Giant magnetoresistance

Magnetic materials in nanoelectronics

- properties and fabrication



Giant magnetoresistance

- 1. Introduction
- 2. Magnetoresistance
- 3. Giant magnetoresistance

Normal metal resistance intrinsic resistivity – due solely to phonons in a perfect lattice [5]

Bloch-Grüneisen formula [5]:

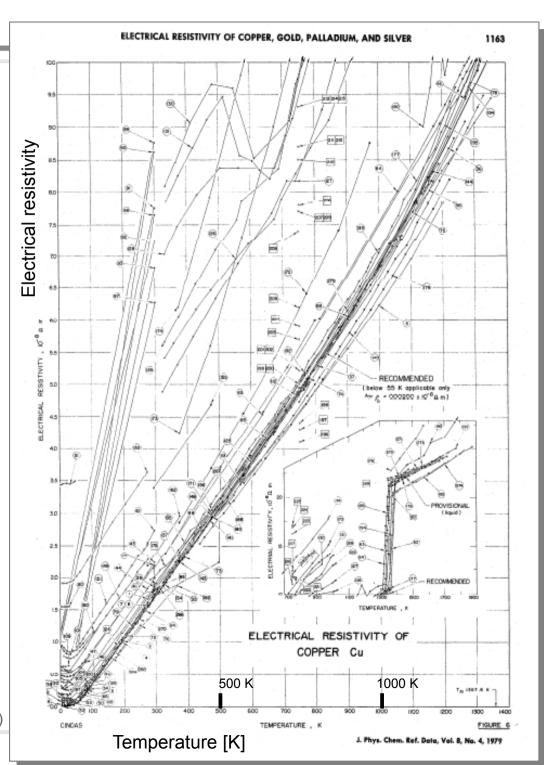
$$\rho_i(T) = \frac{C}{M\theta} \left(\frac{T}{\theta}\right)^5 \int_0^{\theta/T} \frac{z^5 e^z}{(e^z - 1)^2} dz$$

 θ – Debye temperature, M – atomic mass

- the formula was derived for monovalent metal with spherical Fermi surface and phonon spectrum from Debye model
- despite this the formula is useful for initial analysis of experimental results

Note the huge number of references used to create the diagram to the right

image from: R.A. Matula, J.Phys.Chem.Ref.Data 8, 1147 (1979)



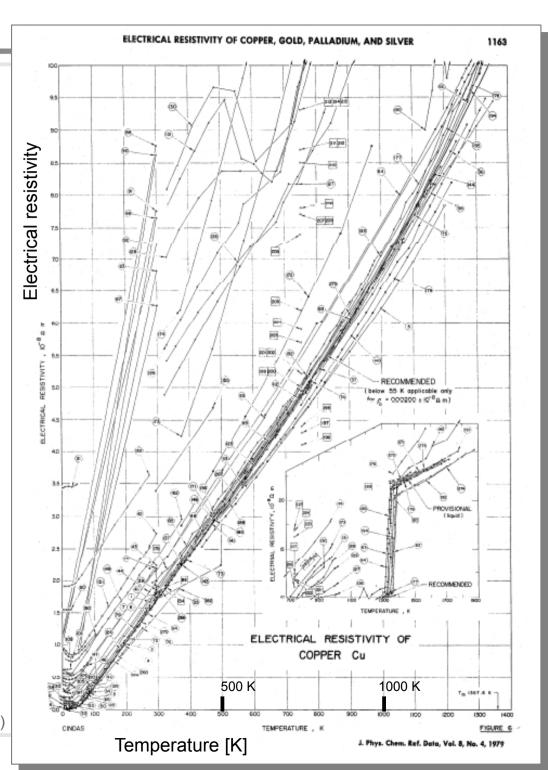
Bloch-Grüneisen equation for limiting temperatures [5]:

$$T \to 0: \rho_i(T) \to 124.431 \left(\frac{T}{\theta}\right)^5$$
$$T \to \infty: \rho_i(T) \to \frac{C}{4M\theta} \left(\frac{T}{\theta}\right)$$

$$T \rightarrow \infty : \rho_i(T) \rightarrow \frac{C}{4M\theta} \left(\frac{T}{\theta}\right)$$

| | θ at 298 K [K] [5,11] | |
|----|---------------------------|--|
| Au | 178±8 | |
| Ag | 221 | |
| Cu | 320 | |
| Fe | 467 [11] | |
| Со | 445 [11] | |
| Ni | 450 [11] | |

image from: R.A. Matula, J.Phys.Chem.Ref.Data 8, 1147 (1979)



