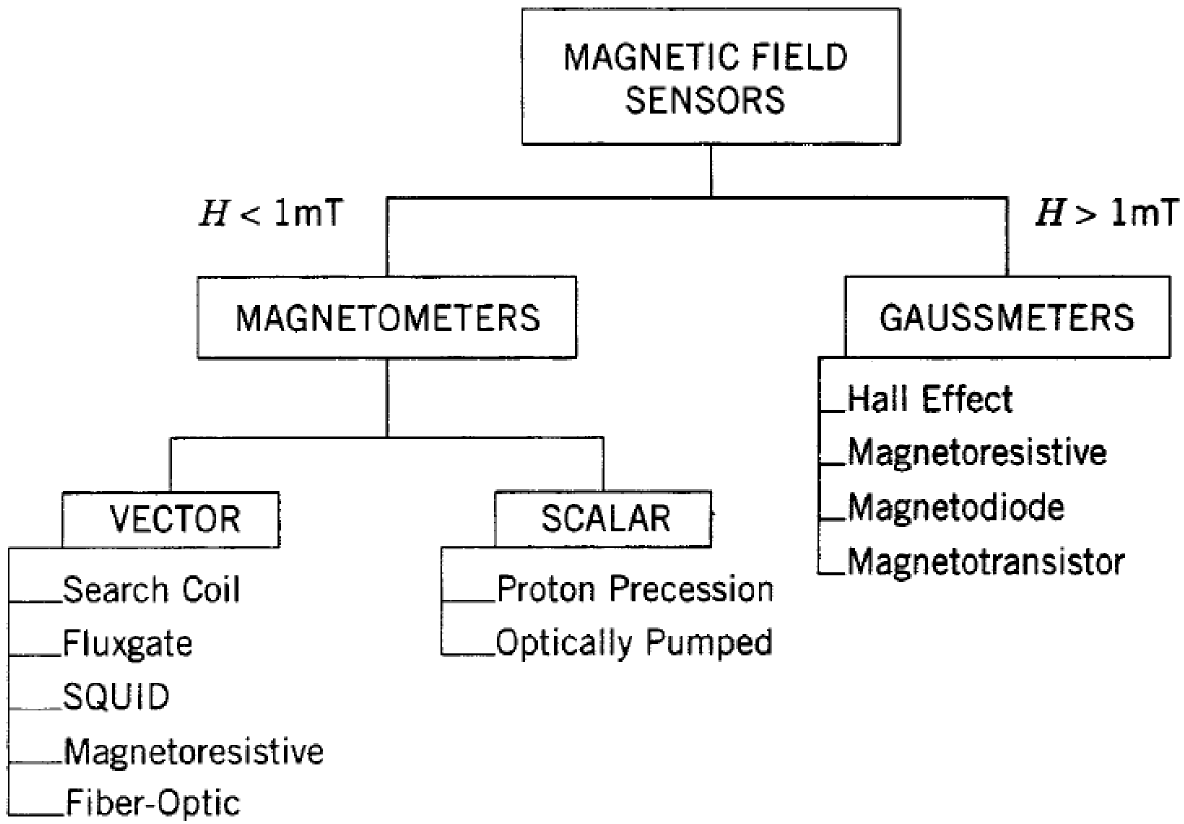


FIGURE 48.1 Magnetic field sensors are divided into two categories based on their field strengths and measurement range: magnetometers measure low fields and gaussmeters measure high fields.

- Distinction magnetometer-gaussmeter is rather arbitrary and not commonly used.

graphics from [18]: S.A. Macintyre, Magnetic Field Measurement

Magnetic field measurement

- Magnetic sensor can be divided according to different criteria:

TABLE 48.1 Field Strength Instrument Characteristics *

Instrument	Range (mT)	Resolution (nT)	Bandwidth (Hz)	Comment
Induction coil	10^{-10} to 10^{6}	Variable	10^{-1} to 10^{6}	Cannot measure static fields
Fluxgate	10^{-4} to 0.5	0.1	dc to 2×10^{3}	General-purpose vector magnetometer
SQUID	10^{-9} to 0.1	10^{-4}	dc to 5	Highest sensitivity magnetometer
Hall effect	0.1 to 3×10^{4}	100	dc to 10^{8}	Best for fields above 1T
Magnetoresistance	10^{-3} to 5	10	dc to 10^{7}	Good for mid-range applications
Proton precession	0.02 to 0.1	0.05	dc to 2	General-purpose scalar magnetometer
Optically pumped	0.01 to 0.1	0.005	dc to 5	Highest resolution scalar magnetometer

