3

Magnetic hysteresis and basic magnetometry

Magnetic reversal in thin films and some relevant experimental methods

Maciej Urbaniak

IFM PAN 2012

Today's plan

- Classification of magnetic materials
- Magnetic hysteresis
- Magnetometry

Classification of magnetic materials

All materials can be classified in terms of their magnetic behavior falling into one of several categories depending on their bulk magnetic susceptibility χ.

$$\chi = \frac{\vec{M}}{\vec{H}}$$

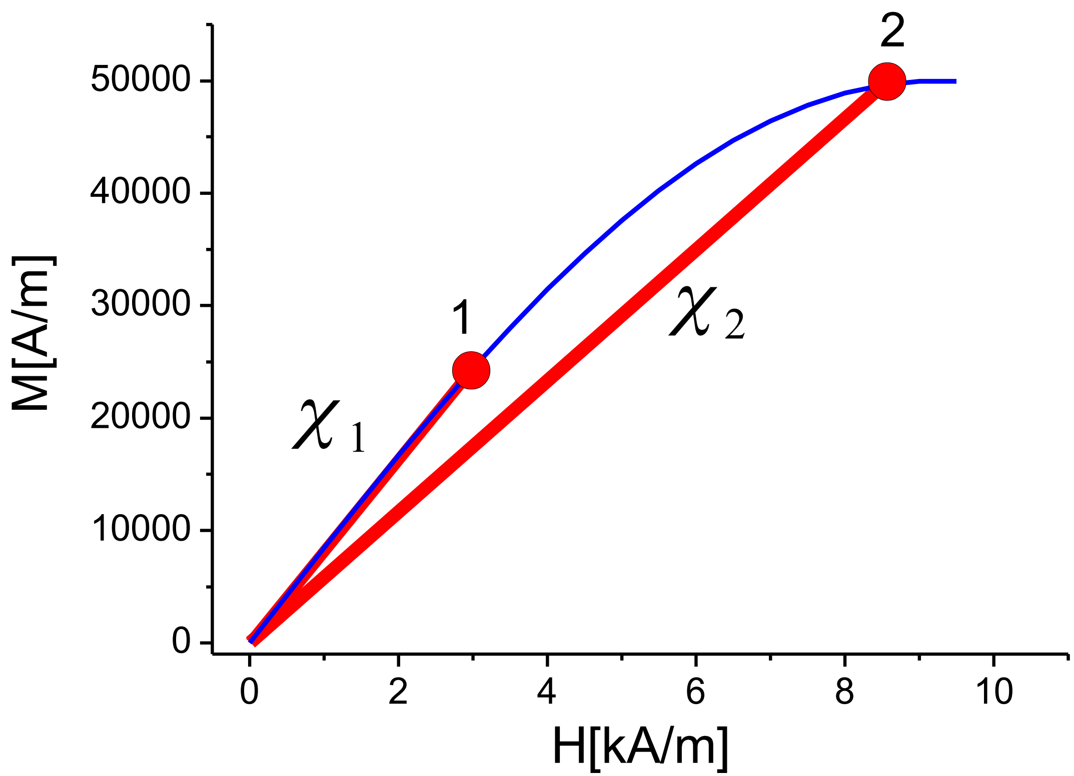
In general the susceptibility is a position dependent tensor

In some materials the magnetization is not a linear function of field strength. In such cases the differential susceptibility is introduced:

$$\chi_d = \frac{d\vec{M}}{d\vec{H}}$$

We usually talk about isothermal susceptibility:

$$\chi_T = \left(\frac{\partial \vec{M}}{\partial \vec{H}}\right)_T$$



Theoreticians define magnetization as:

$$M = -\left(\frac{\partial \vec{F}}{\partial \vec{H}}\right)_T$$

$F = E - TS$ -Helmholtz free energy

Classification of magnetic materials

It is customary to define susceptibility in relation to volume, mass or mole (or spin):

$$\chi = \frac{\vec{M}}{\vec{H}} \quad [dimensionless], \qquad \chi_{\rho} = \frac{(\vec{M}/\rho)}{\vec{H}} \left[\frac{m^3}{kg}\right], \qquad \chi_{mol} = \frac{(\vec{M}/mol)}{\vec{H}} \left[\frac{m^3}{mol}\right]$$

The general classification of materials according to their magnetic properties

μ<1	$\chi<0$	diamagnetic*
μ>1	$\chi>0$	paramagnetic**
μ≫1	$\chi\gg 0$	ferromagnetic***

*dia /daɪəmægˈnɛtɪk/ -Greek: “from, through, across” - repelled by magnets. We have from L2:

$$\vec{F} = \frac{1}{2\mu_0}\chi V \nabla(B^2)$$

The force is directed antiparallel to the gradient of ***B²*** i.e. away from the magnetized bodies

- water is diamagnetic $\chi \approx -10^{-5}$ (see levitating frog from L2)

** para- Greek: beside, near; for most materials $\chi \approx 10^{-5} - 10^{-3}$ [1].

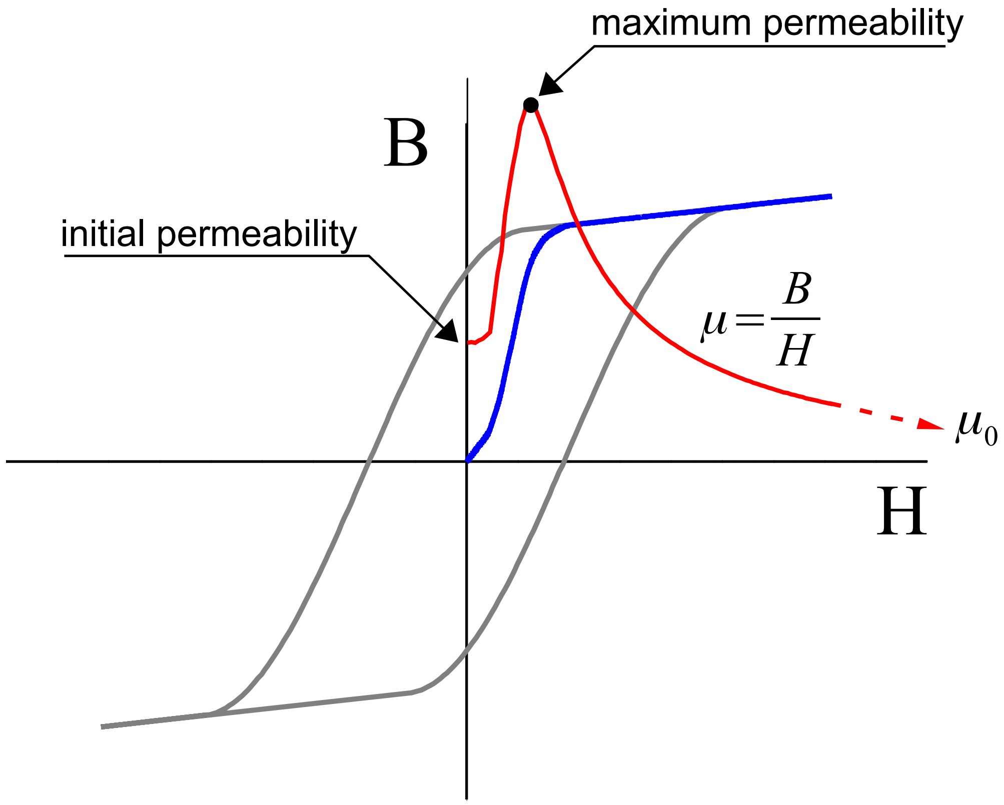
***susceptibility ranges from several hundred for steels to 100,000 for soft magnetic materials (Permalloy)

Classification of magnetic materials

•Feebly magnetic material – a material generally classified as “nonmagnetic” whose maximum normal permeability is less than 4 [5].

•Ferromagnetic materials can be classified according to the magnetic structure on atomic level:

1. Ferromagnets
2. Antiferromagnets
3. Ferrimagnets
4. Asperomagnets -random ferromagnets
5. Sperimagnets – random ferrimagnets



Classification of magnetic materials

In general the susceptibility is frequency dependent and the magnetization depends on the preceding field values [3]:

$$\vec{M}(t)=\int f(t-t')H(t')dt'$$

It is customary to introduce a complex susceptibility:

$$\chi=\chi_{real}+i\,\chi_{imag}$$

$$e^{ix}=\cos(x)+i\sin(x)$$

Then we have:

$$\vec{M}=R.e(\chi\vec{H})=R.e[(\chi_{real}+i\,\chi_{imag})\vec{H}_0 e^{-i\omega t}]=\vec{H}_0(\chi_{real}\cos(\omega t)+\chi_{imag}\sin(\omega t))$$

The imaginary part of susceptibility is responsible for magnetic losses.

Magnetic periodic table of elements

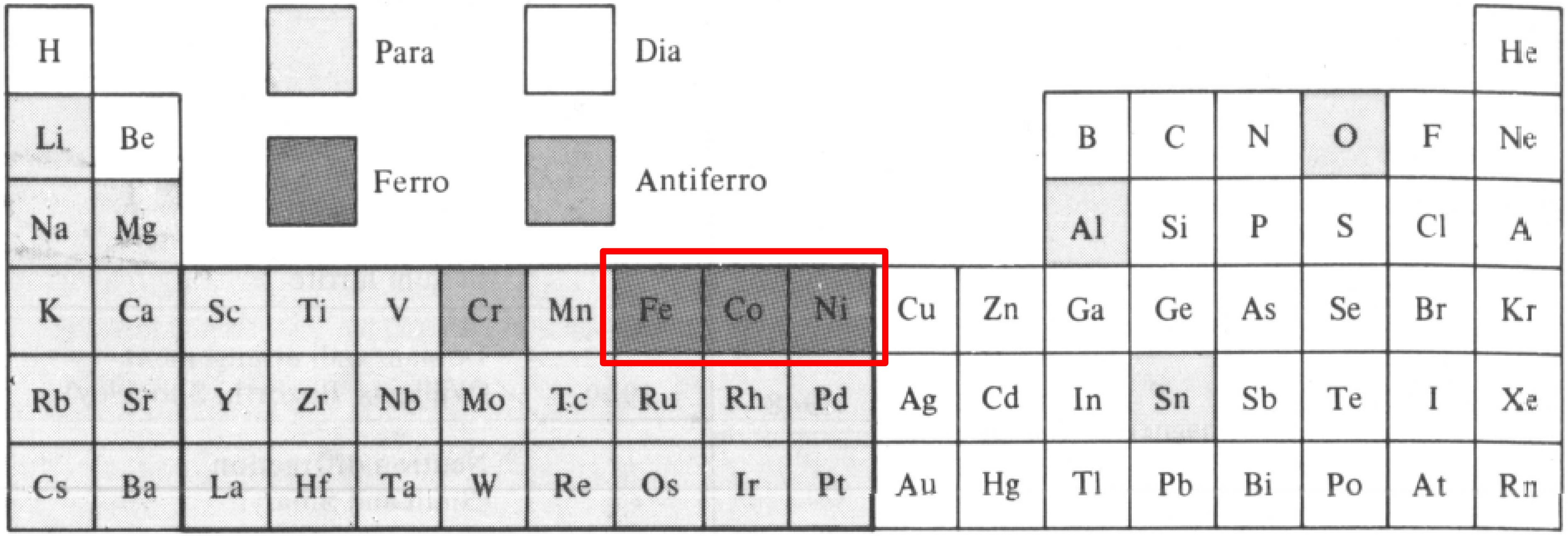
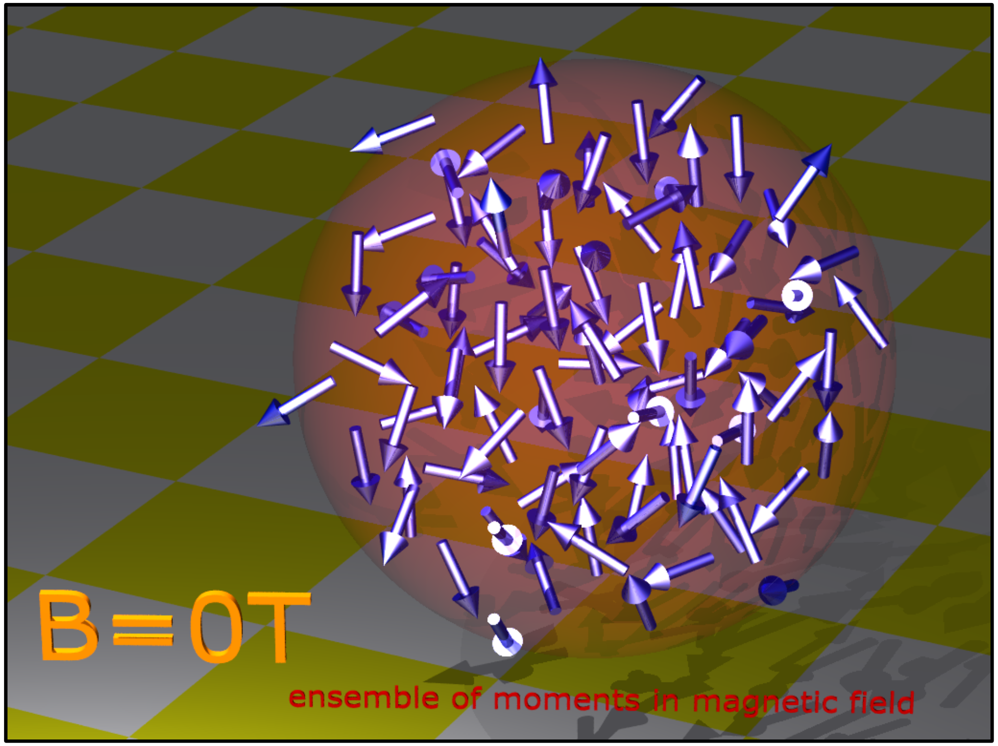


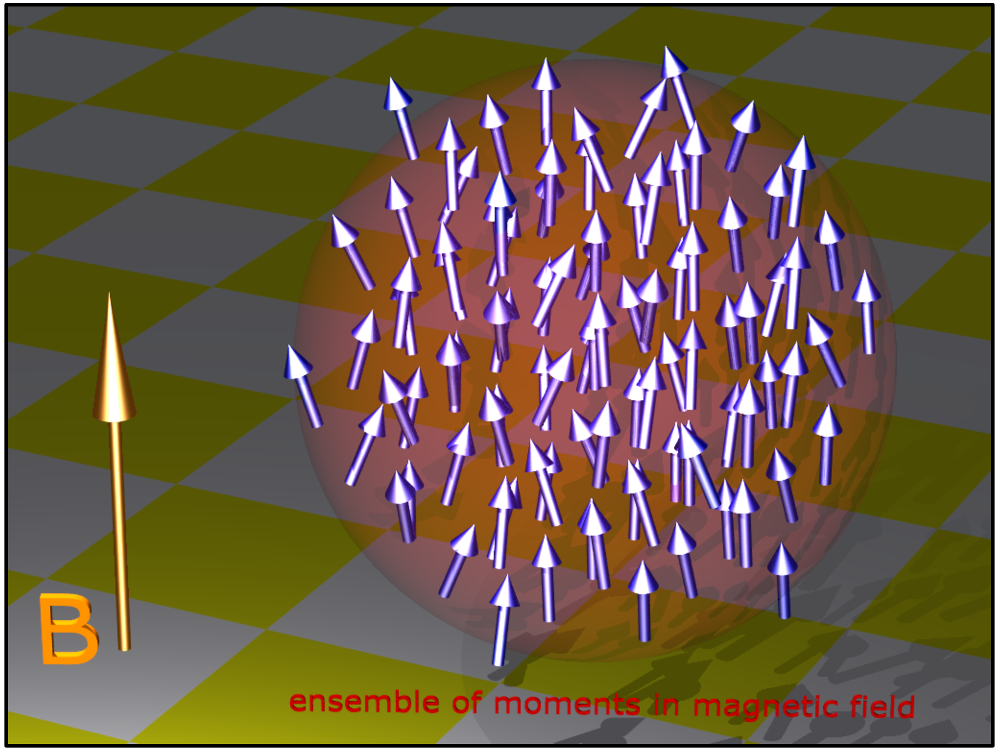
Image source: B.D. Cullity, Introduction to Magnetic Materials, Addison-Wesley 1972, p. 612

The transition elements are enclosed by a heavy line. See Appendix 3 (opposite page) for data on the rare earths.

Paramagnetic materials

• We consider an ensemble of atoms with fixed magnetic moments ***m*** [2]. We neglect here and further the effect of diamagnetism which is negligible in paramagnetic and ferromagnetic materials.





[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996

Paramagnetic materials

• We consider an ensemble of atoms with fixed magnetic moments ***m*** [2]. We neglect here and further the effect of diamagnetism which is negligible in paramagnetic and ferromagnetic materials.