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Introduction

Mathiessen rule (empirical) – the total resistivity of a specimen is a sum of resistivities due to phonons, impurities, defects, etc. (approx. valid if scattering events are independent)

$$\rho = \rho_{phonons} + \rho_{impurities} + \dots$$

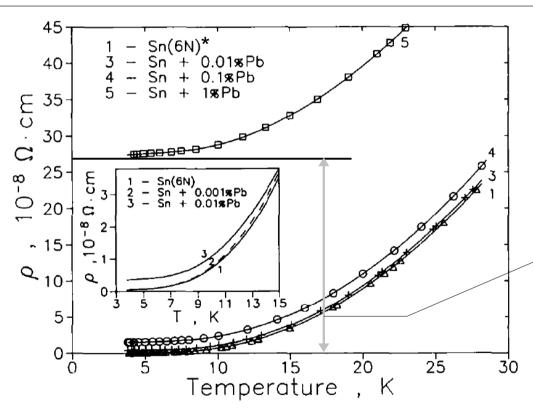


Fig. 1. Dependence of the electrical resistivity of lead-doped tin on temperature at various concentrations of lead admixtures.

- tin + lead, T≤28 K
- the resistivity of the Sn+Pb mixture increases with Pb concentration
- the dependence of a residual resistivity on Pb concentration is not linear

image from: R. Wawryk, J. Rafalowicz, Cz. Marucha, K. Balcerek, International Journal of Thermophysics 15, 379 (1994)

Size effect in resistivity

In thin films* the resistivity depends additionally on the thickness *t* of the layer

- in bulk samples only small fraction of electrons experiences the scattering at the outer boundaries
- in thin films (panel b) the contribution from surface scattering becomes important and the resistivity increases

The theoretical determination of the $\rho(t)$ dependence is very difficult. The approximate Fuchs- Sondheimer theory predicts the following $\rho(t)$ dependence [6,34]:

$$\frac{\sigma_0}{\sigma} = 1 + \frac{3}{8} \frac{\lambda}{t} (1 - p) \qquad (\frac{t}{\lambda} \gg 1)$$

$$\frac{\sigma_0}{\sigma} = \frac{4\lambda}{3t(1+2p)\ln(\lambda/t)} \qquad \left(\frac{t}{\lambda} \ll 1\right)$$

 λ – mean free path (mfp), p – fraction of electrons that are specularly** reflected at the outer boundaries, σ_0 – bulk conductivity

substrate (b substrate **speculum - Latin. mirror

^{*}a film is said to be thin if the mean free path of current carriers is comparable with its thickness (compare the definition of magnetic thin film)

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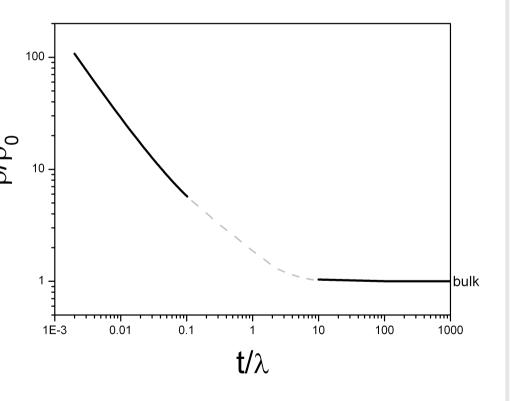
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Introduction

Size effect in resistivity

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in thin films the contribution from surface scattering becomes important and the

resistivity increases

thermal evaporation onto 500-nm thick SiO₂ on Si(100) substrates in an ultra high vacuum UHV

 Note that the crystalline structure of the films can change with thickness too.