*as of year 1999 (M.U)

table from [18]: S.A. Macintyre, Magnetic Field Measurement

Vibrating coils magnetometer

- Coil magnetometers are usually used to measure varying field.
- The situation can be reversed: direct use of Faraday law.
- Piezoelectric sheets are used to excite the cantilever bending
- Two individual sensing coils are orthogonally fastened at the tip of cantilever
- Rotation of the coils allows the measurements of field components perpendicular to rotation axis
- Very high spatial resolution of measurements - coils virtually at the same position (**3-axis measurement**)

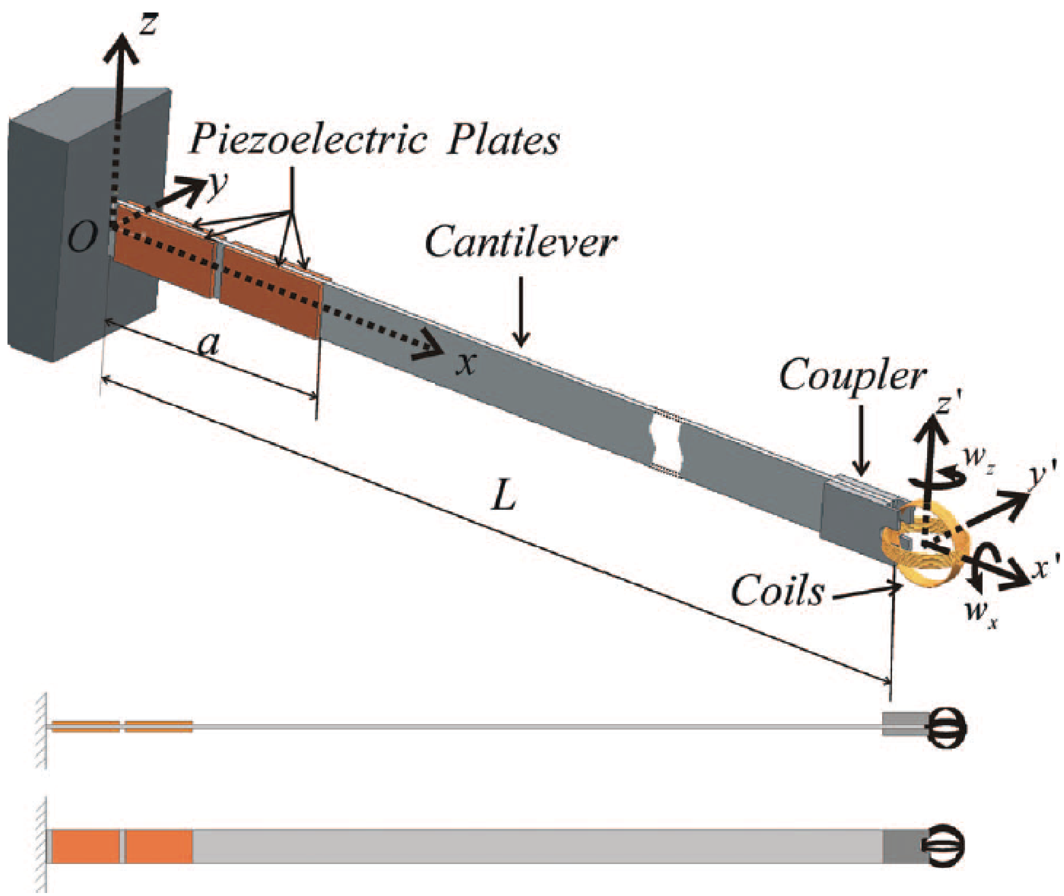


FIG. 1. (Color online) Schematic diagram of device configuration.