• From quantum mechanics (QM) we adopt the expression for ***m***:

$$\vec{m} = g\,\mu_B \vec{S} \qquad \mu_B = \frac{e\,h}{4\pi m_e} = 9.27400968(20)\times 10^{-24}\,A\,m^2$$

• From QM again we know that S_z can assume only 2S+1discrete values

• The moments ***m*** interact with the external field but not with each other. We use the Boltzmann distribution to get the average moment of the ensemble along the field:

$$\langle m_z \rangle = \frac{\sum m_z e^{m_z B / k_B T}}{\sum e^{m_z B / k_B T}}$$

• Using the expression for m and remembering that S is discrete we get:

$$\langle m_z \rangle = \frac{\sum_{n=-S}^{S} g\,\mu_B n\, e^{g \mu_B n B / k_B T}}{\sum_{n=-S}^{S} e^{g \mu_B n B / k_B T}}$$

[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996

Paramagnetic materials

The previous expression can be shown to give [2]:

$$\frac{\langle S_z \rangle}{S} = B_S\left(\frac{g\,\mu_B S\,B}{k_B T}\right)$$

, where B_S is a Brillouin function depending on S:

$$B_S(x) = \frac{2S+1}{2S}\coth\left(\frac{2S+1}{2S}x\right) - \frac{1}{2S}\coth\left(\frac{x}{2S}\right)$$

For S=1/2 we get: $B_{1/2}(x) = \tanh(x)$

For small values of *x* Taylor expansion leads to: $\coth(x) = \frac{1}{x} + \frac{x}{3} + \ldots$ and after tedious arithmetic one gets:

$$B_S(x) = \frac{S+1}{3S}x \quad \rightarrow \quad \langle S_z \rangle = \left(\frac{g\,\mu_B S(S+1)}{3 k_B T}\right) B$$

Substituting $m = g\,\mu_B \langle S_z \rangle$ we get (we omit vector notation as system is isotropic):

$$M = \left(\frac{N S(S+1)}{3 k_B T}\right)(g\,\mu_B)^2 B \quad \longleftarrow \quad \langle M_z \rangle = N \langle m_z \rangle$$

, where N is a number of spins per unit volume

[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996

Paramagnetic materials

From the previous expression we get **Curie Law** [2]:

$$\chi = \frac{\vec{M}}{\vec{H}} = \frac{C}{T}$$

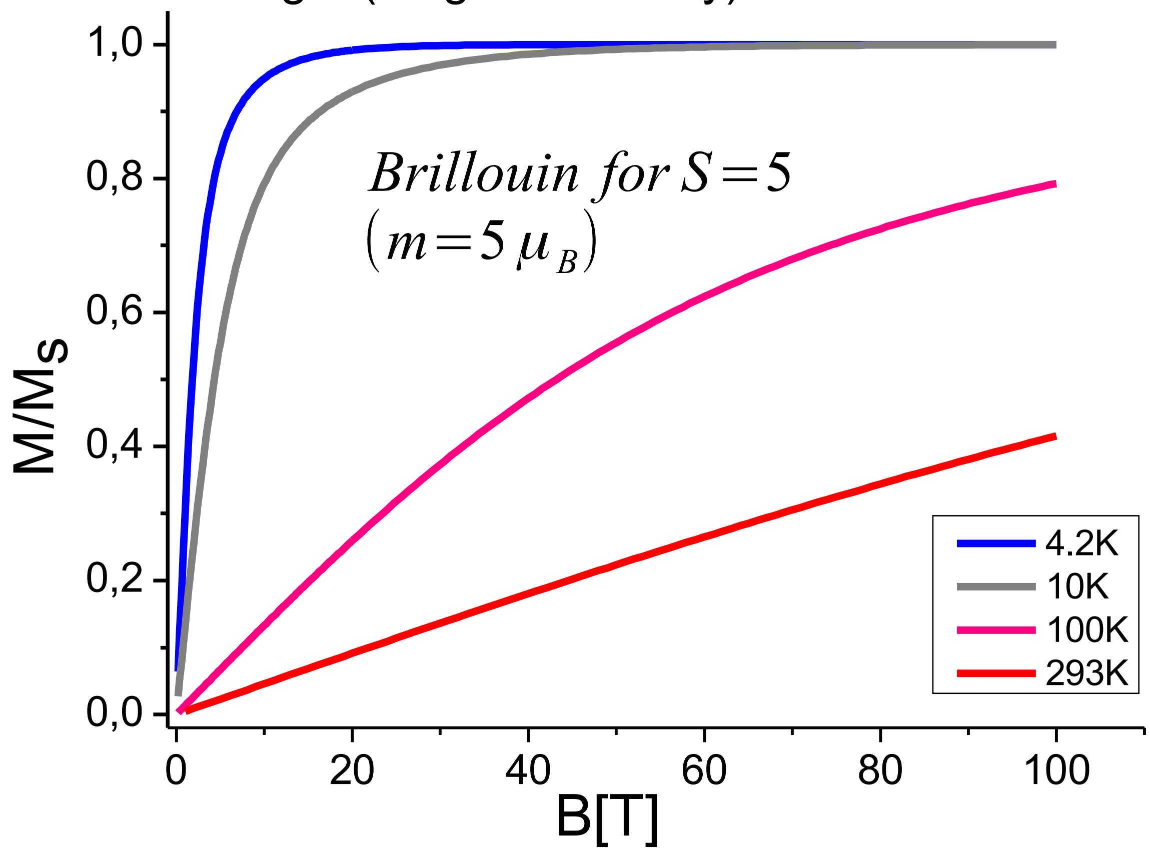
with Curie constant $$C = \mu_0 \left(\frac{N\,S(S+1)}{3\,k_B} \right) (g\,\mu_B)^2$$

In paramagnets the susceptibility may depend on ***H*** but it is a single-valued function; it does not depend on the previous values of field strength (magnetic history).

- Accessible magnetic fields (97 T as of 2011) do not allow reaching the magnetic saturation of most paramagnets
- Lowering temperature allows considerable decrease of essential fields

$$\frac{\langle S_z \rangle}{S} = B_S \left(\frac{g\,\mu_B\,S\,B}{k_B\,\mathbf{T}} \right)$$

!



[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996

Paramagnetic materials

Curie Law: $\chi = \frac{C}{T}$

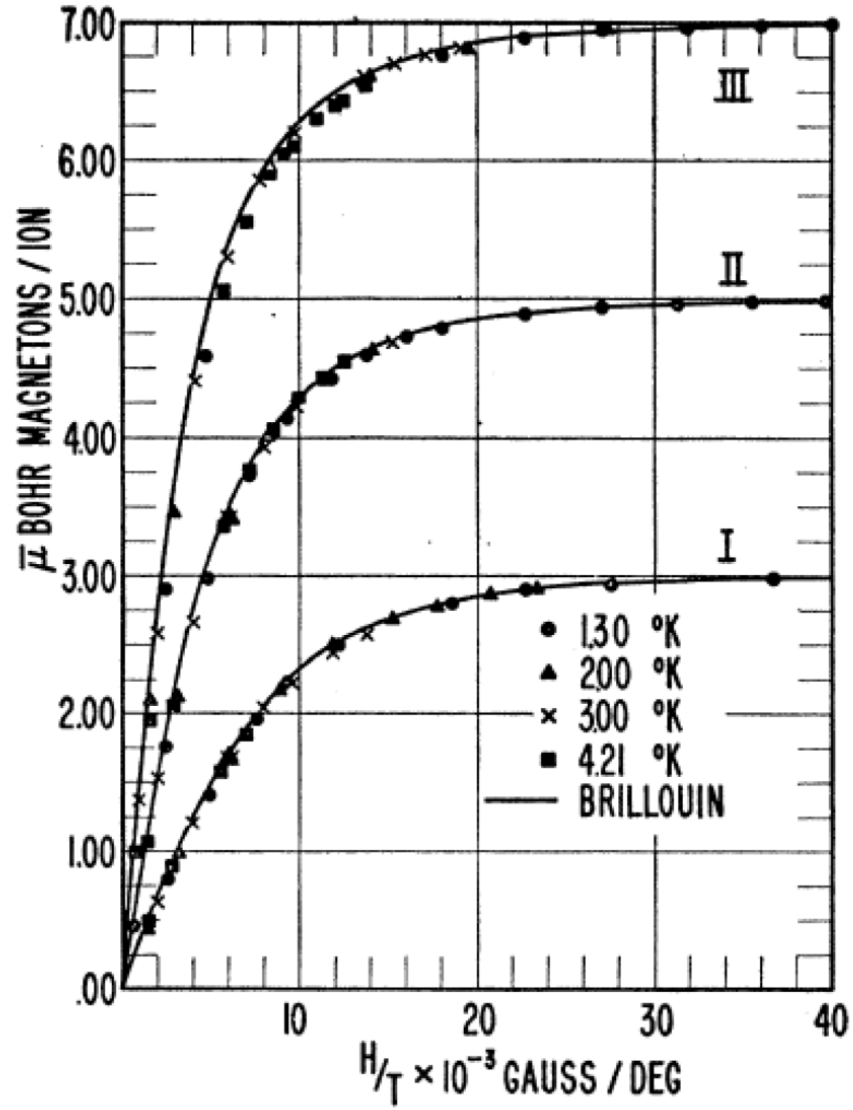


FIG. 3. Plot of average magnetic moment per ion, $\bar{\mu}$ *vs* H/T for (I) potassium chromium alum ($J=S=3/2$), (II) iron ammonium alum ($J=S=5/2$), and (III) gadolinium sulfate octahydrate ($J=S=7/2$). $g=2$ in all cases, the normalizing point is at the highest value of H/T.

- J=S=3/2, 5/2, 7/2
- g=2 in all cases
- note **high fields and very low temperatures**

W.E. Henry, Phys.Rev. **88** 559 (1952)

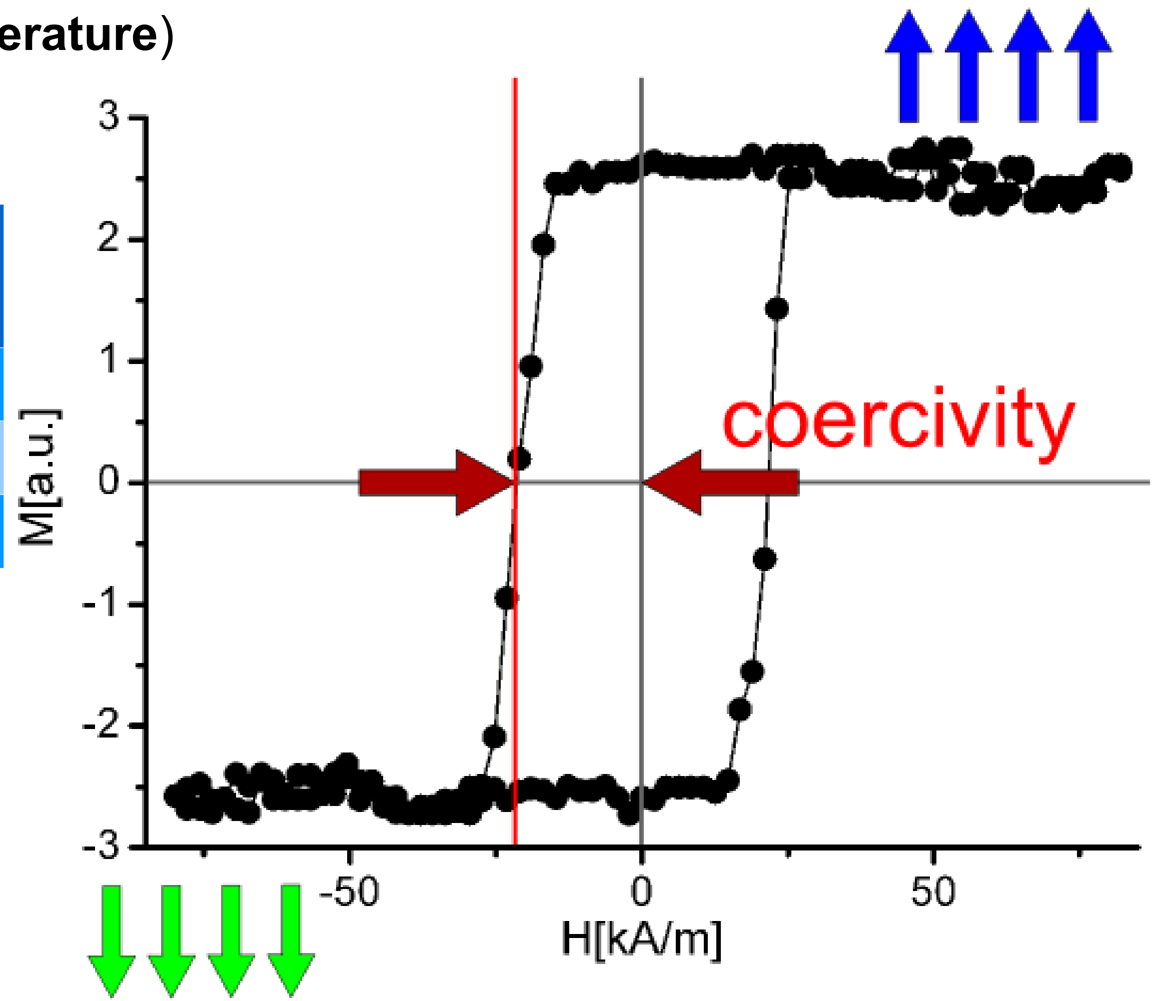
Ferromagnetic materials

Most notable features of ferromagnetic materials:
- high initial susceptibility/permeability
- they usually retain magnetization after the removal of the external field – **remanence**
- the magnetization curve (B-H or M-H) is nonlinear and hysteretic
- they lose ferromagnetic properties at elevated temperatures (**Curie temperature**)

The notable examples [4]:

	M_S[kA/m] @RT	T_C[K]
Fe	1714	1043
Co	1433	1403
Ni	485	630

M_S[Am^2/kg] @RT
217.75
161
54.39



Ferromagnetic materials

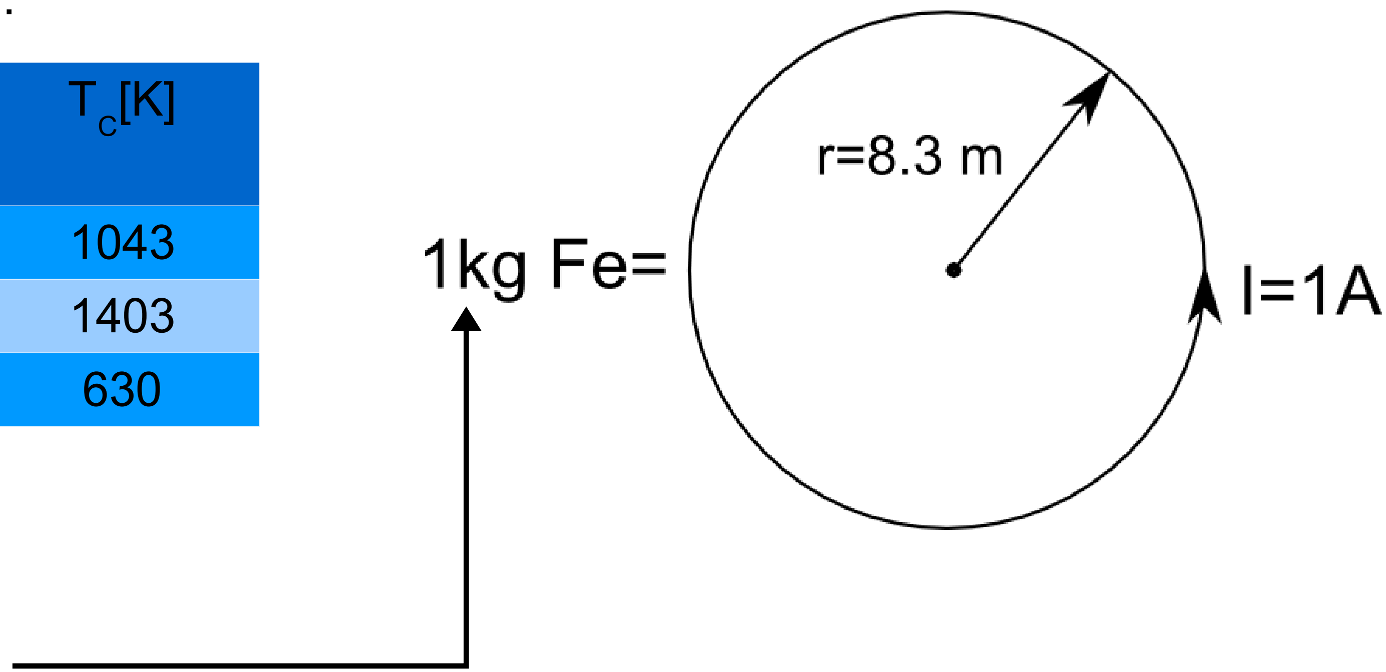
Most notable features of magnetic materials:

- high initial susceptibility/permeability
- they usually retain magnetization after the removal of the external field – **remanence**
- the magnetization curve (B-H or M_H) is nonlinear and hysteretic
- they lose ferromagnetic properties at elevated temperatures (**Curie temperature**)

The notable examples [4]:

	M_s[kA/m] @RT	T_C[K]
Fe	1714	1043
Co	1433	1403
Ni	485	630

M_s[Am^2/kg] @RT
217.75
161
54.39



Ferromagnetism – Weiss molecular field approximation

- in contrast to the description of paramagnets we assume that magnetic moments interact via the exchange energy
- the description is appropriate for rare earth ions (in crystal lattice) in which the uncompensated spins are in inner atomic shells
- the model is inappropriate for the description of the magnetization of metals

We describe the total energy of a system in magnetic field ***B*** by [2]:

$$E=-\sum_{ij,i\neq j} J_{ij}\vec{S}_i\cdot\vec{S}_j-\sum_i g\mu_B\vec{S}_i\cdot\vec{B} \qquad J_{ij}-exchange\,integral$$

We tag one spin and replace other spins by their mean value. The energy terms in which spin *i* is is involved sum to:

$$E_i=-2\sum_j J_{ij}\vec{S}_i\cdot\langle\vec{S}_j\rangle-g\mu_B\vec{S}_i\cdot\vec{B}=-\vec{S}_i\cdot\vec{B}_i \qquad ,with \quad B_i=2\sum_j J_{ij}\langle\vec{S}_j\rangle+g\mu_B\vec{B}$$

	1	2	3	4
1		•	•	•
2	•		•	•
3	•	•		•
4	•	•	•	

terms involving *i*=2 (for 4 spins)

	1	2	3	4
1		•		
2	•		•	•
3		•		
4		•		

[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996

Ferromagnetism – Weiss molecular field approximation

The problem of ferromagnetism is now changed into the problem of **isolated spins interacting with an applied field** – paramagnetism. From there we have:

$$\langle S_z \rangle = S\,B_S\left(\frac{g\,\mu_B S\,B}{k_B T}\right) \rightarrow \quad (with \quad g\,\mu_B \vec{S}\cdot\vec{B} \rightarrow \vec{S}\cdot\vec{B}) \qquad \langle S_{iz} \rangle = S\,B_S\left(\frac{S\,B_i}{k_B T}\right)$$

Substituting B_i from previous page we obtain:

$$\langle S_{iz} \rangle = S\,B_S\left(\frac{S}{k_B T}\left[2\sum_j J_{ij}\langle \vec{S}_j \rangle + g\,\mu_B B\right]\right)$$

Dropping indexes (on average spins are equal) and summing only over nearest neighbors the expression can be rewritten [2]:

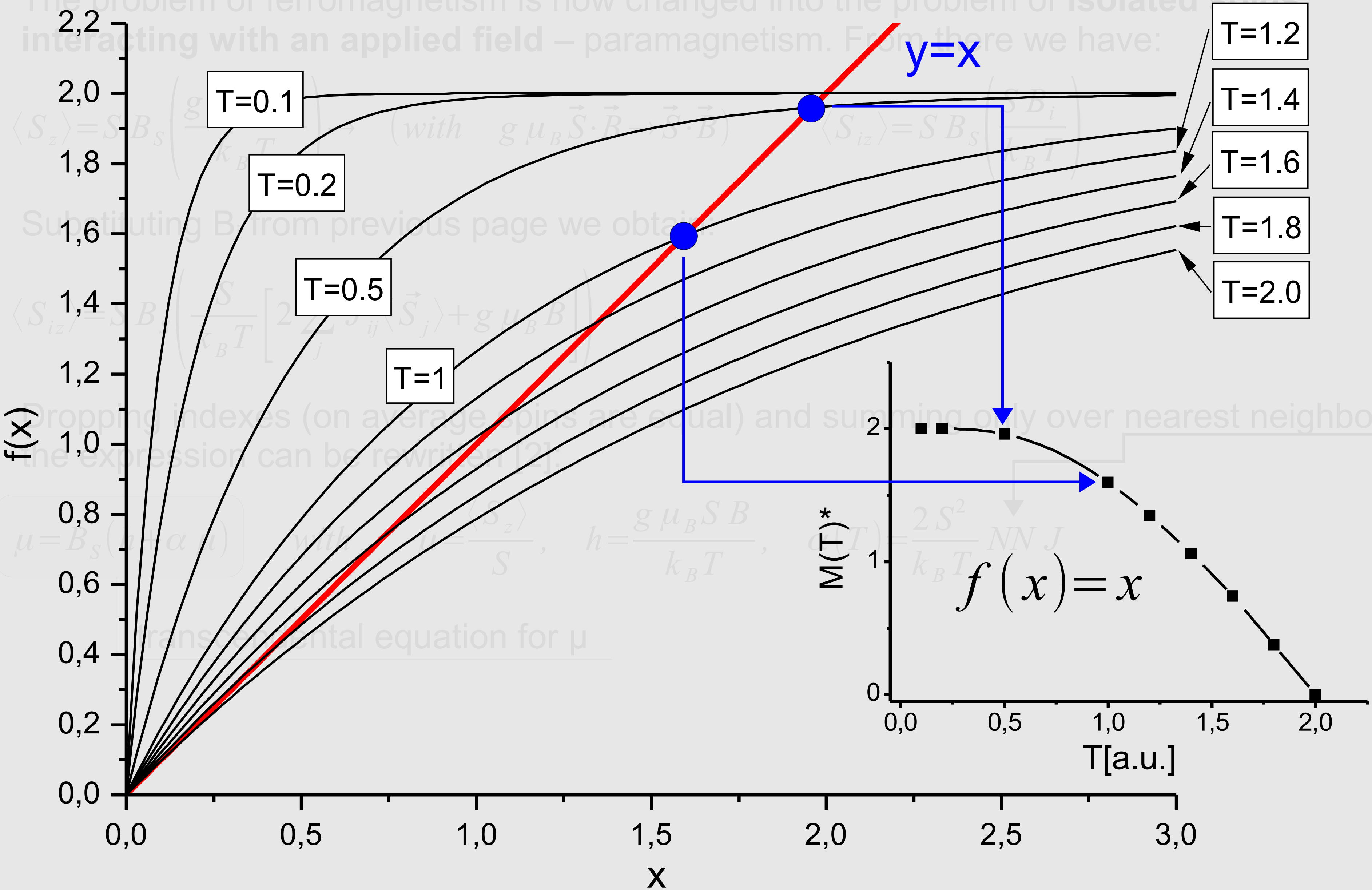
$$\mu = B_S(h + \alpha\,\mu) \qquad with \qquad \mu = \frac{\langle S_z \rangle}{S}, \quad h = \frac{g\,\mu_B S\,B}{k_B T}, \quad \alpha(T) = \frac{2\,S^2}{k_B T} NN\,J$$

transcendental equation for μ

[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996

$$f(x)=2-2e^{(-(x+h)/T)}$$

Arbitrary function depending on 1/T, not related to Brillouin function, gives ***M***(***T***) dependence with “Curie temperature”.