> image (fragment) from: H.-D. Liu, Y.-P. Zhao, G. Ramanath, S.P. Murarka, G.-C. Wang, Thin Solid Films **384**, 151 (2001)

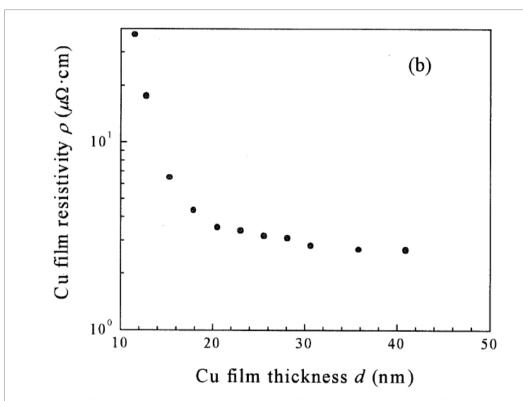


Fig. 2. (a) Sheet resistance R of Cu films vs. deposition time $t_{\rm d}$ plotted in semi-log scale. and (b) Cu film resistivity ρ plotted as a function of film thickness d on a semi-log scale. The error bars are less than the legend size.

Dependence of resistivity on temperature in magnetic metals:

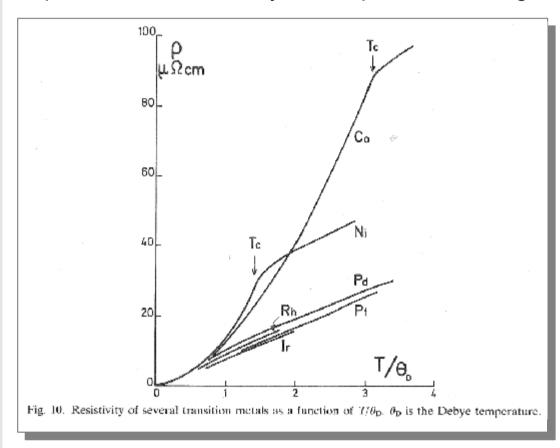
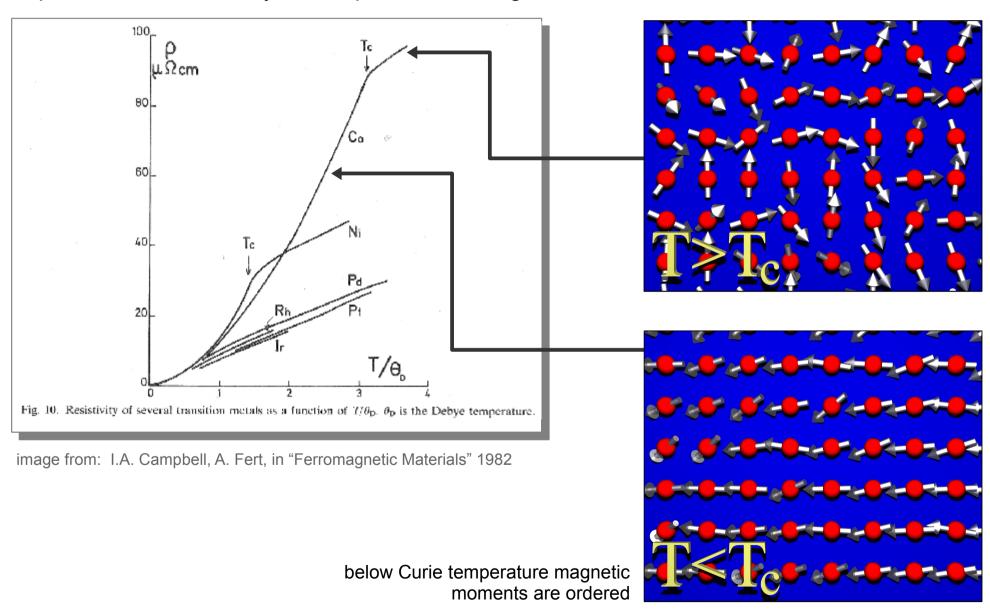


image from: I.A. Campbell, A. Fert, in "Ferromagnetic Materials" 1982

- below Curie temperature T_c
 resistivity of a magnetic metals
 increases with temperature faster
 than above it
- below T_c temperature increase leads to increased magnetic disorder
- resistivity and magnetic order correlate