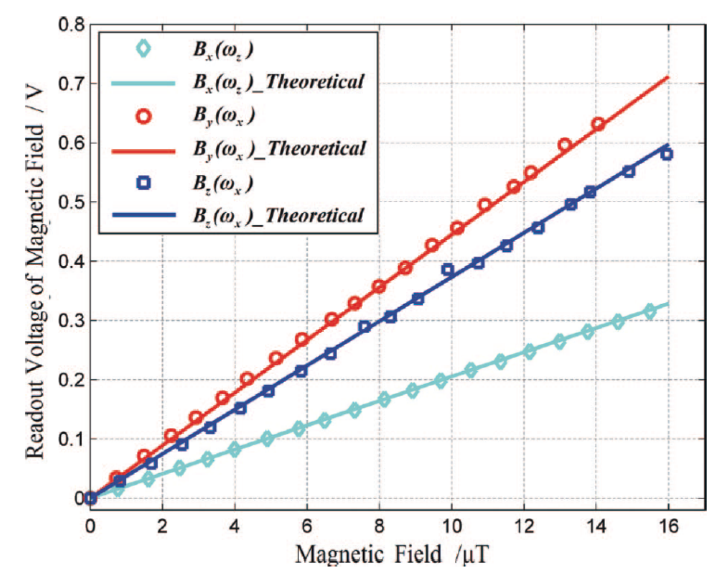
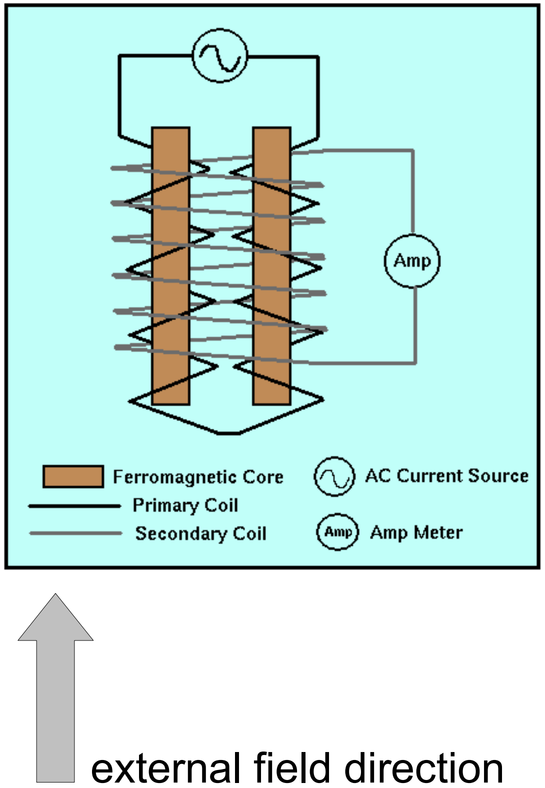


FIG. 6. (Color online) Relationships of measured readout voltages and magnetic fields in the single-mode.

Jing Yin, Cheng Liang Pan, Hong Bo Wang, and Zhi Hua Feng, Rev. Sci. Instrum. 82, 124702 (2011)

Fluxgate magnetometer

- Fluxgate magnetometers use the Faraday's law of induction to measure static magnetic field; used in II World War for submarine hunt.
- Induction coil magnetometers utilize the principle of induction to measure varying fields.

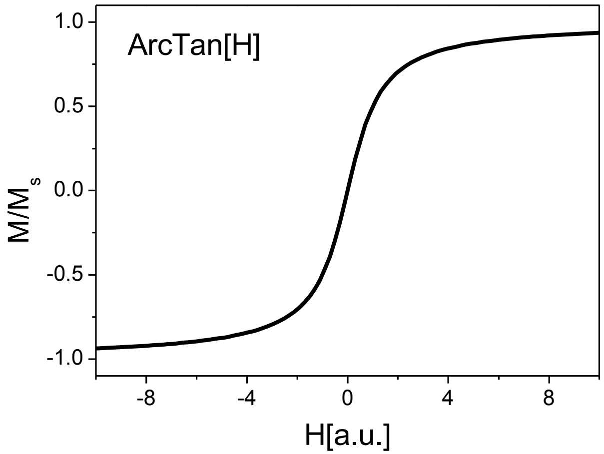


- Both coils of ferromagnetic cores are fed with alternating current.
- The pick-up coil (gray) wound around cores receives signal from magnetization changes in both cores.
- With no external field the signals are opposite and the output signal is null.
- When the **external field** is present the induction in one coil is increased and in the other decreased; the signal appears in pick-up coils.
- The fluxgate utilizes field dependence of magnetic permeability ($\vec{B} = \mu \vec{H}$) of ferromagnetic core: in higher fields the magnetic response of the core to the external field is weaker.
- The signal in pick-up coils non-linearly related to the external field strength.
- Three such devices combined can give vector magnetometer.