* M:=x(T)

Ferromagnetism – Weiss molecular field approximation

Equation $\mu = B_S(h + \alpha\,\mu)$ can be solved numerically giving:

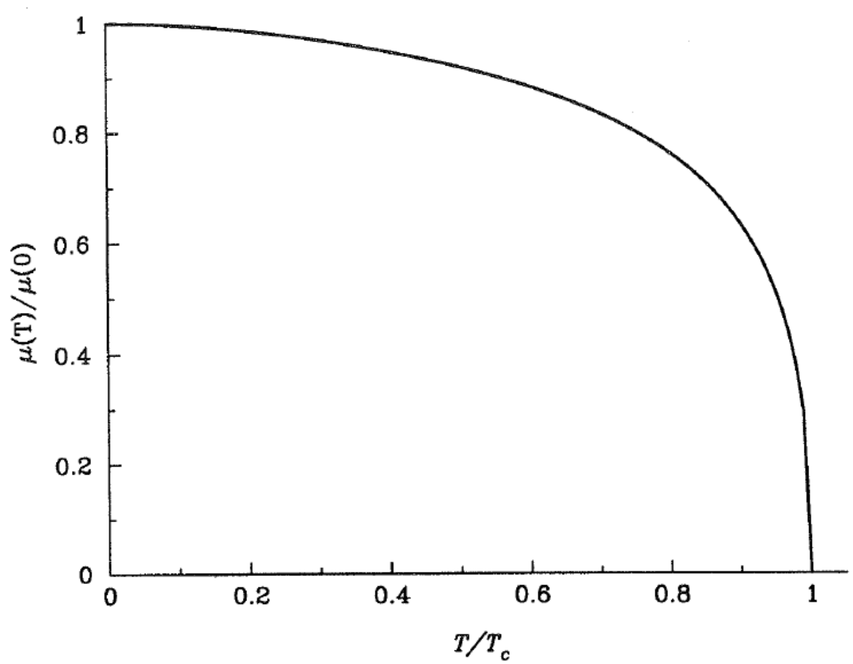


FIG. 2.1. An approximate shape of the solution of eqn (2.2.33) for the case $h = 0$. The temperature, T, is normalized to the Curie temperature, T_c, above which the only solution of this equation is $\mu = 0$.

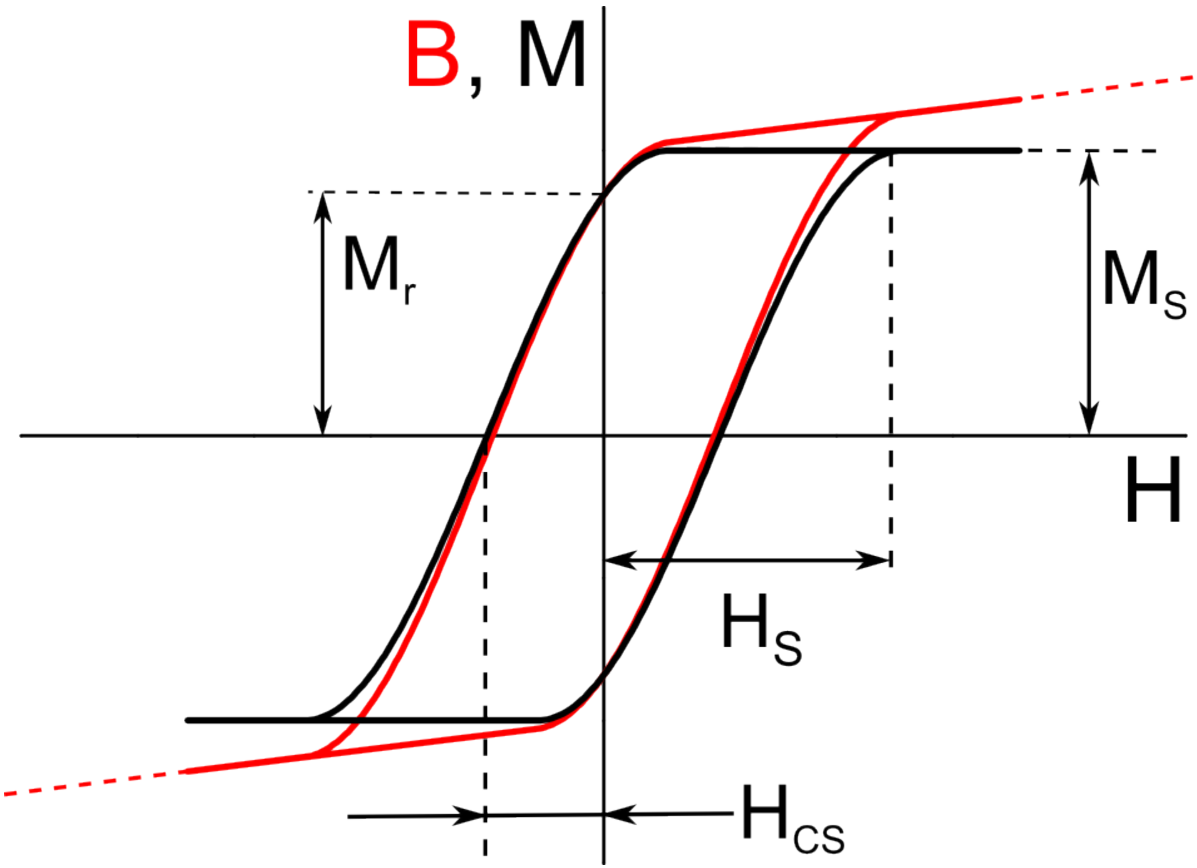
- Above critical temperature (paramagnetic Curie temperature) there is no spin order

- Weiss model describes weakly interacting localized moment

[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996

Hysteresis nomenclature

The magnetic hysteresis can be presented both as ***B***(***H***) and ***M***(***H***)* dependencies.



intrinsic induction:

$$\vec{B}_i = \vec{B} - \mu_0 \vec{H} = \mu_0 \vec{M}$$

coercive field strength – field required to reduce the **magnetic induction** to zero after the material has been symmetrically cyclically magnetized.

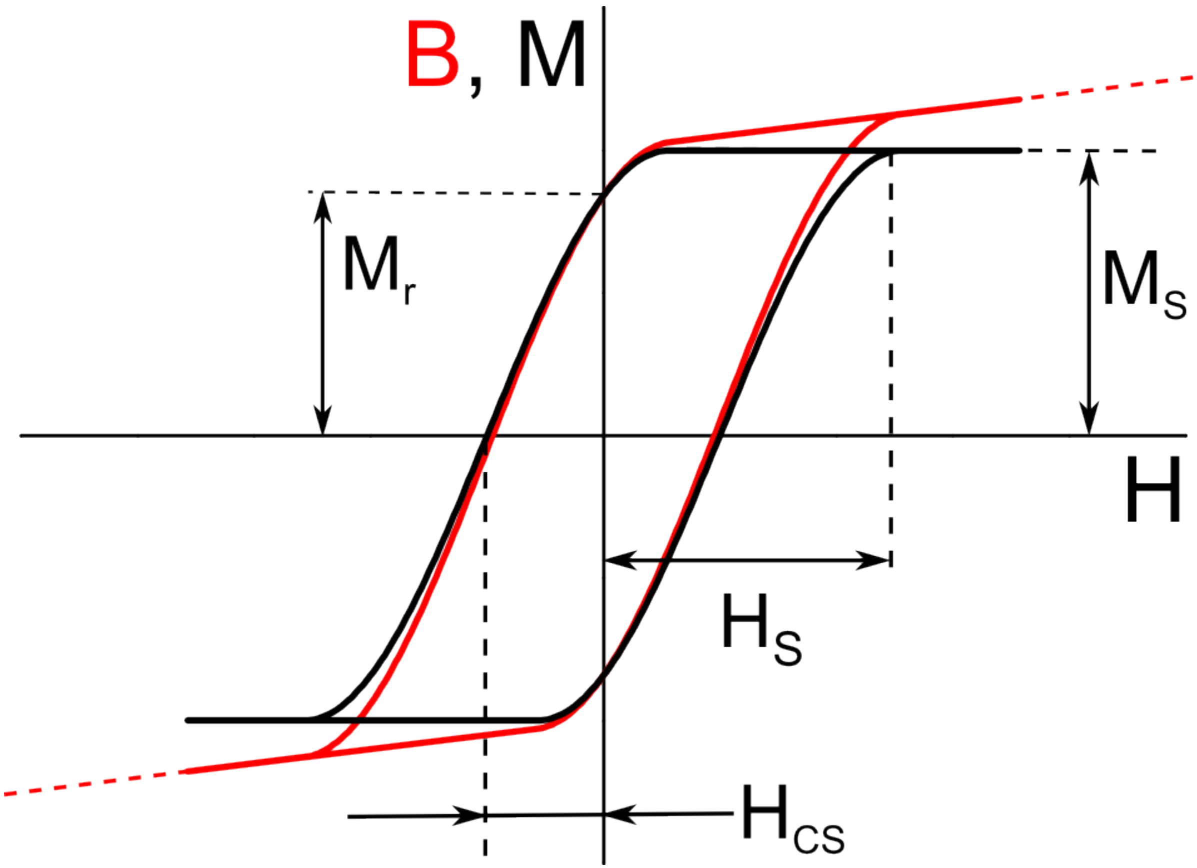
intrinsic coercive field strength – field required to reduce the **intrinsic induction** to zero after...

coercivity, *H*cs—the maximum value of coercive field strength that can be attained when the magnetic material is symmetrically cyclically magnetized to *saturation induction*, B_S.

*μ_0 ***M***(H) dependence is called a intrinsic hysteretis loop [5]

Hysteresis nomenclature

The magnetic hysteresis can be presented both as ***B***(***H***) and ***M***(***H***) dependencies.



saturation induction, *Bs*—the maximum intrinsic induction possible in a material
saturation magnetization, **M***s:*

$$\vec{M}_s = \vec{B}_s / \mu_0$$

demagnetization curve—the portion of a dc hysteresis loop that lies in the second (or fourth quadrant). Points on this curve are designated by the coordinates, ***B***d and ***H***d.

remanence, *Bdm*—the maximum value of the remanent induction for a given geometry of the magnetic circuit.

*for the glossary of magnetic measurements terms see ASTM, A 340 – 03a, 2003 [5]

Hysteresis losses

From Faraday's law it follows [4] that the change in a current in circuit 1 produces *emf* in the second circuit:

$$emf_{21} = -M_{21} \frac{dI_1}{dt}$$

At any instant of time the following relation is fulfilled:

$$emf^{appl.} + emf^{ind.} = IR$$

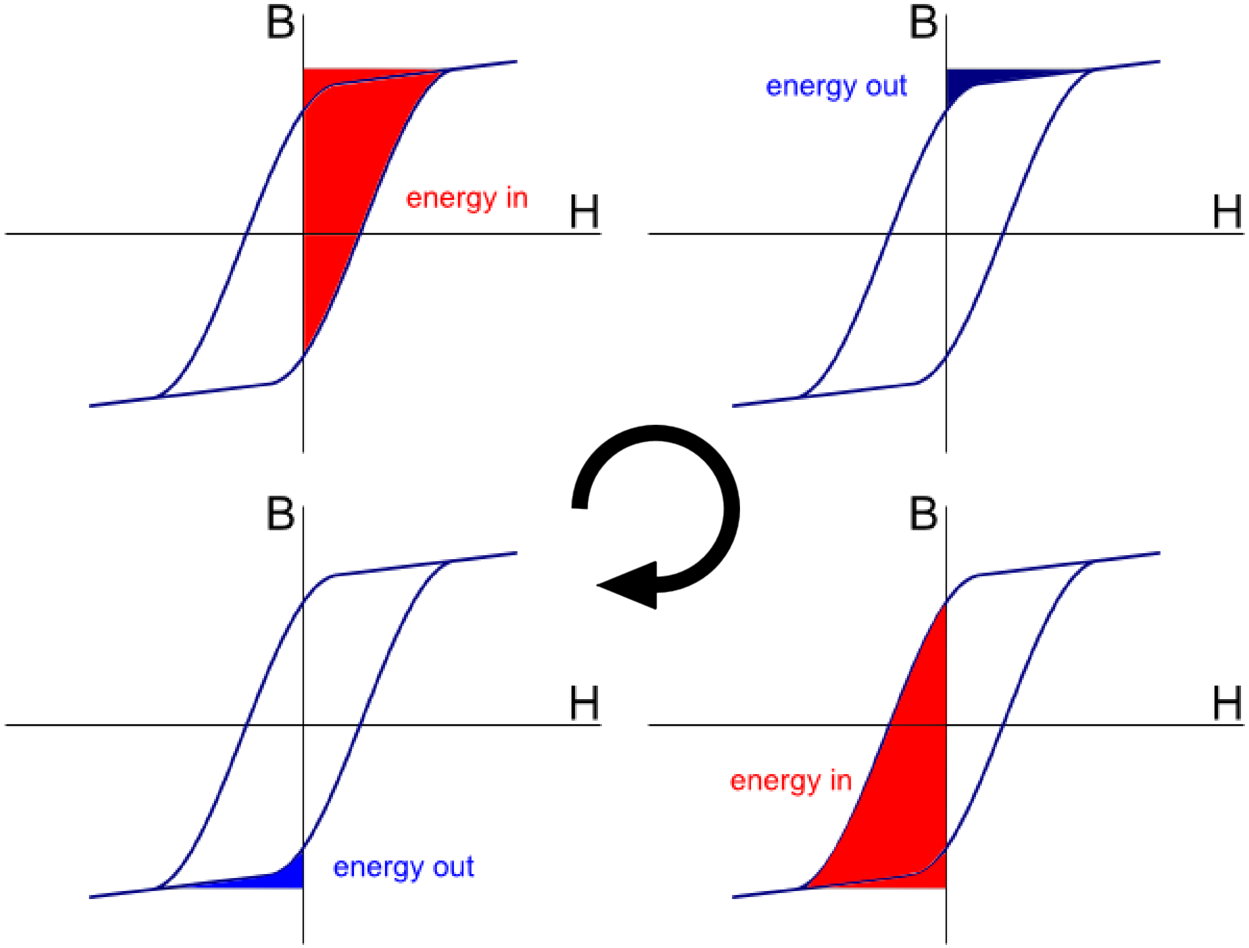
It can be shown [4] that the total energy required to establish a currents in an ensemble of coils fixed in space is:

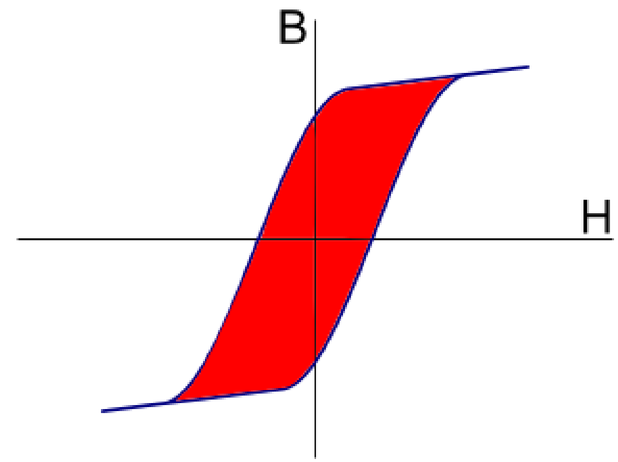
$$W = \frac{1}{2} \sum_i \Phi_i I_i, \qquad where\ \Phi_i = \int_S \vec{B} \cdot dS \quad is\ the\ flux\ \ enclosed\ \ by\ \ i-th\ \ circuit$$

Further it can be shown that the total energy may be expressed by:

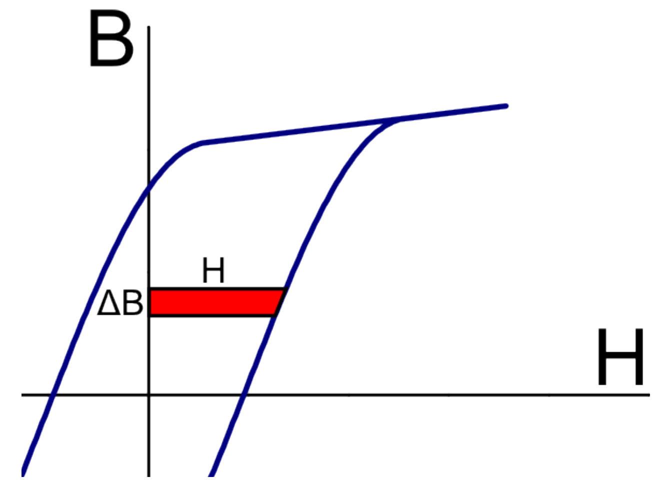
$$W = \int \Delta \vec{B} \cdot \vec{H}\, dV$$