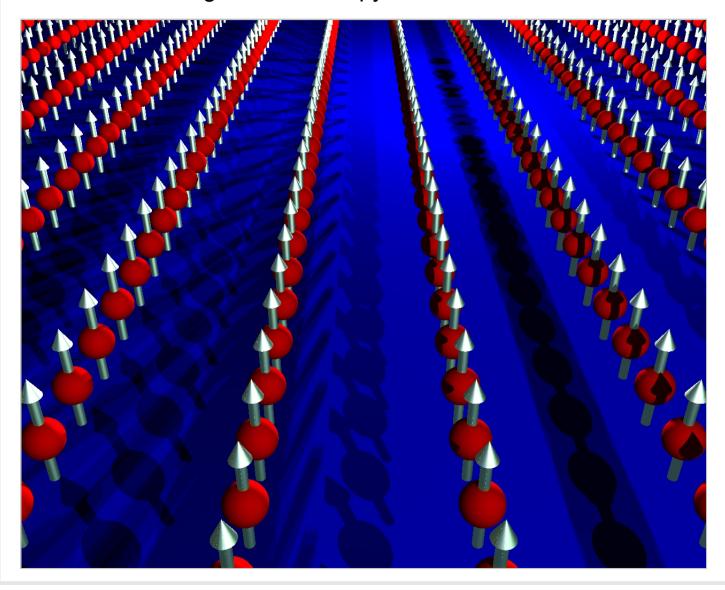
T_{Curie}:
Fe 1044 K
Co 1388 K
Ni 627 K

Dependence of resistivity on temperature in magnetic metals:



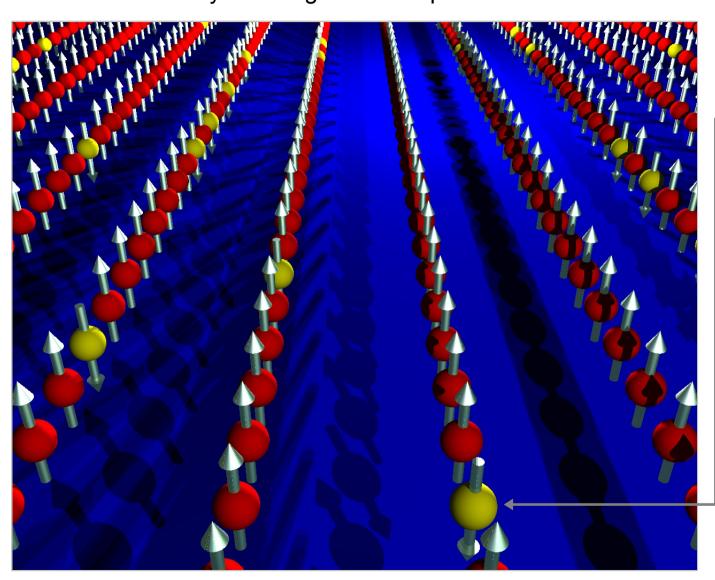
Magnons

 at very low temperatures all spins point in almost exactly one direction* determined by the effective magnetic anisotropy



Magnons

 at higher temperatures the thermal energy coming from spin-phonon coupling must be accommodated by rearrangement of spins*



It is energetically costly to reverse single spins S [9]:

$$\underline{\Delta E} = N_{nn} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

 $N_{\it nn}$ - number of nearest neighbors

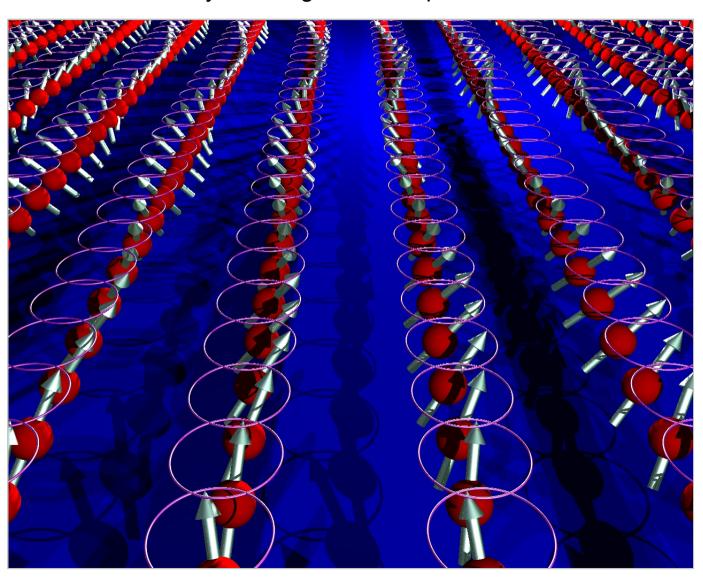
 $\boldsymbol{J}_{\boldsymbol{i}\boldsymbol{i}}$ - exchange integral