F. Primdahl, J. Phys. E: Sci. Instrum. **12** 241 (1979)
image taken from http://www.earthsci.unimelb.edu.au/ES304/MODULES/MAG/NOTES/flxgate.html

Fluxgate principle

- We imitate the real magnetization curve of fluxgate cores by ***M***(***H***)=ArcTan[***H***]:



- Both ferromagnetic cores (1, 2) are placed in equal alternating fields of opposite sign:

$$H_1(t) = A\cos(t) \qquad H_2(t) = -A\cos(t)$$

- **External** positive field *h* is added to the system:

$$H_1(t) = A\cos(t) + h \qquad H_2(t) = -A\cos(t) + h$$

- The magnetization versus time is now (no hysteresis in our model!):

$$M_1(t) = \arctan(A\cos(t) + h)$$

- Because of ***B***= μ$_0$(***H***+***M***), the varying magnetization induces varying voltage in pick-up coils (we neglect geometrical prefactor and ***H*** which is much smaller than ***M****):*

$$V_1(t) = \frac{d}{dt}\arctan(A\cos(t) + h) = -\frac{A\sin(t)}{1 + (A\cos(x) + h)^2}$$

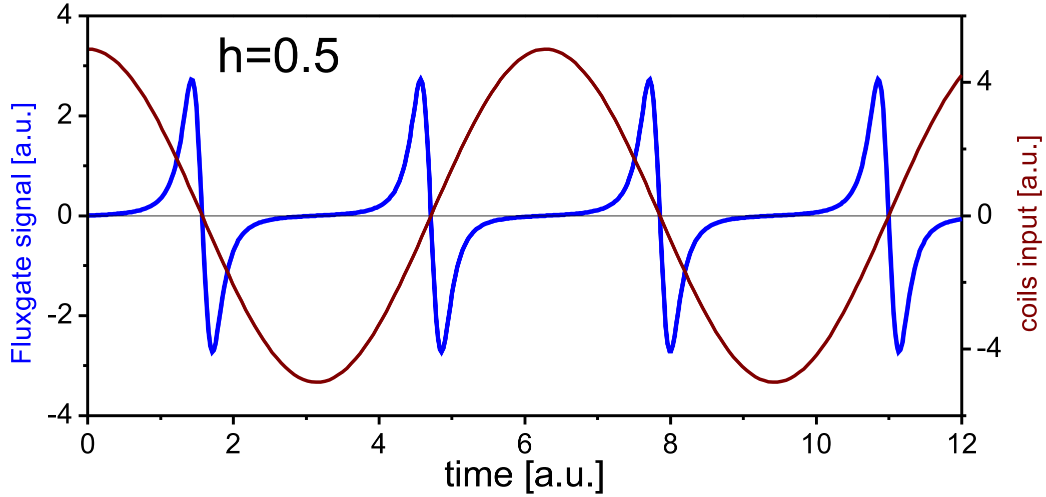
- The output of the fluxgate is:

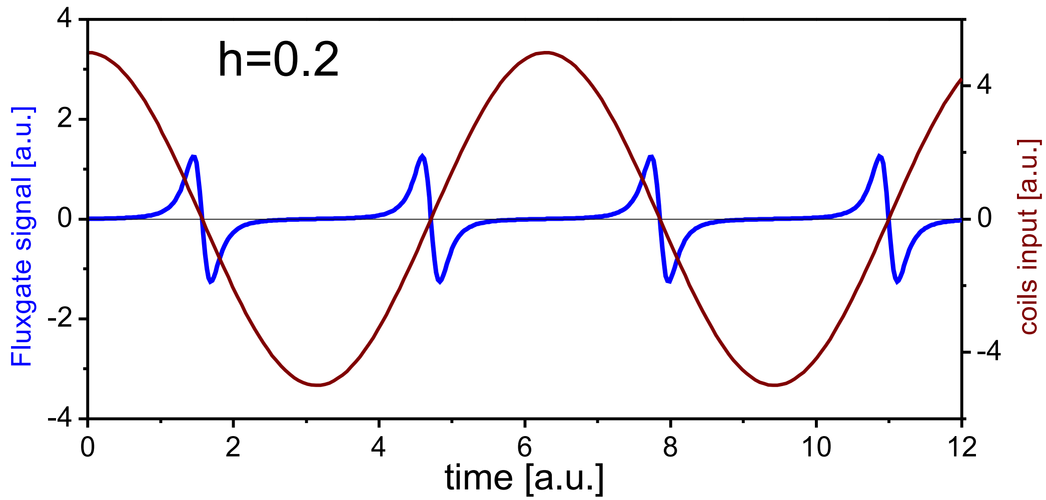
$$V(t) = V_1(t) + V_2(t)$$

Fluxgate principle

•Output signal of the fluxgate (A=5):

h=0.5;A=5;
Plot[{-(A Sin[x])/(1+(h+A Cos[x])^2)+(A Sin[x])/(1+(h-A Cos[x])^2),A Cos[x]},{x,0,6 Pi}]





•Fluxgate signal depends on the strength of the external field but is not proportional to it

•To improve performance feedback circuits are used which “produce a magnetic field inside the sensor that opposes the external field. This keeps the field inside the sensor near zero and in a linear portion of the magnetization curve of the ferromagnetic core.”-S. A. Macintyre [18]

Fluxgate magnetometer

- Fluxgate magnetometers use the Faraday's law of induction to measure static magnetic field; used in II World War for submarine hunt.
- Fluxgate magnetometers are used for measurements of *weak fields with high sensitivity*.

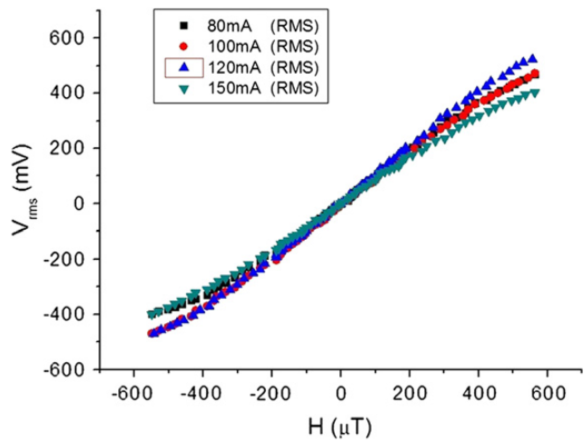


Fig. 8. Output signals of the fluxgate sensor vs. external magnetic field for different excitation currents.

fluxgate

airborne magnetic survey

MEMS (micro-electro mechanical system)

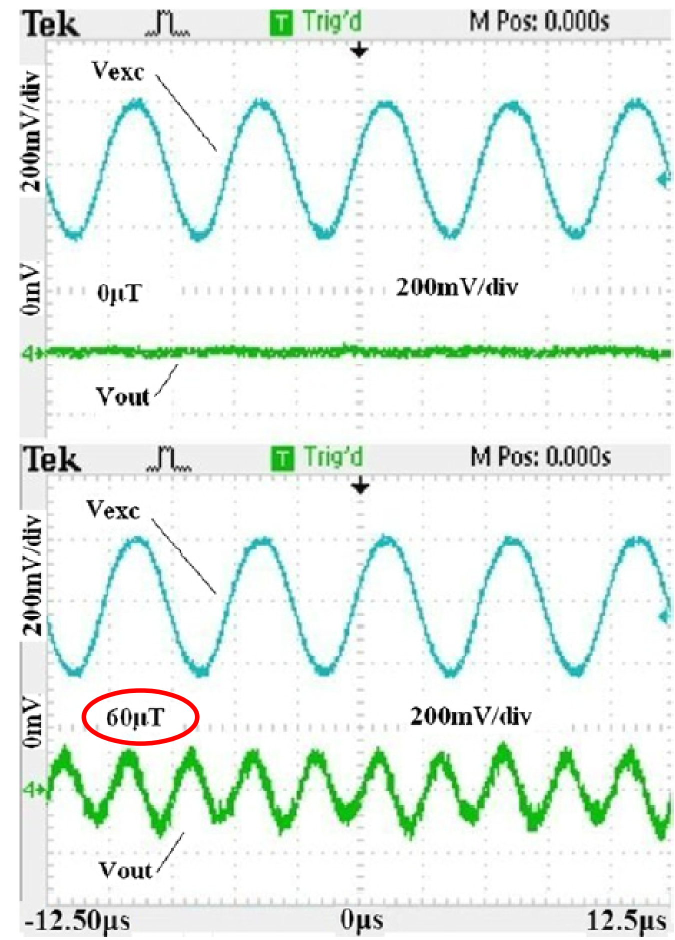


Fig. 7. Output signals of the fluxgate sensor with/without external magnetic field.