Hysteresis losses

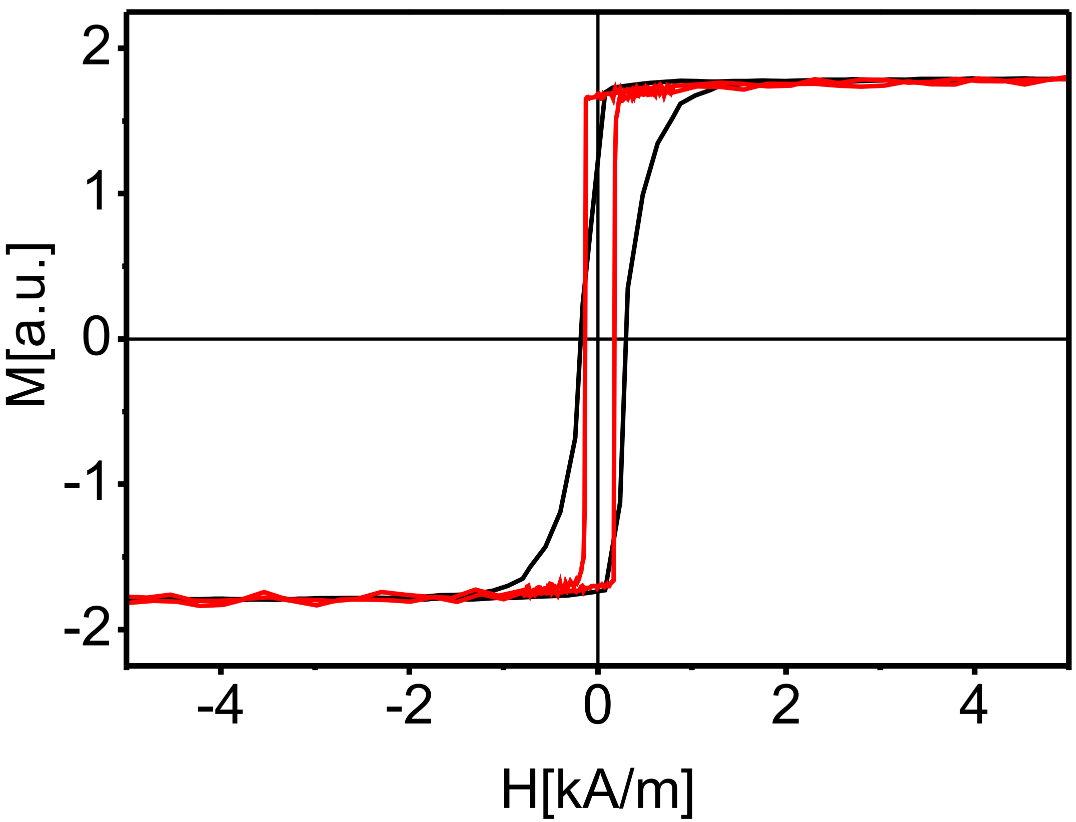




$$W = \int \Delta\vec{B}\cdot\vec{H}\, dV$$



Hysteresis curves



•Hysteresis is not an intrinsic property of the material

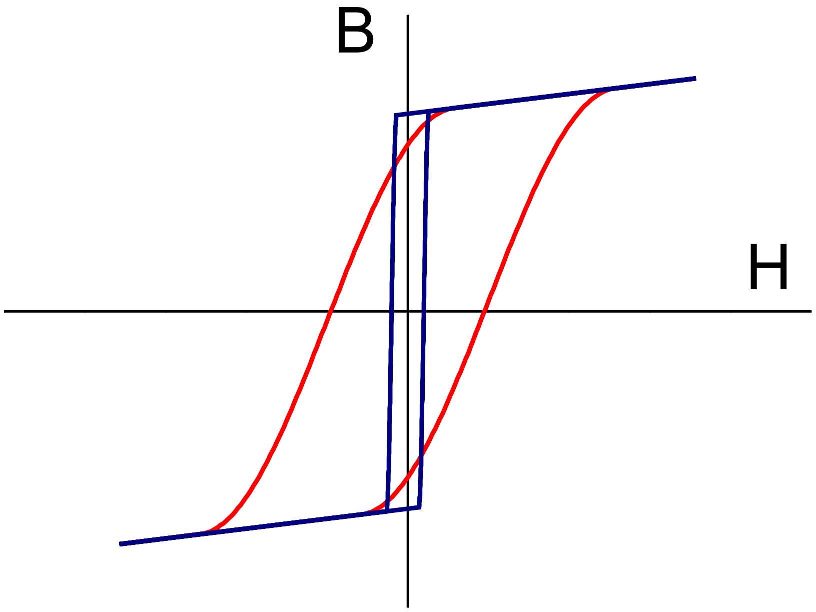
•The character of magnetization depends on sample preparation and the shape of the sample.

$[Ni_{80}Fe_{20}(2\ nm)/Au(5\ nm)]_{15}$

$Ni_{80}Fe_{20}(38\ nm)$

$Ni_{80}Fe_{20}$- soft magnetic material, $H_C \approx 160$ A/m

Hysteresis curves – magnetically soft and hard materials



•Magnetically soft (hard) materials are usually called soft (hard) magnetic materials

•Soft magnetic materials – coercivity of the order of 1 kA/m (≈12 Oe)

Magnetically soft materials	Magnetically hard materials
Fe-Si alloys (several % Si)	Alnico (Al-Ni-Co)
Permalloy (Ni-Fe (ca. 20%) alloys)	samarium–cobalt magnet alloys
Sendust (Fe-Al-Si)	neodymium magnet (NdFeB)
Amorphous alloys	
Nanocrystals	

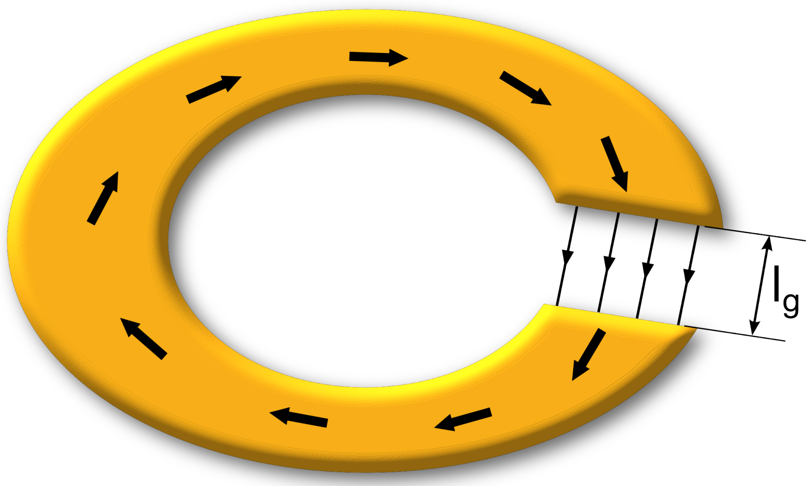
Hysteresis curves – energy product

Consider a ring magnet of length l_m with a gap of length l_g (an corresponding cross-section areas A_m and A_g. We have (since there are no external currents):

$$\oint \vec{H}\, dl = 0 \qquad \text{or} \qquad H_m l_m + H_g l_g = 0$$

Assuming no flux-leakage and because of continuity of ***B*** we obtain:

$$\mu_0 H_g A_g = B_m A_m$$



Hysteresis curves – energy product

Consider a ring magnet of length l_m with a gap of length l_g (an corresponding cross-section areas A_m and A_g. We have (since there are no external currents):

$$\oint \vec{H}\,dl = 0 \qquad \text{or} \qquad H_m l_m + H_g l_g = 0$$

Assuming no flux-leakage and because of continuity of ***B*** we obtain:

$$\mu_0 H_g A_g = B_m A_m$$

The above two relations combine to give [6] (V_g, V_m – volumes of magnetic material and the gap):

$$\mu_0 H_g^2 = -H_m B_m \left(\frac{V_m}{V_g}\right)$$

Multiplication of bothe expressions:

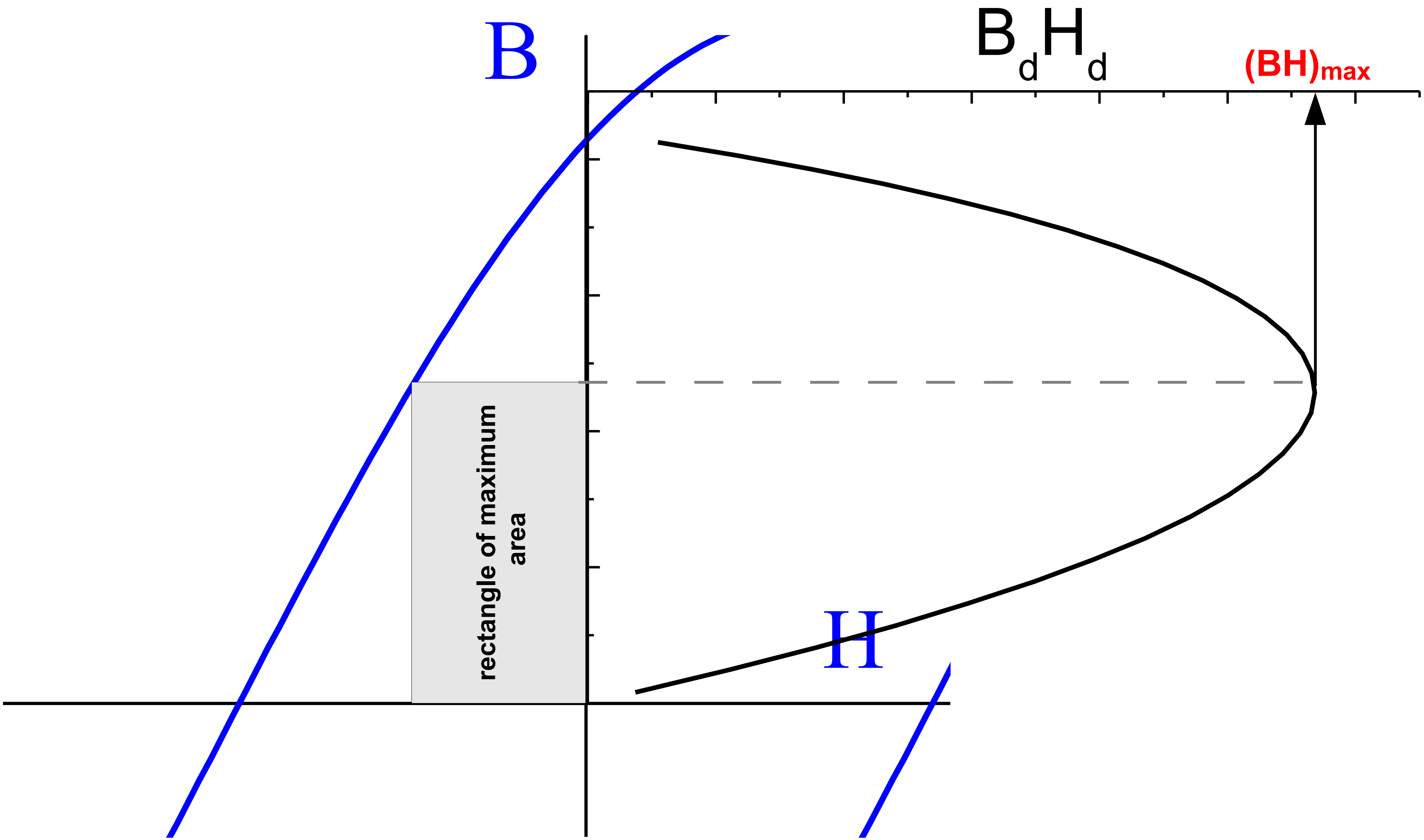
$$\leftarrow \quad H_g l_g (\mu_0 H_g A_g) = -H_m l_m (B_m A_m)$$

- For a given volumes of material and gap width the field strength in the gap is maximum when (BH) is maximum
- (BH) is called energy* product (the B and H values in the second quadrant are usually subscripted with d)
- Maximum attainable value of the product, i.e., $(BH)_{max}$ is a figure of merit of materials for permanent magnets

$^*\vec{H}\cdot\vec{B}\left[\frac{A}{m}\frac{Wb}{m^2} = \frac{A}{m}\frac{V\,s}{m^2} = \frac{J}{m^3}\right]$

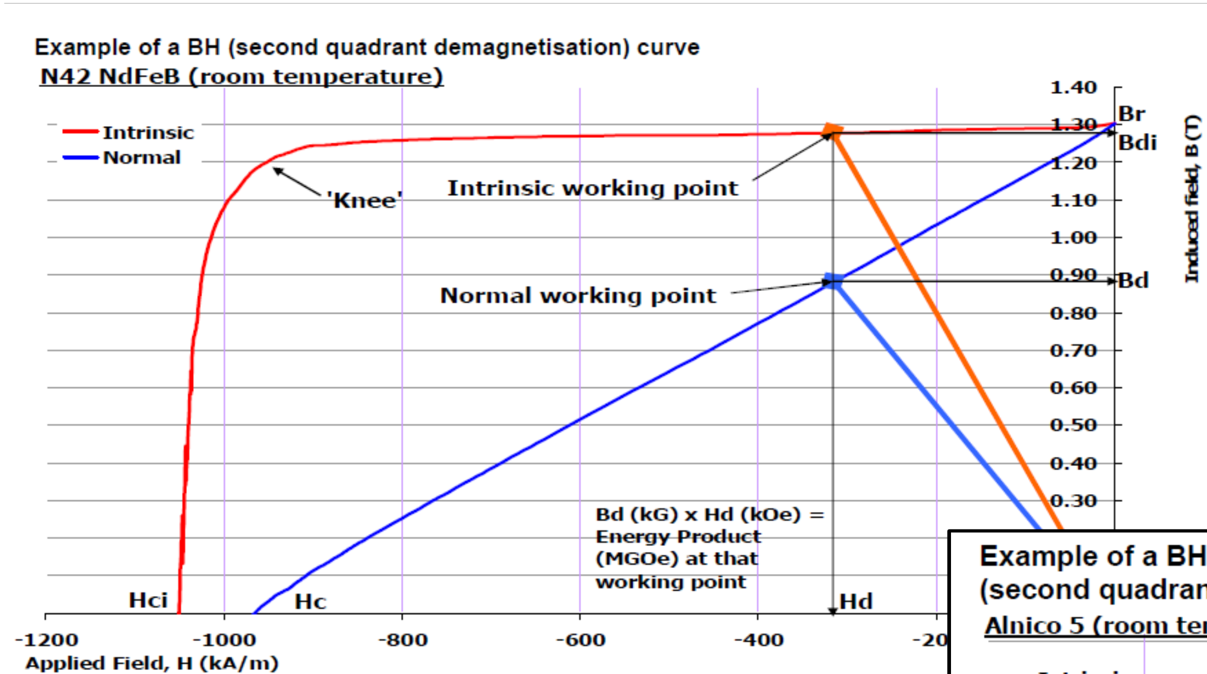
Hysteresis curves – energy product

$(BH)_{max}$ is an intrinsic property of the material

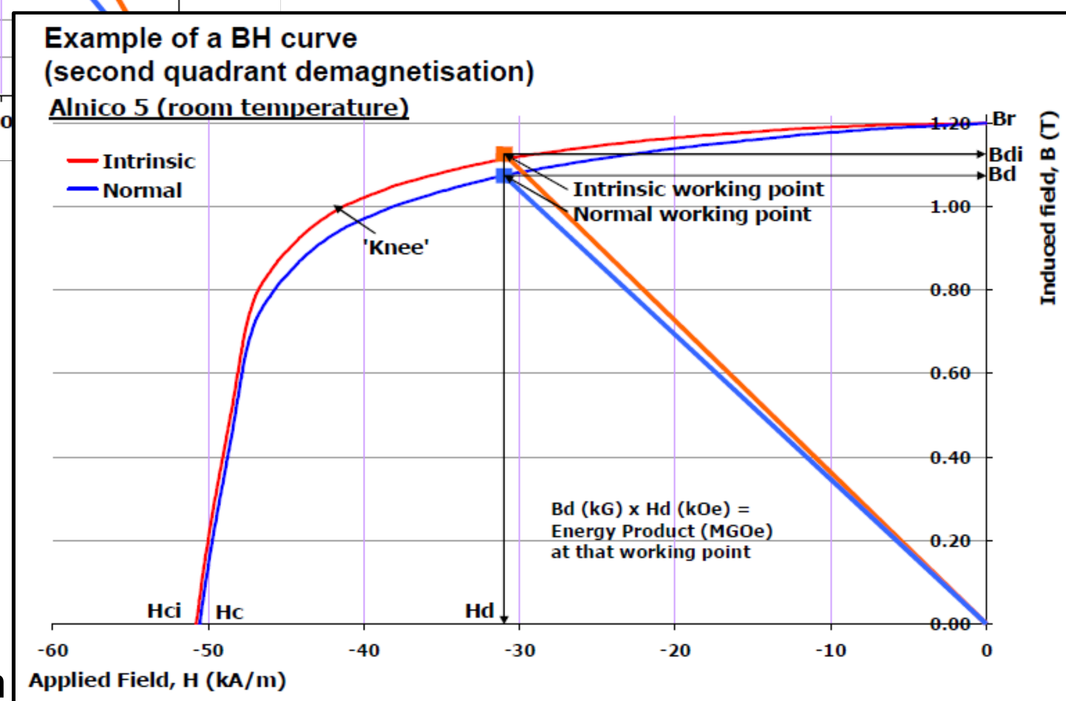


Hysteresis curves – energy product

Square hysteresis loop materials are optimal for attaining high $(BH)_{max}$.



$(BH)_{max}$ up to 80 kJm^{-3}



$(BH)_{max}$ up to 400 kJm^{-3} at RT in commercialy available magnets

graphics source: data sheets from e-magnetsuk.com

Hysteresis curves – load line

- Magnetic materials used as permanent magnets operate on demagnetization curve (second quadrant of the hysteresis loop)
- In the absence of external field the demagnetizing field is the primary factor that influences the magnetization

Within the magnet we have:

$\vec{B}=\mu_0(\vec{H}+\vec{M})$ and $\vec{H}_{demag}=-N\vec{M}$

Substituting ***M*** from the second expression to the first one we obtain:

$$\frac{B}{-H}=\mu_0\frac{1-N}{N}$$

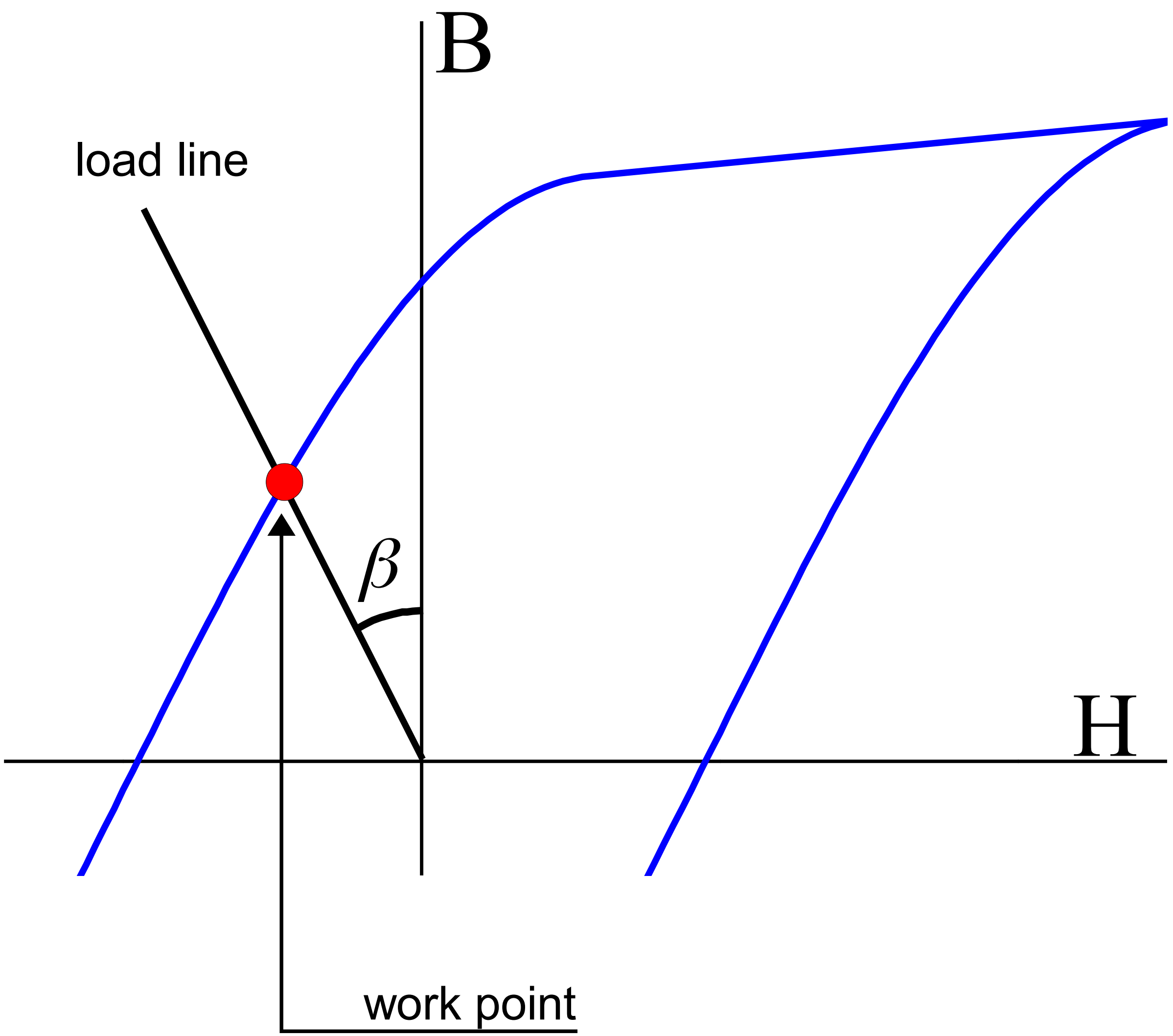
This expresses the relation between H and M for a given demagnetizing factor (in general N is position dependent)