...and a state with single flipped spin is not an eigenstate of Hamiltonian [35]

^{*}the spins may be excited directly by magnetic field too.

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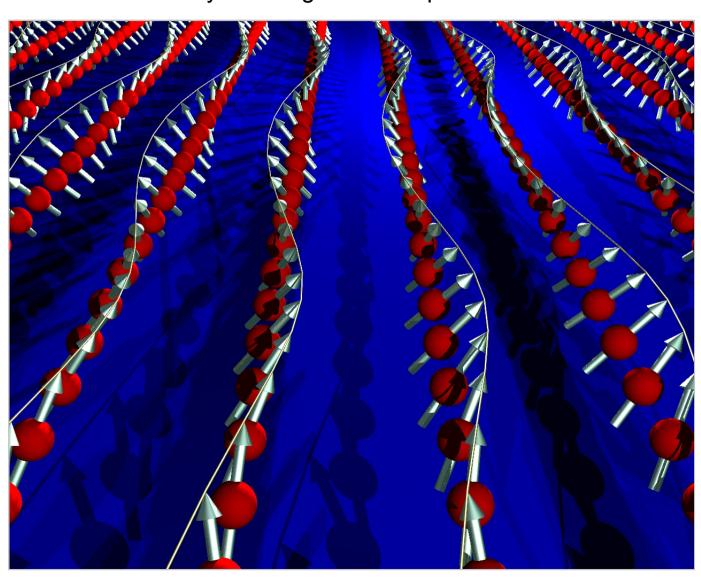
Correlated movement/ precession of spins (pictured here as classical moments) reduces the energy of a spin system by [9]:

$$\Delta E = \left(\frac{h}{2\pi}\right) \omega_q \approx 2SJq^2 a^2$$

^{*}the spins may be excited directly by magnetic field too.

Magnons

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$$\Delta E = \left(\frac{h}{2\pi}\right) \omega_q \approx 2SJq^2 a^2$$

Each elementary excitation reduces the total spin NS of the system by one unit.

^{*}the spins may be excited directly by magnetic field too.

Magnon contribution to resistivity

- measurement in pulsed magnetic fields
- almost linear and non-saturating decrease of resistivity in fields above technical saturation (paraprocess)
- spin-flip and non-spin-flip (on phonons, other electrons [8]) scattering events are responsible for resistance
- spin-flip scattering on magnons (s-d interband transitions) is responsible for magnetoresistance
- "effect results from a reduction of electronmagnon scattering processes due to a damping of the spin waves at high fields"
 [8]
- magnon magnetoresistance is estimated to saturate at 80T at 50K and 2000 T [sic] at 450 K