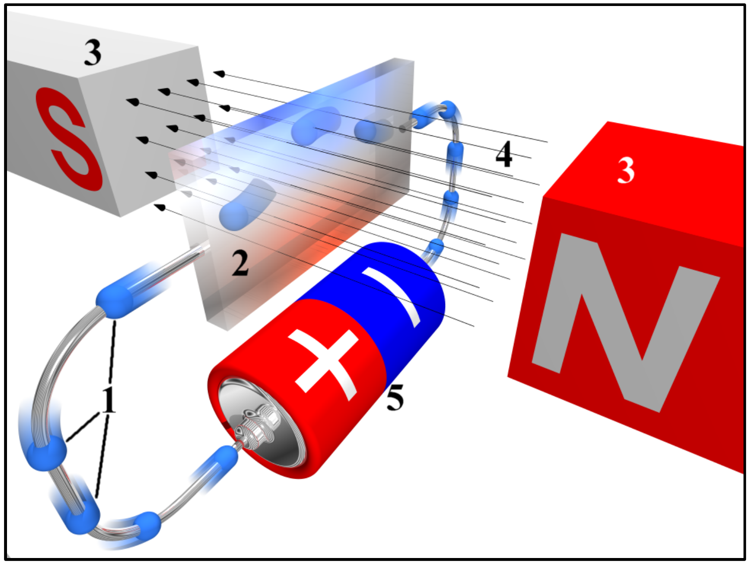
Jian Lei, Chong Lei, Yong Zhou, Measurement **45**, 535 (2012)

image taken from http://www.uh.edu/engines/epi2381.htm

Hall magnetometer

•Lorentz force (see next lecture) acting on electrons in a circuit deflects them perpendicularly to drift direction:

$$\vec{F}_{Lorentz} = q\vec{E} + q\vec{v}\times\vec{B}$$



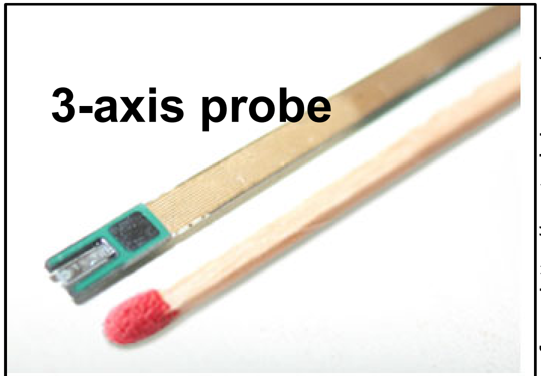
•The build-up of charges on outer limits of the circuit induces Hall voltage which depends on the field strength and is used to sense it.

•The Hall voltage is given by (t-film thickness, R_H-Hall coefficient*):

$$U_y = R_H \frac{I}{t} B_z$$

•The main figure of interest is field sensitivity of the sensor (for a given driving current I_c):

$$\gamma_b = \frac{U_y}{B_z} = \frac{R_H I_c}{t}$$



The new HFC probe is able to measure intense fields in tiny gaps.

image from http://metrolab.com/

Hall sensors are relatively easy to miniaturize

•Semiconductors are used to obtain high sensitivity combined with temperature stability (InAs)

•The Hall sensors have a limited use at high fields and low temperatures (conductivity quantization)

image from Wikimedia Commons; authors: Peo (modification by Church of emacs)

*for InAs R_H is about 0.0001 m^3/As

Most important facts from todays talk:

- Static magnetic field sources are electric currents and intrinsic magnetic moments of elementary particles

- At distances large in comparison to its spatial extension every current distribution produces magnetic induction which can be approximated by magnetic dipole.

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- POV-Ray www.povray.org
- Blender www.blender.org
- Paint.Net www.getpaint.net

Special thanks are due to Google team as I used the searche engine but bought nothing from the ads.