Schliesch, Working Point, Load Lines, Temperature and Stability, 2008.01.28

Hysteresis curves – load line

The above expression can be used to determine the work point of a magnet for a given N.

$$\frac{B}{-H} = \mu_0 \frac{1-N}{N}$$



$$\cot \beta = \mu_0 \frac{1-N}{N}$$

Minor hysteresis curves

- Minor hysteresis – external field does not saturate the sample
- First order reversal curves allow the characterization of interactions between magnetic particles in particulate media (magnetic recording)

J. Appl. Phys., Vol. 85, No. 9, 1 May 1999

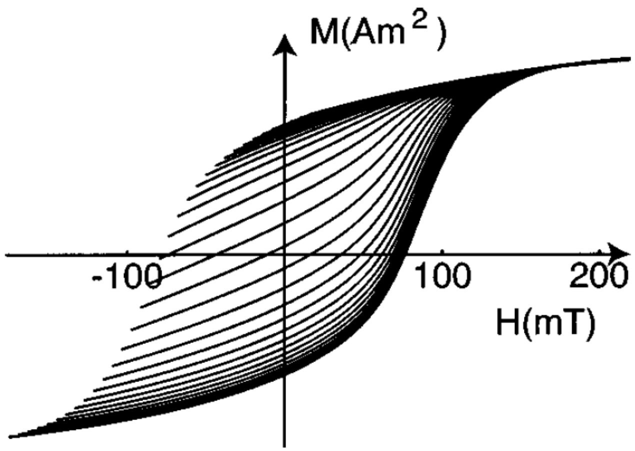
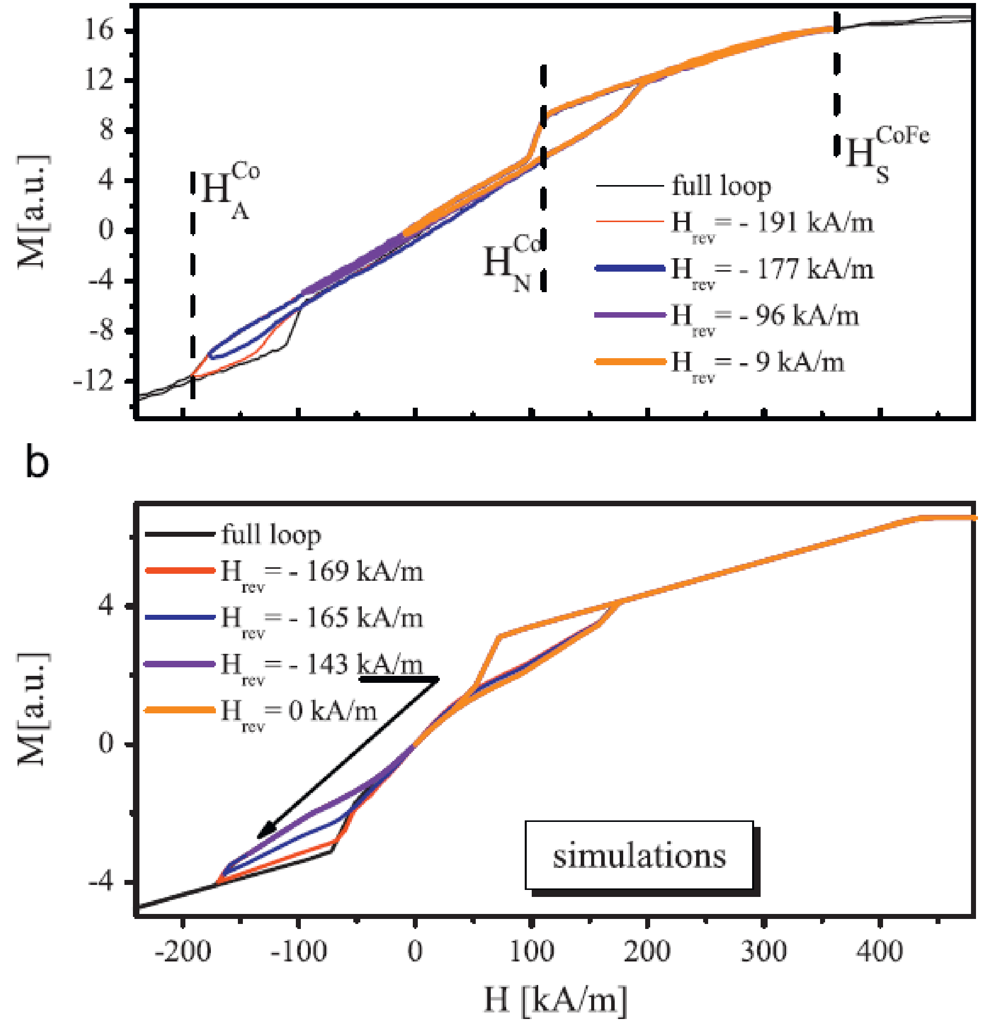


FIG. 1. A set of first order reversal curves (FORCs) for a piece of a typical floppy magnetic recording disk.

Christopher R. Pike, Andrew P. Roberts, and Kenneth L. Verosub



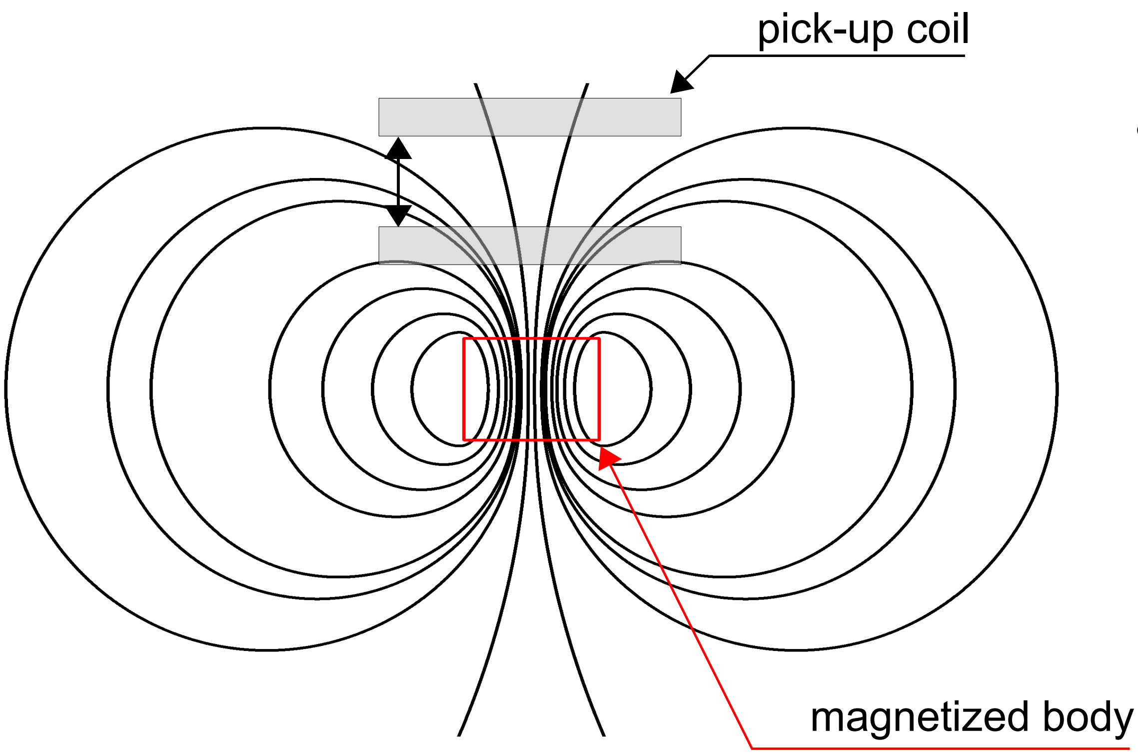
Major and minor hysteresis loops obtained for [CoFe(1.2nm)/ Au(1.2 nm)/Co(0.6nm)/Au(1.2nm)]$_{10}$ ML

Induction methods magnetometry

- In induction methods the Faraday induction is used to measure the magnitude of magnetic moment of the specimen.
- The method is based on the Maxwell equation:

$$\nabla \times \vec{E} = \frac{\partial}{\partial t} \vec{B}$$

- The electromotive force generated in the pick-up coils is proportional to the magnetization of the sample; it depends to on the orientation of the magnetic moment relative to the coils:



- Depending on position of the coils the integral of the induction through the any surface bounded by the coils changes; the voltage (or integral of ***E*** along the coil perimeter) depends on the rate of change of induction ***B***:

$$U = \oint \vec{E}\, dl = \iint \frac{\partial}{\partial t} \vec{B}\; dS$$

*field lines of the magnet of height 3 and width 4 which is infinite in the direction perpendicular to the image

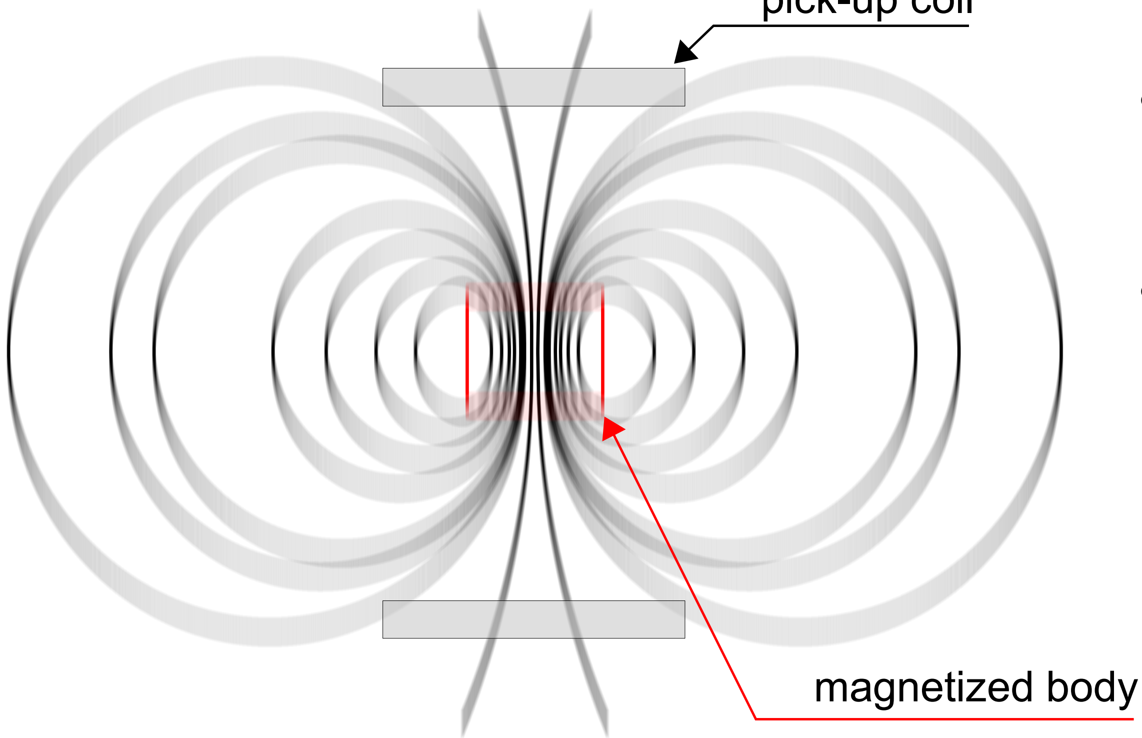
Induction methods magnetometry

•In induction methods the Faraday induction is used to measure the magnitude of magnetic moment of the specimen.

•The method is based on the Maxwell equation:

$$\nabla \times \vec{E} = \frac{\partial}{\partial t} \vec{B}$$

•The electromotive force generated in the pick-up coils is proportional to the magnetization of the sample; it depends to on the orientation of the magnetic moment relative to the coils:



•In measurement one usually uses static pick-up coils while the sample (and its magnetic field) vibrates

•To minimize the influence of the external sources of magnetic field pairs of coils ares used: the variations of the external field add to the signal in one coil and subtract from the signal of the other coil.

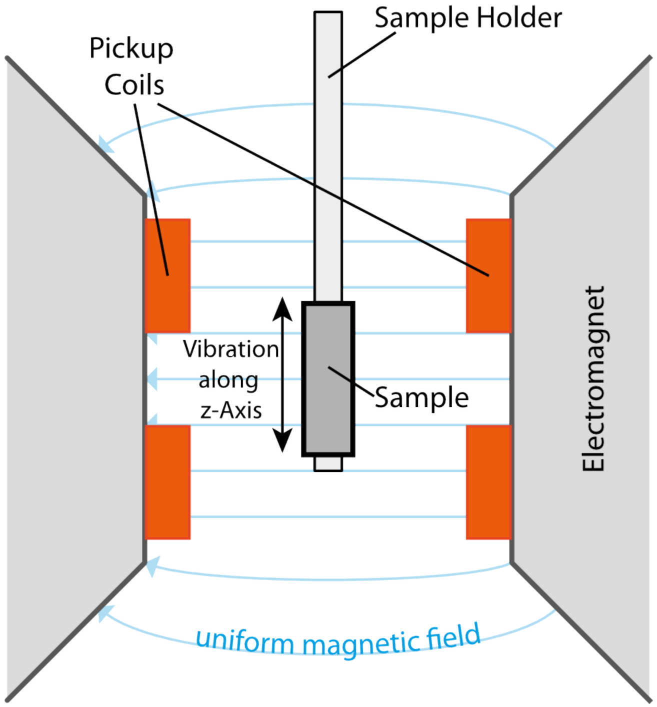
*field lines of the magnet of height 3 and width 4 which is infinite in the direction perpendicular to the image

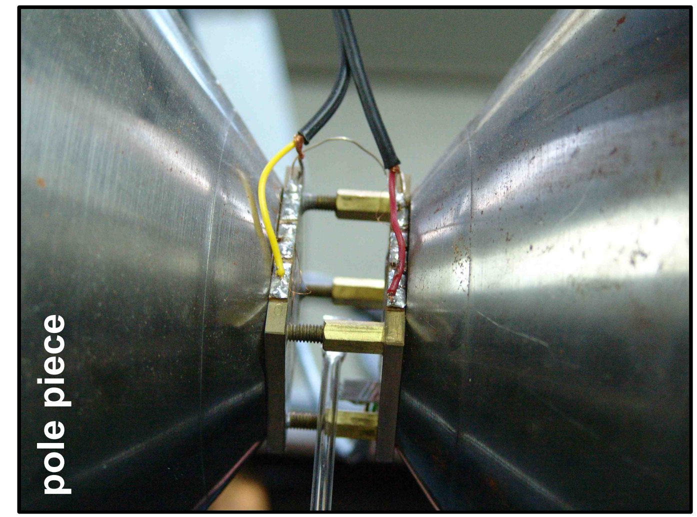
Vibrating sample magnetometer

•VSM is a device used to measure magnetic moment and hysteresis

•It uses the electromagnetic induction and lock-in principle of measurement

Image source - Wikimedia Commons; author: R0oland





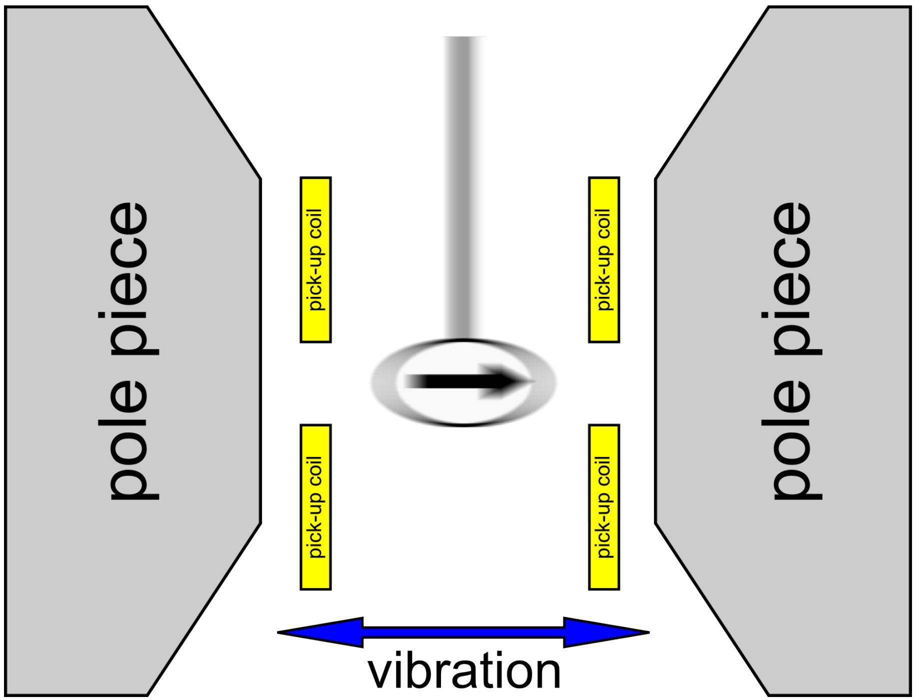
“nonmagnetic” sample holder

•Commercially available VSM magnetometers have sensitivity below 10^{-9} Am^{-2} (depending on acquisition time)