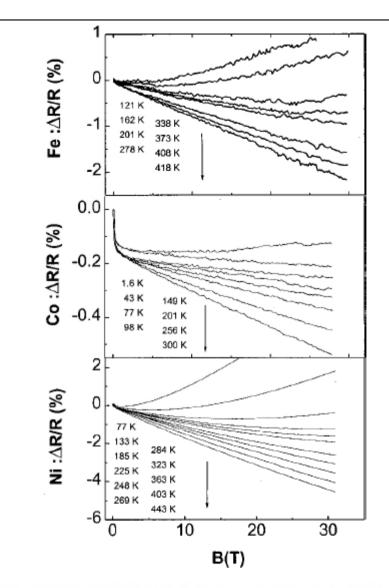
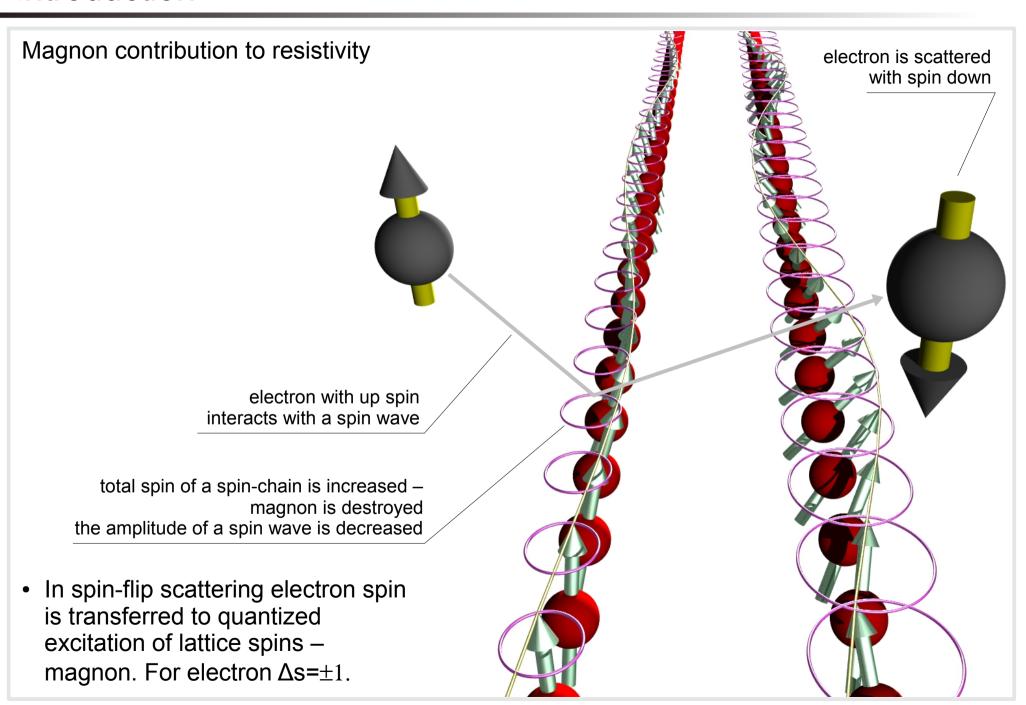
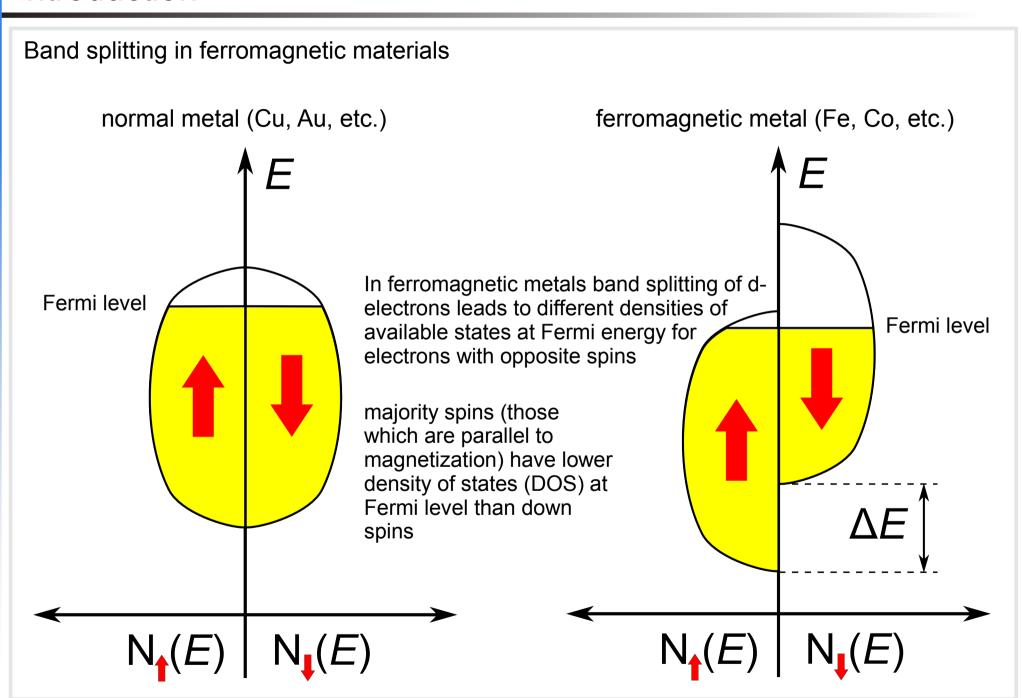