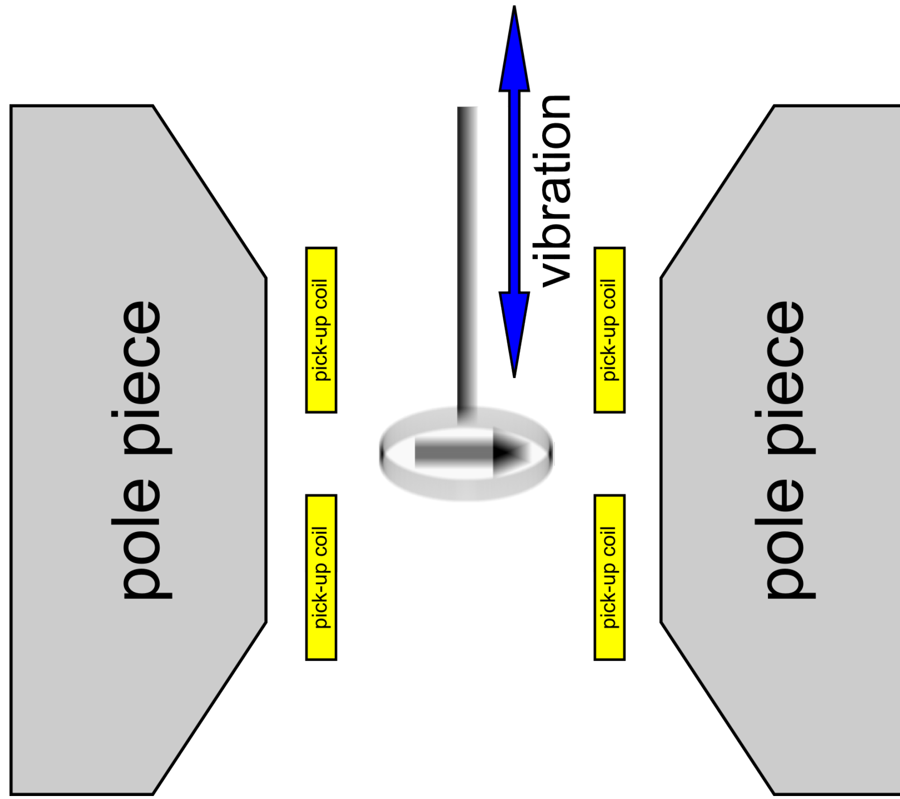
VSM (properly calibrated) measures the absolute value of magnetic moment of the sample

Vibrating sample magnetometer

•VSM can be used in different configurations of sample vibration relative to the external field direction:



parallel configuration
(vibration parallel to the field)



transverse configuration

A. Zieba and S. Foner, Rev. Sci. Instrum. 53, 1344 (1982)

Vibrating sample magnetometer – principle of operation

Lock-in principle of measurement allows the measurement of signals weaker then the noise. We start from the trigonometric identity:

$$\cos(\alpha)\cos(\beta)=\frac{1}{2}\cos(\alpha-\beta)+\frac{1}{2}\cos(\alpha+\beta)$$

•We assume that the sample vibrates with frequency ω giving the signal of that frequency and amplitude A in pick-up coils (through magnetic induction)
•We mix* (multiply) that signal with the signal of frequency ω_1 taken from the generator that drives the sample (we can have some phase difference φ). Using the above identity we have:

$$A\cos(\omega t)B\cos(\omega_1 t+\varphi)=\frac{1}{2}AB\cos(\omega t-\omega_1 t-\varphi)+\frac{1}{2}AB\cos(\omega t+\omega_1 t+\varphi)$$

But $\omega_1=\omega$ (the same generator) so we obtain:

$$A\cos(\omega t)B\cos(\omega_1 t+\varphi)=\frac{1}{2}AB\cos(\varphi)+\frac{1}{2}AB\cos(2\omega t+\varphi)$$

constant in time

fast varying component

Using **low pass-filter** we can filter out the varying component.
There remain only constant voltage which is proportional to the signal from the sample and which is maximum if the phase difference is a multiple of π:

$$\frac{1}{2}AB\cos(\varphi)$$

*mixing can also mean adding signals - additive mixers in audio electronics

Vibrating sample magnetometer – principle of operation cont'd

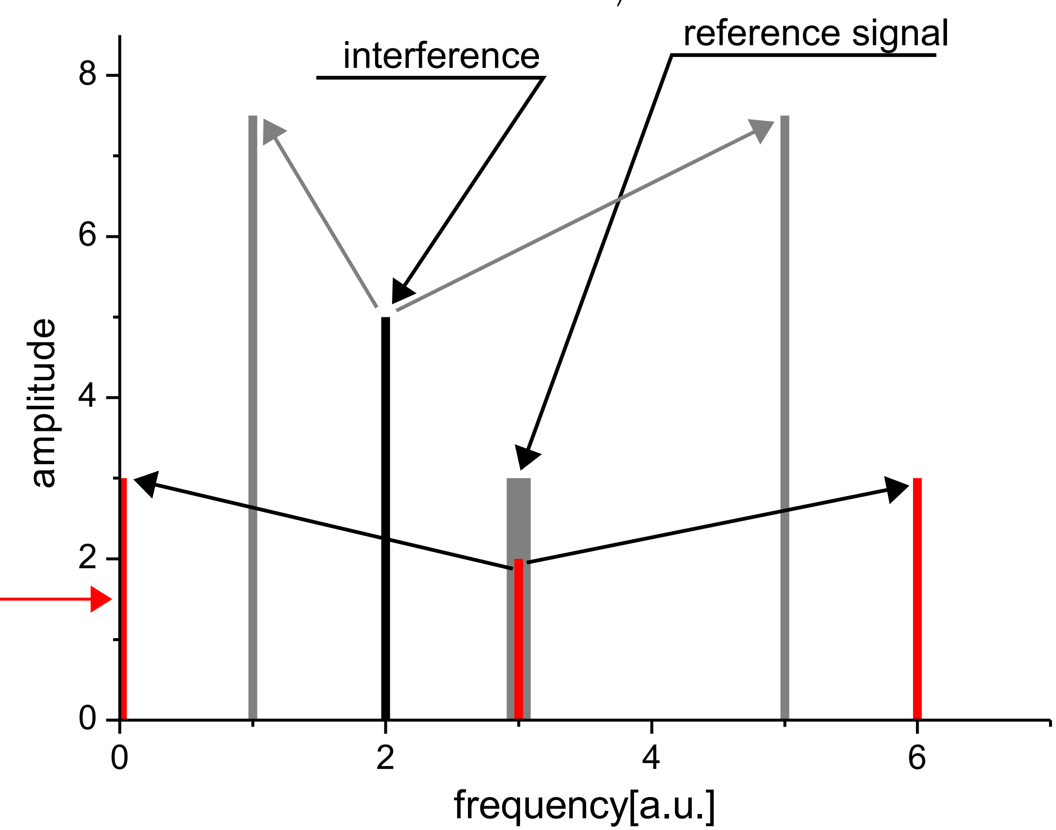
The signal from the pick-up coils can be interfered by external sources of electromagnetic radiation (50 Hz and its harmonics from power lines, car ignition circuits etc.). The signal can be expressed now as:

$$S_{coil} = \sum_i A_i \cos(\omega_i t + \varphi_i)$$

Multiplying again by the reference signal (from generator) we get:

$$S_{coil} B \cos(\omega_r t + \varphi) = \sum_i \left(\frac{1}{2} A_i B \cos((\omega_i - \omega_r) t - \varphi_i) + \frac{1}{2} A B \cos((\omega_i + \omega_r) t + \varphi_i) \right)$$

Only those signals which have a reference frequency contribute to constant voltage received from mixer



Vibrating sample magnetometer – the sensitivity function

- Sensitivity function ***G***(***r***) represents the spatial distribution of detection coil sensitivity – the dependence of VSM signal on sample position. The function G is calculated for given direction of sample motion and given set of detection coils.
- For the moment moving with velocity v(t) the signal induced in coils is:

$$U(t) = \mu G(\vec{r}) v(t)$$

- To obtain time dependence of the signal the above expression must be integrated over the volume of the sample for given amplitude and frequency of oscillations.

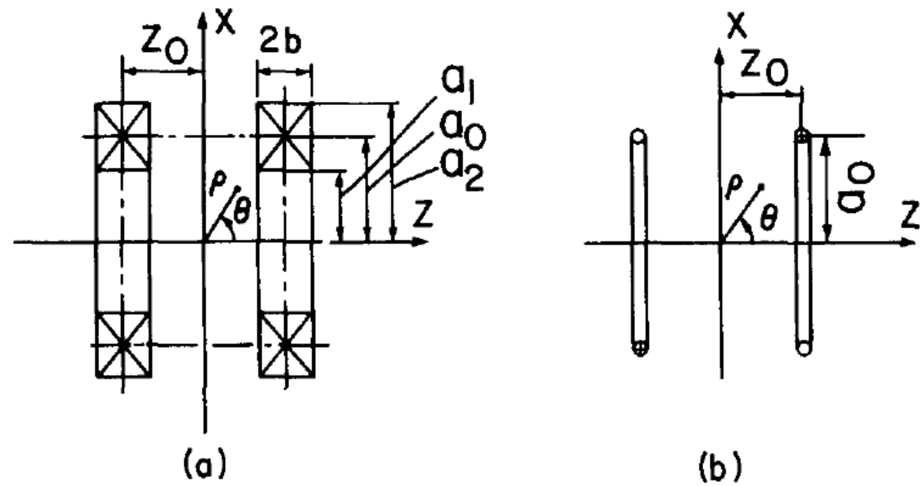


FIG. 1. Axial detection coils: (a) thick rectangular cross-section coils, (b) thin coils. The spherical coordinates ρ and θ give the position of vibrating dipole.

- For *thin coils* spaced 0.866..times their diameter apart the sensitivity function is not maximal but is very flat at center of the coils system – the signal does not depend much on the dipole position.

A. Zieba and S. Foner, Rev. Sci. Instrum. 53, 1344 (1982)

Vibrating sample magnetometer – the sensitivity function

- Sensitivity function ***G***(***r***) represents the spatial distribution of detection coil sensitivity – the dependence of VSM signal on sample position. The function G is calculated for given direction of sample motion and given set of detection coils.

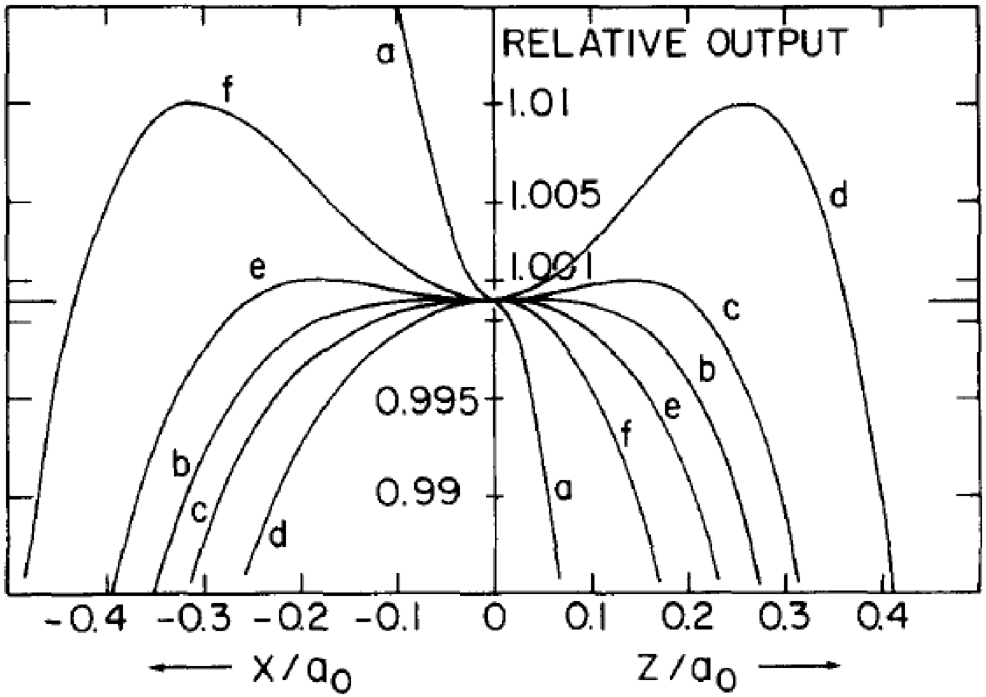


FIG. 3. Relative sensitivity function vs displacement of small sample along the z axis (right-hand side of the figure), and along the x axis (in the perpendicular symmetry plane, left-hand side of the figure) for thin-coil pairs with intercoil distances z_0/a_0 equal to: (a) ½; (b) $\sqrt{3}/2 \cong 0.8660$; (c) 0.8841; (d) 0.9244; (e) 0.8444; (f) 0.7992. The curves (c) and (d) correspond to coils with elongated homogeneity along the z axis, and curves (e) and (f) have elongated homogeneity in the perpendicular plane. Note that the elongated homogeneity corresponds to overcompensation of 0.1% for (c) and (e) and 1% for (d) and (f), respectively.

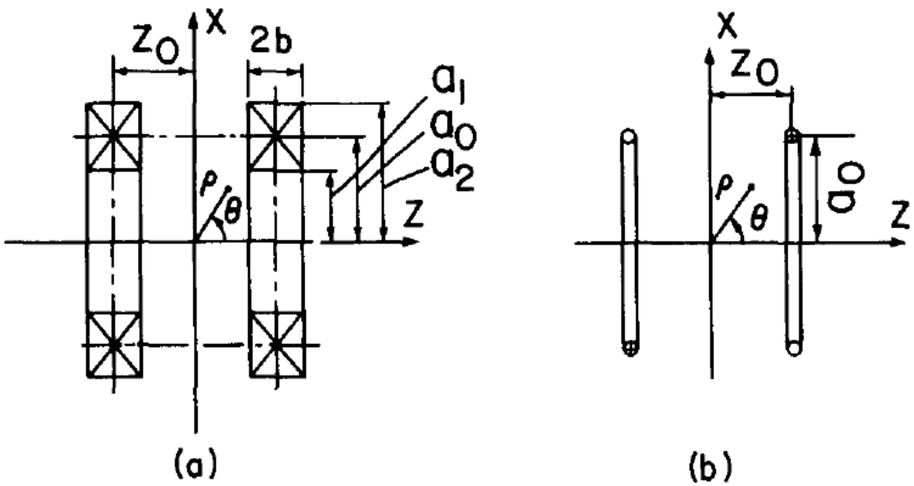


FIG. 1. Axial detection coils: (a) thick rectangular cross-section coils, (b) thin coils. The spherical coordinates ρ and θ give the position of vibrating dipole.

- For *thin coils* spaced 0.866..times their diameter apart the sensitivity function is not maximal but is very flat at center of the coils system – the signal does not depend much on the dipole position.

A. Zieba and S. Foner, Rev. Sci. Instrum. 53, 1344 (1982)

Vibrating sample magnetometer

- There are several common coils configurations each of which is characterized by different sensitivity function.

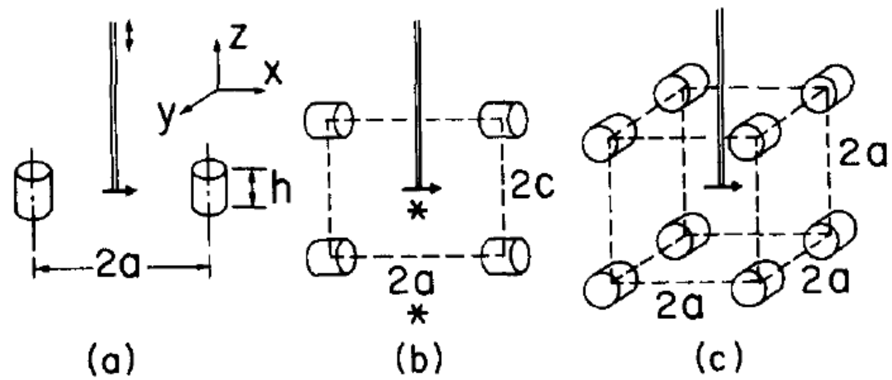


FIG. 5. Three principal transverse detection coil configurations for a VSM with sample vibration perpendicular to the direction of the dipole moment and exhibiting the saddle point at the symmetry center. The axes of the coils are directed along the *z*, *x*, and *y* axes, respectively. The asterisks in Fig. 5(b) indicate the position of the accidental saddle points for that geometry when the two upper coils are removed.

- Depending on the shape of the sample one usually need corrections factors to obtain the signal independent of the shape.

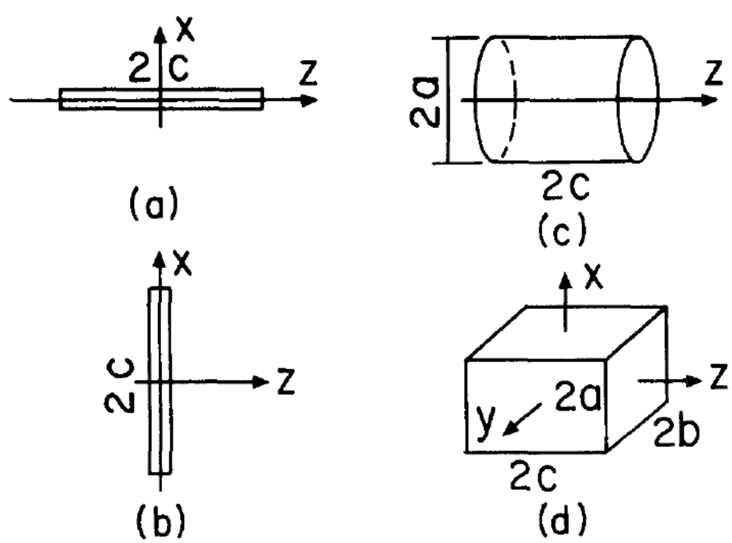
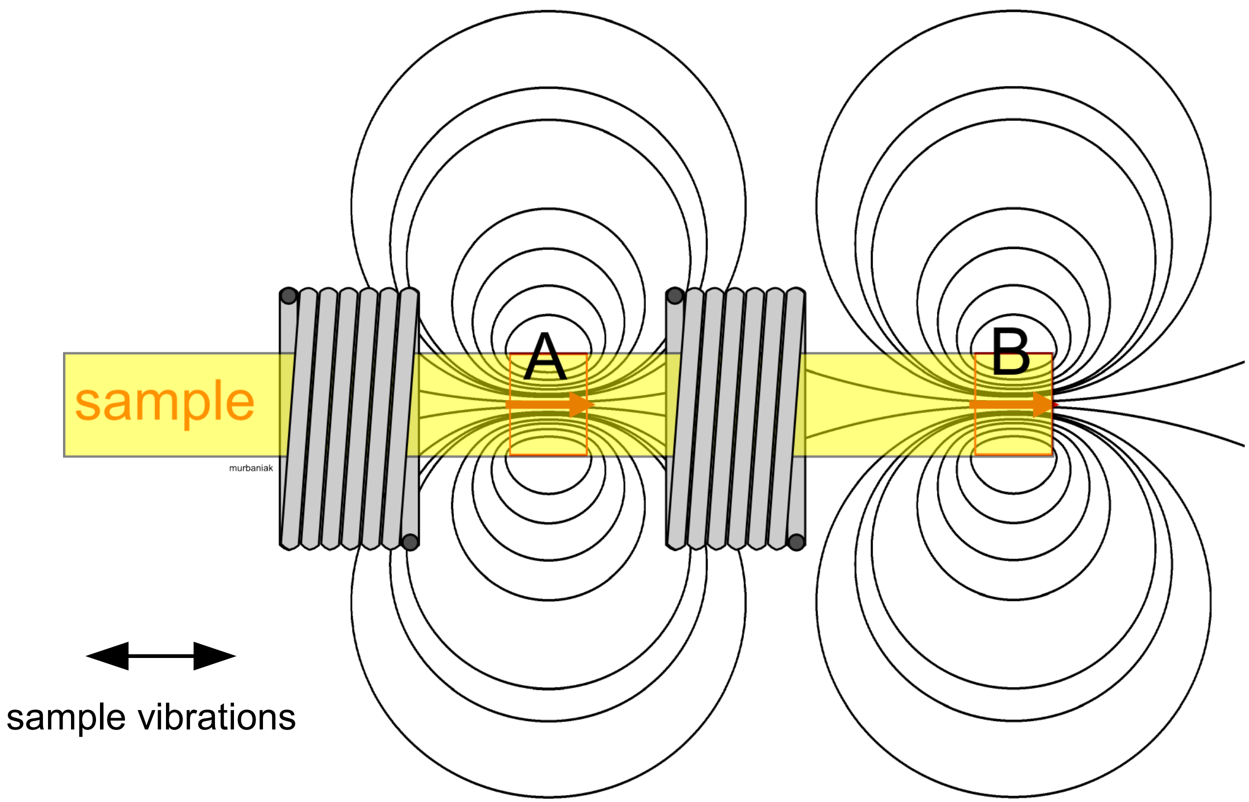


FIG. 7. Examples of regular sample shapes: (a) thin rod (arbitrary cross section) parallel to the *z* axis; (b) thin rod perpendicular to the *z* axis; (c) cylinder; (d) rectangular parallelepiped.

A. Zieba and S. Foner, Rev. Sci. Instrum. 53, 1344 (1982)

Vibrating sample magnetometer – sample size

- The ideal measurement of the M(H) dependence is performed using ellipsoidal (or spherical) samples; in that case the sample can be represented by the point dipole.
- It is often desirable to maintain the integrity of the sample for further measurements so the size of the sample cannot be made small compared to detection coils.



- Parts **A** and **B** of the sample (yellowish bar) contribute oppositely to the signal in the right coil
- For infinitely long sample the signal would be zero
- The sample size should be possibly small (which increases the signal to noise ratio)

A. Zieba and S. Foner, Rev. Sci. Instrum. 53, 1344 (1982)

Vibrating sample magnetometer – sample size

- The ideal measurement of the M(H) dependence is performed using ellipsoidal (or spherical) samples; in that case the sample can be represented by the point dipole.
- It is often desirable to maintain the integrity of the sample for further measurements so the size of the sample cannot be made small compared to detection coils.

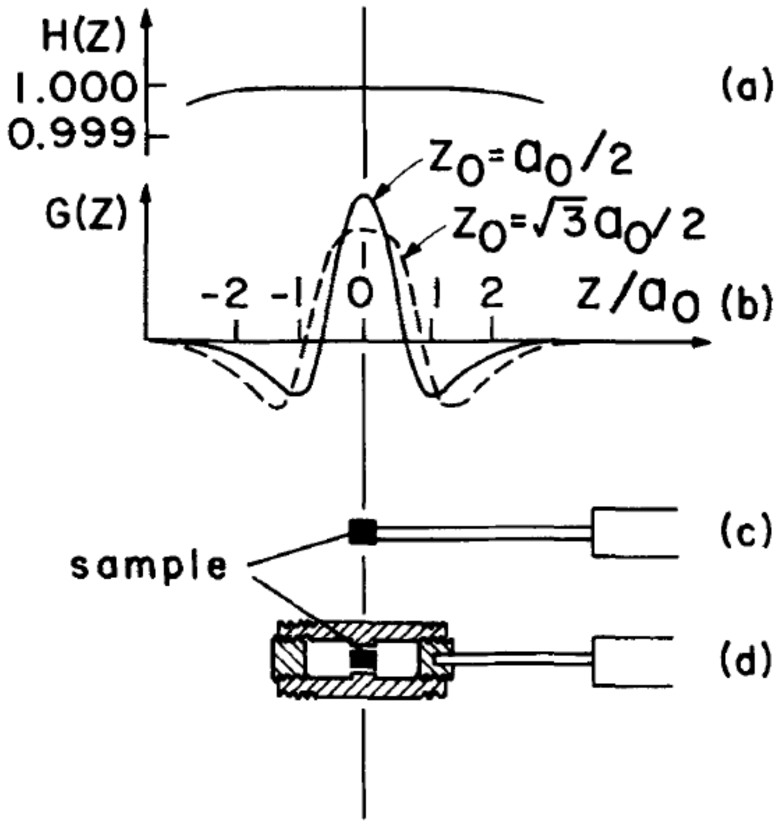


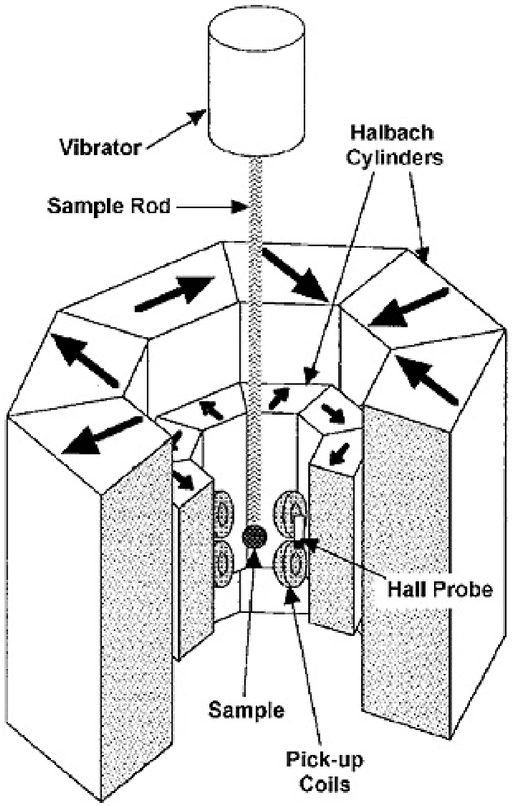
FIG. 11. Sketch comparing relative spatial distribution of: (a) relative external field $B(z)$; (b) sensitivity function $G(z)$ for $z_0 = a_0/2$ and $z_0 = a_0\sqrt{3}/2$; (c) sample support rod; (d) pressure clamp and support rod.

- For typical two coils configurations the material beyond about $z/a_0=1$ produces an output of opposite sign to that of $z/a_0<1$.
- Large support rod structures may interfere with the signal of the sample – the symmetric arrangement of holders will reduce/cancell that effect

A. Zieba and S. Foner, Rev. Sci. Instrum. 53, 1344 (1982)

Vibrating sample magnetometer – cont'd

- VSM is a standard method of measuring hysteresis of thin magnetic films (other popular method is a Kerr effect magnetometry)
- The VSM may use permanent magnets configurations like Halbch cylinders (see L.2) instead of electromagnets.



- There is an interesting development of VSM called an alternating gradient magnetometer (**AGM**):

-the sample is placed in the static magnetic field which is locally modified by a small varying field of current coils

-this field creates field gradient which exerts a sinusoidally varying force on the sample

-the displacement of the sample is sensed by the piezoelement which is a part of the sample holder

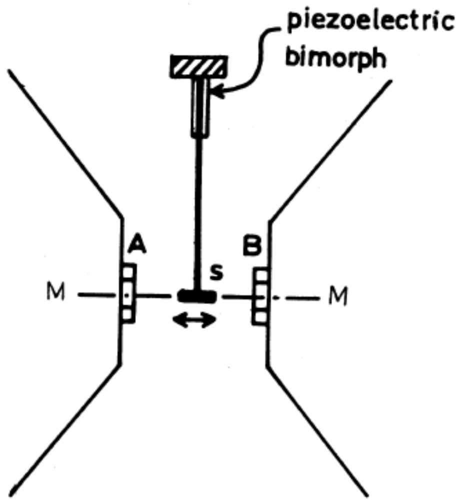


Fig.2 Simple AGM. M, M=electromagnet pole pieces; A, B=gradient field coils, connected series opposing; S=sample

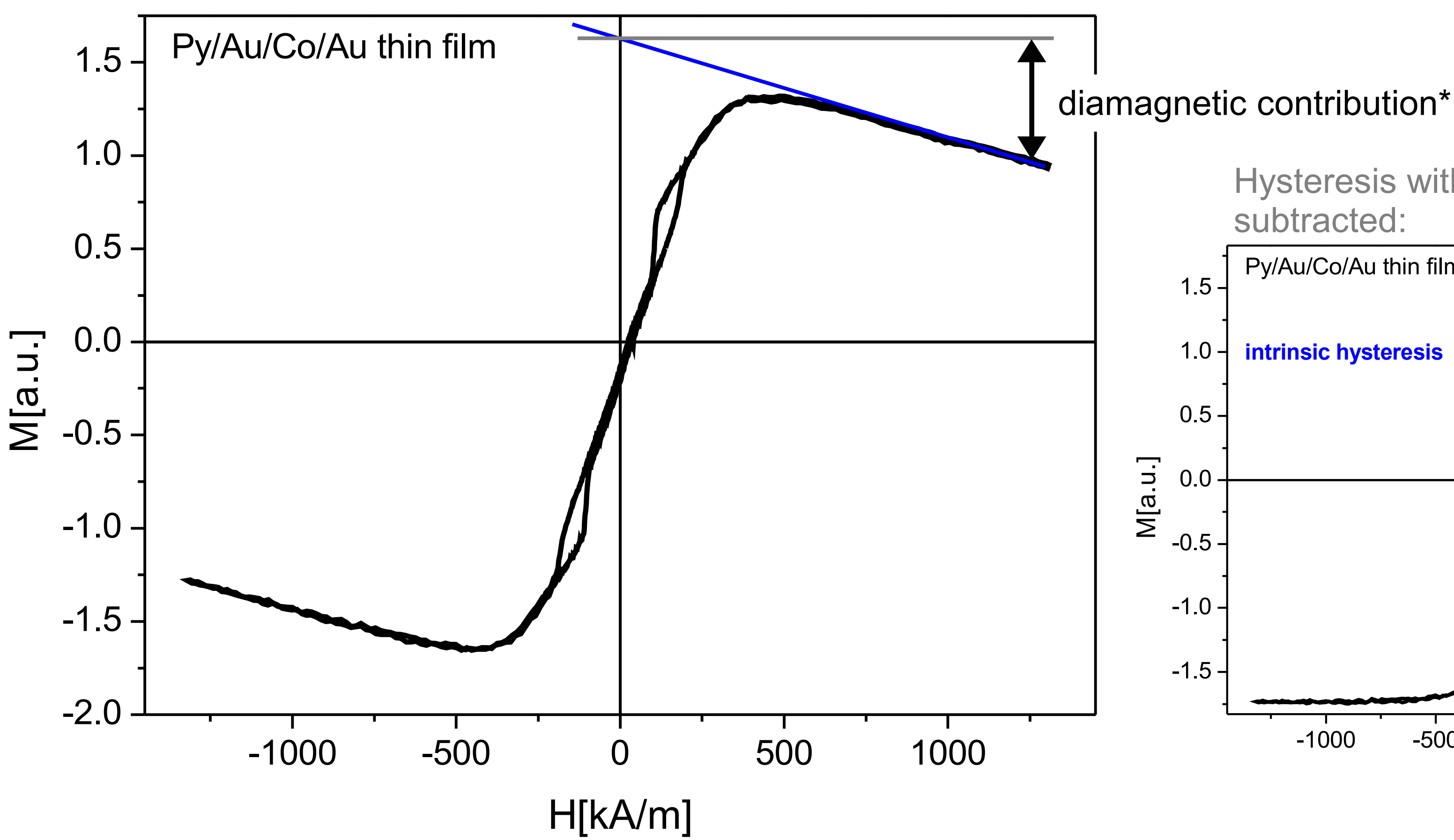
-AGM is not necessarily suitable for soft magnetic materials

[7] C.D. Graham, J. Mater. Sci. Technol. **16**, 100 (2000)

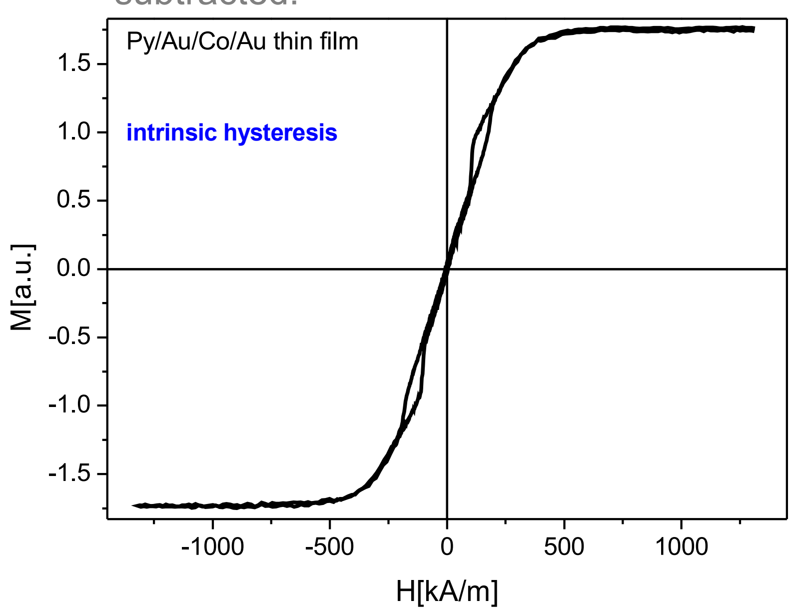
image source: http://www.tcd.ie/Physics/Magnetism/Research/instrumentation.php

Vibrating sample magnetometer – cont'd

- The shape of the sample influences the measurement of hysteresis with VSM (demagnetizing field) – intrinsic field differs from the applied field; the effect of demagnetizing field can be properly taken into account in ellipsoidal samples (see L2) or in their limiting cases (elongated rod, thin film)
- With VSM measurements of small volume samples (like thin films) it is often necessary to subtract the diamagnetic contribution from the sample holder



Hysteresis with diamagnetic contribution subtracted:



*in typical macroscopic thin film samples the volume of the substrate may be 10^6 times the volume of the magnetic film

Vibrating sample magnetometer with SQUID

•The superconducting quantum interference device (SQUID) can be used as a flux to voltage converter in the VSM magnetometer.

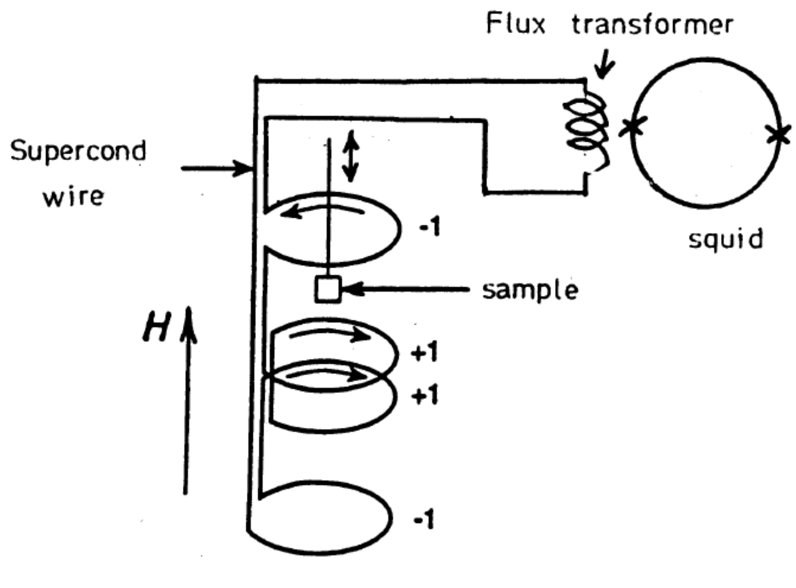


Fig.3 Diagram of SQUID magnetometer

Image from: [7] C.D. Graham, J. Mater. Sci. Technol. **16**, 100 (2000)

•The sample vibrates between pick-up coils placed in an external magnetic field

•The coils are connected so as to insure that the change in the applied field produces no net flux in the coils (second order gradiometer [7])

•The coils are *coupled inductively to SQUID* element which converts flux changes caused by the movement of the sample to voltage

•The voltage is measured with lock-in principle

•The sensitivity of commercial VSMs wit SQUID can exceeds 10^{-11}Am^2*.

It corresponds roughly to 0.1mm×0.1mm×1nm piece of iron

•Resolution is field dependent (noise from field source)

*MPMS SQUID VSM from **Quantum Design**

Vibrating sample magnetometer with SQUID

- The superconducting quantum interference device (SQUID) can be used as a flux to voltage converter in the VSM magnetometer.

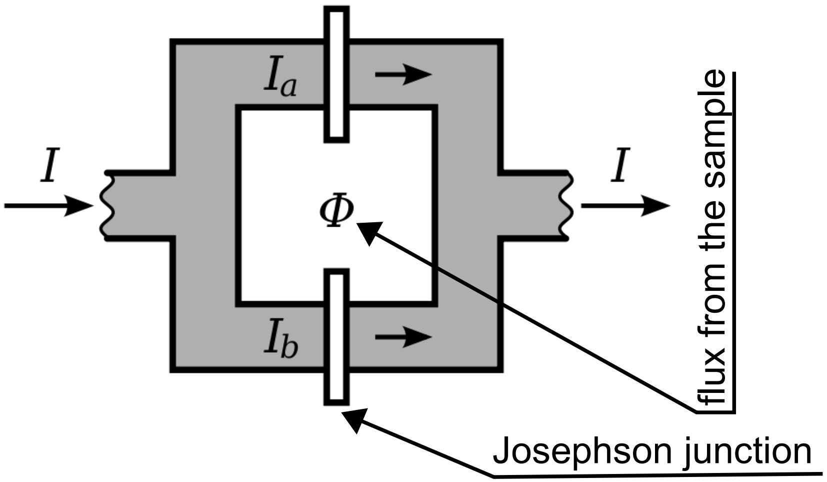


Image source - Wikimedia Commons; author: Miraceti

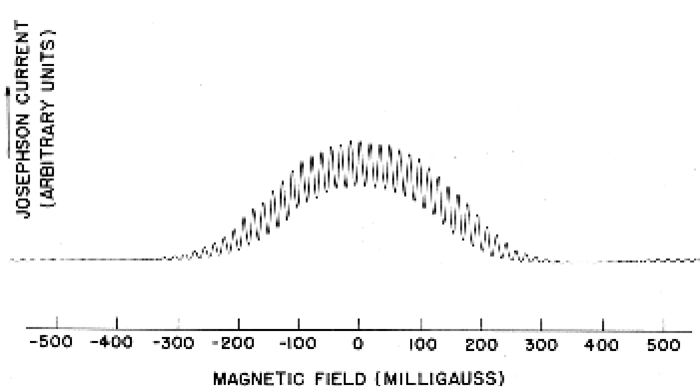
- Magnetic flux passing through a superconducting current circuit is quantized:

$$\varphi_0 = \frac{h}{2e} \approx 2.067 \times 10^{-15}\, T\, m^2$$

- The total current going through the junction can be show to be [8]:

$$J_{tot} = J_0 \cos\left(\frac{2\pi e}{h}\varphi\right)$$

- The current through the junction oscillates as the function of the flux through the superconducting coil. Since the junctions have a resistance we can measure the voltage drop across the device:



Counting oscillations one can determine the flux through the Josephson device

[8] SQUIDs: A Technical Report, http://rich.phekda.org/squid/technical/index.html#toc

Vibrating sample magnetometer with SQUID

- The magnetometers with SQUID are the most sensitive devices of this kind. They can measure field as low as 10^{-14} T* which is *less than fields associated with human brain activities*.
- The resolution of the device can be much better than the magnetic flux quantum.
- The sensitivity of the device allow the measurement of hysteresis loops of single particles:

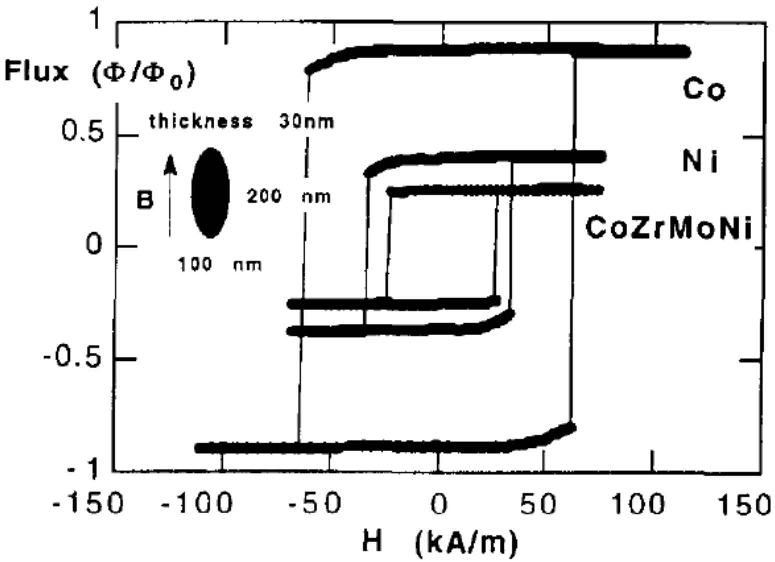


Fig. 3. Hysteresis loops of type A – Co, Ni and CoZrMoNi particles, ellipticity 200×100 nm, thickness 30 nm, $T = 0.2$ K.

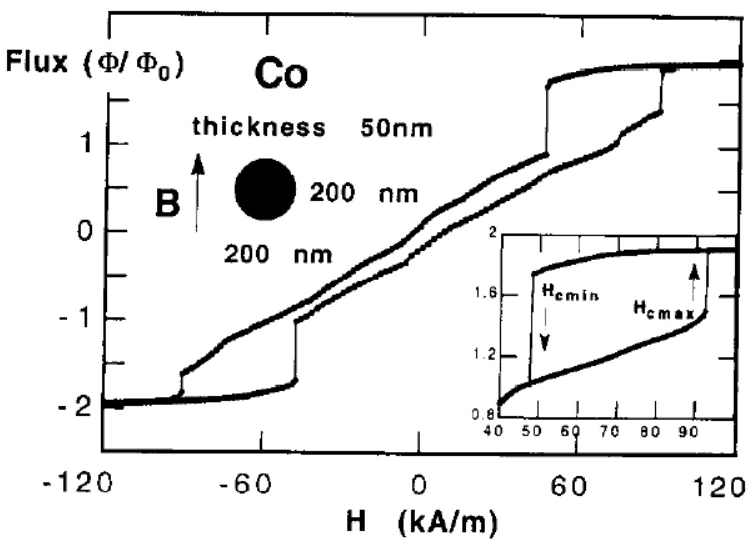


Fig. 6. Hysteresis loop of type C – Co particles, ellipticity 200×200 nm, thickness 50 nm, $T = 0.2$ K. The inset shows a minor loop of this particle.

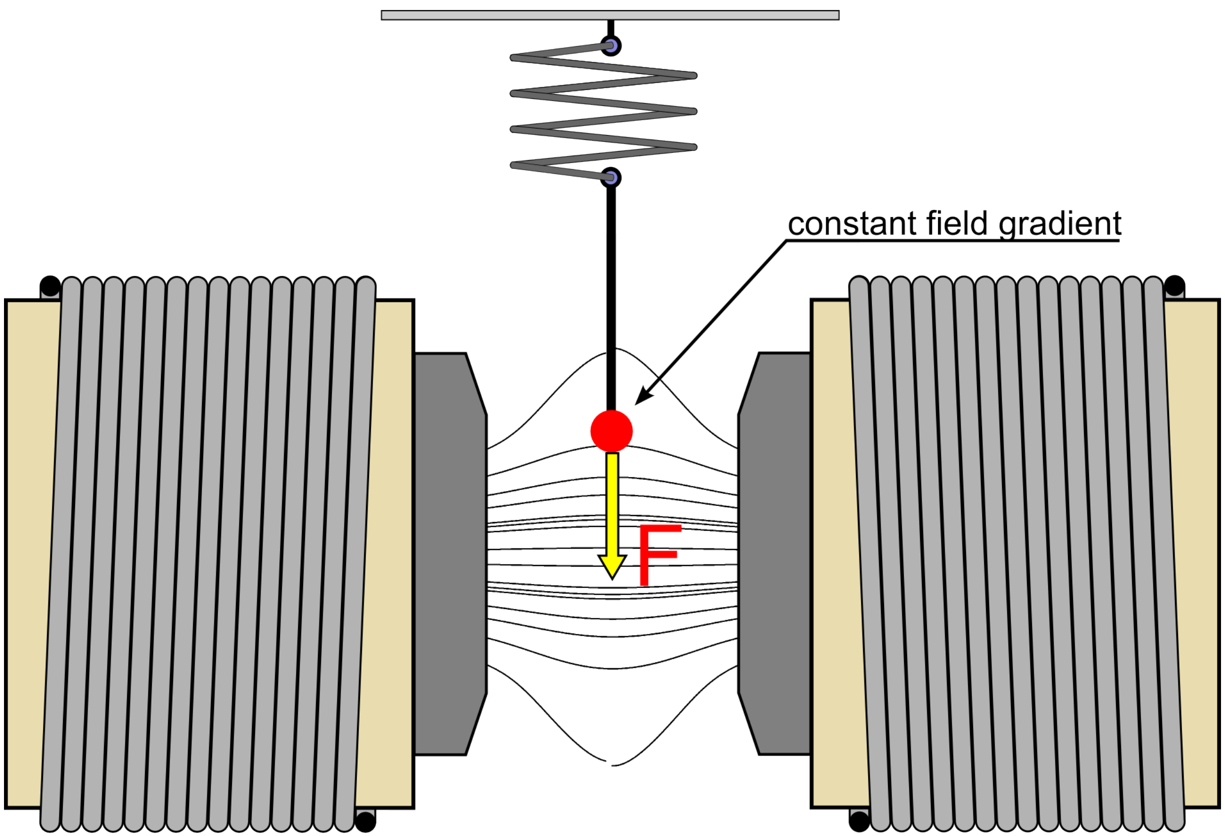
Resolution of $10^{-4}\Phi_0$ – behavior of about 10^6 spins.

W. Wernsdorfer et al., Journal of Magnetism and Magnetic Materials 145 (1995) 33-39

*http://hyperphysics.phy-astr.gsu.edu/hbase/solids/squid.html#c2

Magnetic scales

- The magnetic moment and susceptibility can be measured with magnetic scales [10].
- They utilize the force exerted by a magnetic field with a gradient (see L.2) on magnetized body.
- The **Faraday method** utilizes the force on a small sample placed in virtually constant field gradient:

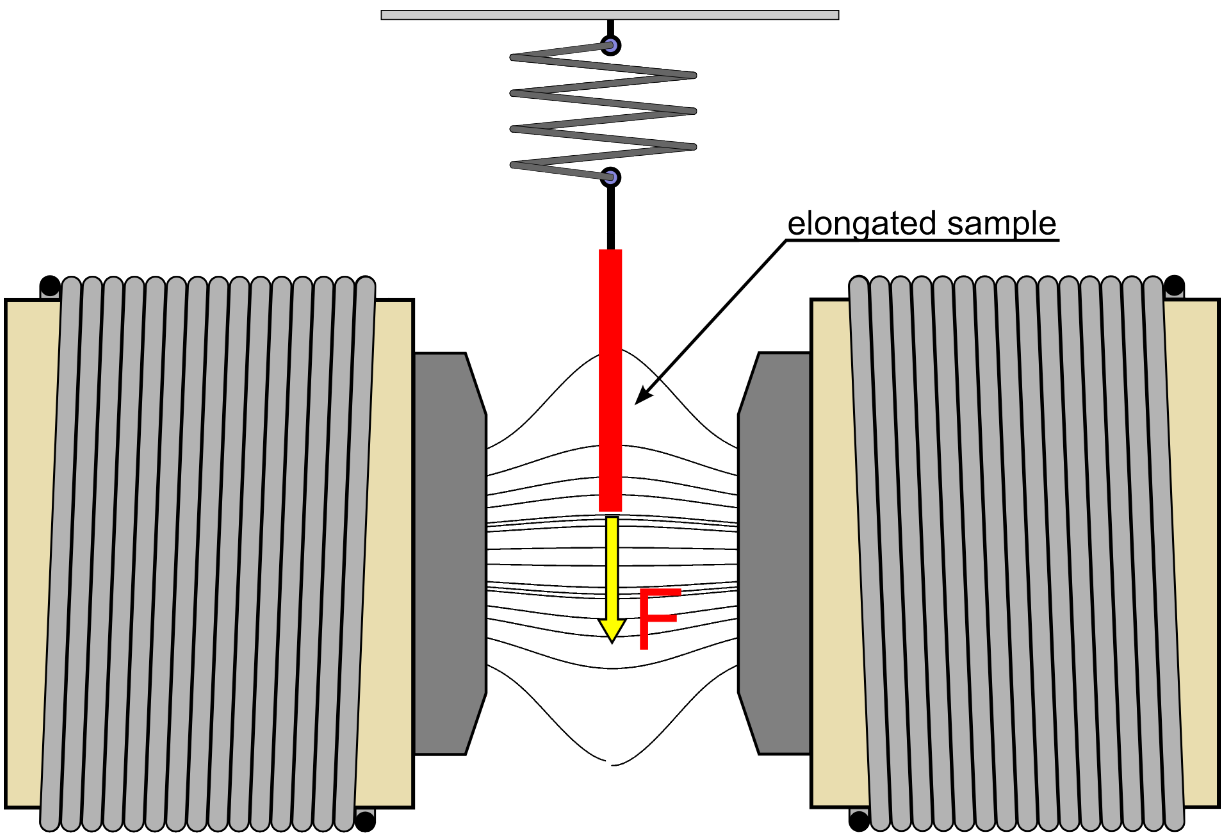


The force on magnetic particle is (L.2) (*V* - volume):

$$\vec{F} = \frac{1}{2\mu_0} \chi V \nabla\left(B^2\right)$$

Magnetic scales

- The **Gouy method** utilizes the force on a long bar sample with one end placed in electromagnet and the other one outside, in very small field.
- The method is used mainly for diamagnetic and paramagnetic substances.



The force on paramagnetic particle is (L.2) (*V* - volume):

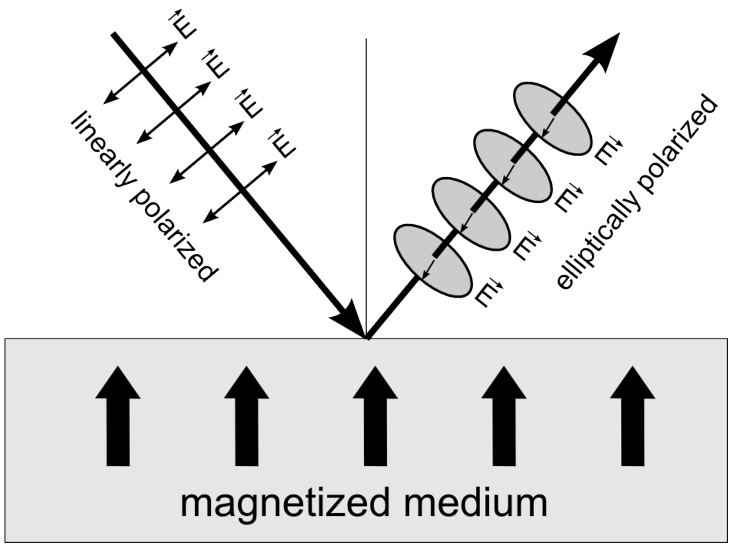
$$\vec{F} = \frac{1}{2\mu_0}\chi V \nabla(B^2)$$

Integrating this along the length of the sample we get [10]:

$$\vec{F} = \frac{1}{2}\chi V \left(H_{max}^2 - H_{min}^2\right) \approx \frac{1}{2}\chi V H_{max}^2$$

Kerr effect magnetometers

Kerr effect – change of polarization of light reflected from the surface of magnetized material*



•Using polarizer for incident light and analyzer for reflected light one obtains voltage which is, for small angles of rotation *proportional to the magnetization*:

$$I = I_0 \cos^2(\theta)$$ Malus' law

$$\cos^2(\theta) = 1 - \theta^2 + \ldots$$

•The effect can be used to measure magnetic hysteresis of thin films (or near surface layers of bulk materials).

•The penetration depth is determined by skin depth for a given radiation frequency** and resistivity of the material.

•The Kerr magnetometers can be extremely sensitive (1.2×10^{-18} Am^2 **!** [9])

**$\delta = \sqrt{\dfrac{2\rho}{2\pi \mu_0 \mu_r f}}$

*the presence of external field and nonmagnetic conductor is enough

Kerr effect magnetometers – cont'd

Exemplary Kerr effect magnetometer data.

Nanotechnology **22** (2011) 095302 P Kuświk *et al*

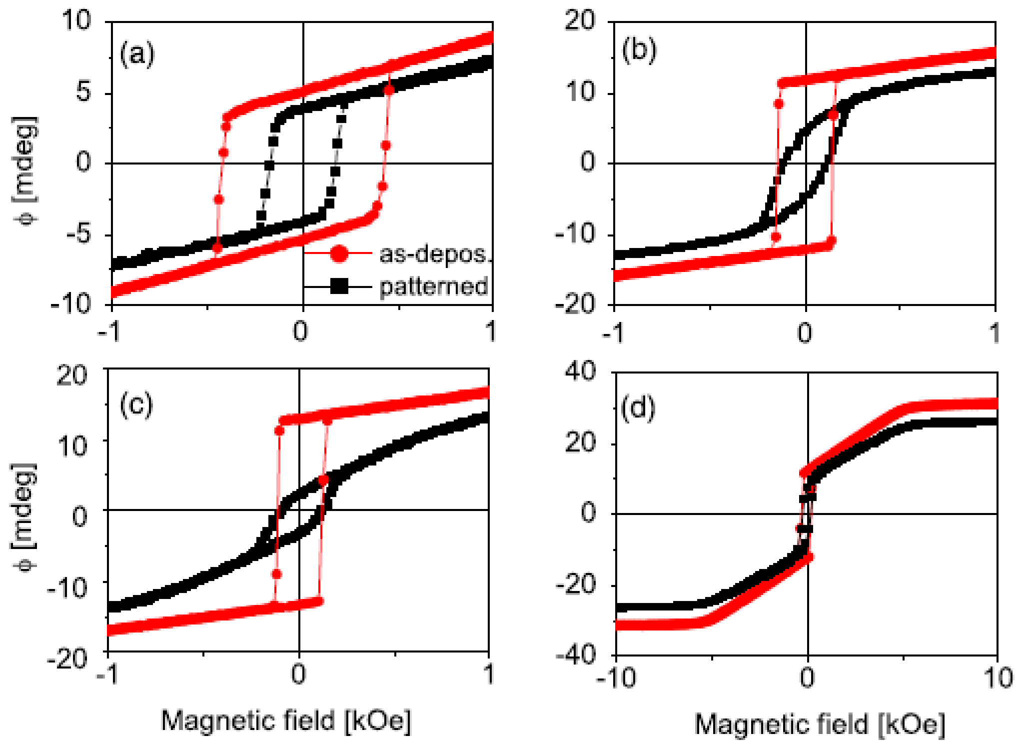
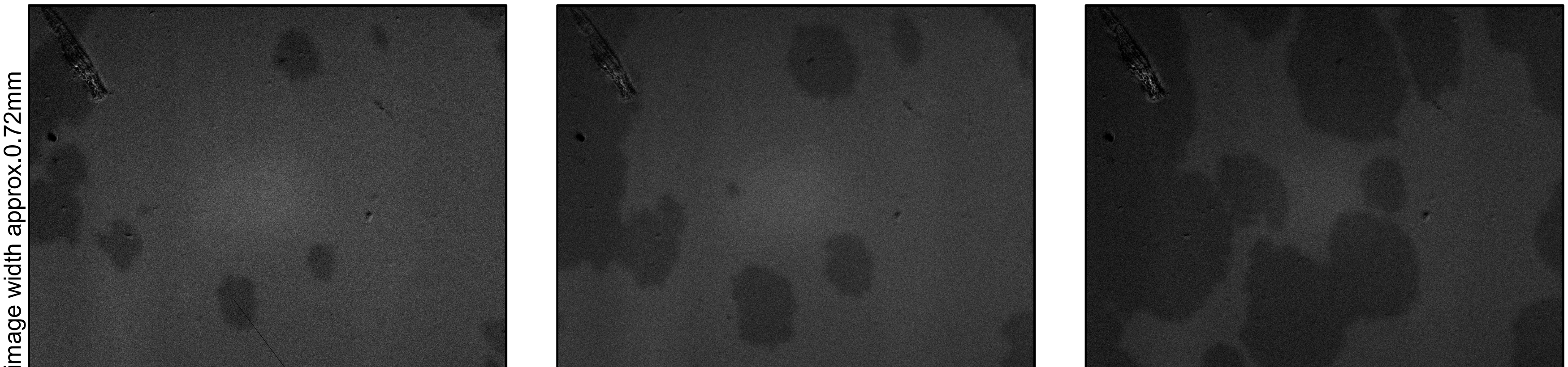
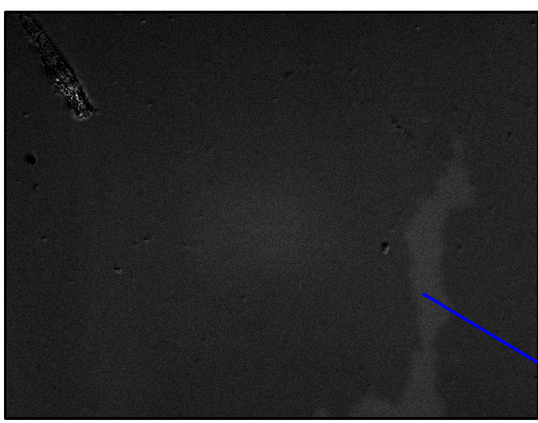


Figure 4. Exemplary P-MOKE hysteresis loops measured for different t_{Co} before and after He^+ ion bombardment through the colloidal mask with $D = 10^{16}$ He^+ cm^{-2}. (a) $t_{Co} = 0.56$ nm; (b) $t_{Co} = 1.2$ nm; (c) $t_{Co} = 1.36$ nm; (d) $t_{Co} = 1.2$ nm in an extended magnetic field range. The linear part of (d) with saturation at 6 kOe corresponds to magnetization reversal of the buffer layer $[Ni_{80}Fe_{20}(2\text{ nm})/Au(3\text{ nm})]_{11}$, the central part to the one of the Co layer.

Kerr effect magnetometers – cont

Combined with optical microscopy Kerr magnetometry can provide information on the magnetic structure of the materials and/or about the dynamics of magnetization processes.





Dynamics of magnetization in graded Co/Au/Co multilayer with perpendicular magnetic anisotropy* and in constant magnetic field.

*natural contrast

Bibliography:

[1] A. Weiss, H. Witte, Magnetochemie, Verlag Chemie, Weinheim 1973
[2] A. Aharoni, Introduction to the Theory of Ferromagnetism, Clarendon Press, Oxford 1996
[3] K. Fredenhagen, Elektrodynamik, University Hamburg, notes
[4] W. Hauser, Introduction to the Principles of Electromagnetism, Addison-Wesley, 1971
[5] ASTM, Standard Terminology of Symbols and Definitions Relating to Magnetic Testing, A 340 – 03a, 2003
[6] R.M. Bozorth, Ferromagnetism, D.van Nostrand Company, 1951
[7] C.D. Graham, J. Mater. Sci. Technol. **16**, 100 (2000)
[8] SQUIDs: A Technical Report, http://rich.phekda.org/squid/technical/index.html#toc
[9] M. Cormier, J. Ferré, A. Mougin, J.-P. Cromières, and V. Klein, Rev. Sci. Instruments **79**, 033706 (2008)
[10] S. Tumanski, Handbook of Magnetic Measurements, CRC Press, Boca Raton, FL 2011