FIG. 1. Longitudinal high-field magnetoresistance ($B\parallel i$) at various temperatures for Fe, Co, and Ni epitaxial films $-\text{Fe}_{80 \text{ nm}}/\text{MgO}$, $\text{Co}_{7 \text{ nm}}/\text{Al}_2\text{O}_3$, and $\text{Ni}_{20 \text{ nm}}/\text{MgO}$. Note the extrinsic negative MR below the technical saturation of the magnetization for Co due to grain boundary effects.



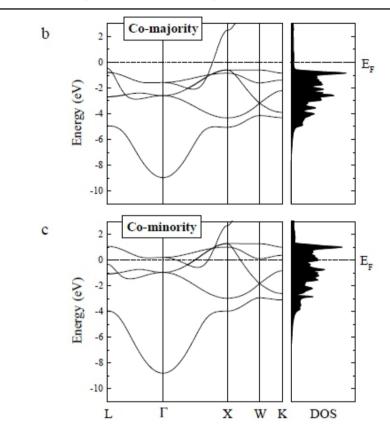
spin-chain prior to the scattering



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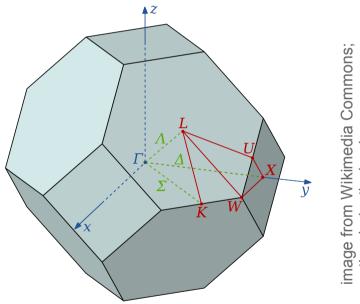
Introduction

Band splitting in ferromagnetic materials



Electronic band structures (left panels) and the density of states (right panels) of Cu (a) and fcc Co for the majority-spin (b) and minority-spin (c) electrons. The band structure of non-magnetic Cu is same for the up-spin and down-spin electrons. It is characterized by the fully occupied d bands and the presence of a dispersive sp band at the Fermi energy, which result in high conductivity of Cu. The electronic structure of ferromagnetic Co is different for the two spin orientations and is characterized by the exchange-split d bands. The Fermi level lies within the sp band for the majority-spin electrons, which leads to high conductivity of majority-spin channel. The Fermi level lies, however, within the d band for the minority-spin electrons resulting in low conductivity of the minority-spin channel. In the latter case the sp electrons are strongly hybridized with the d electrons, which diminishes their contribution to conduction.

- in some ferromagnetic materials the density of states of up-spins may be close to zero
- in Co the d-band DOS for spin-up electrons is about 10 times lower than that of down-spin electrons



graphics from: Perspectives of Giant Magnetoresistance, E. Y. Tsymbal, D. G. Pettifor, published in Solid State Physics, ed. by H. Ehrenreich and F. Spaepen, Vol. 56 (Academic Press, 2001) pp.113-237

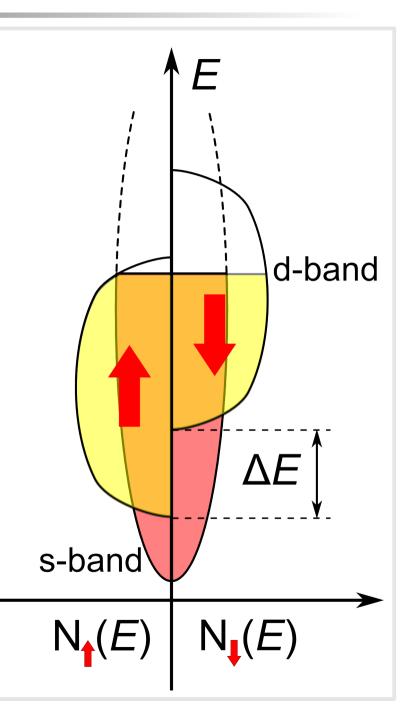
author Inductiveload

Mott – two current model, 1936.

- mobile sp-electrons are responsible for electronic conductivity [3]
- s- electrons can be scattered to free states near Fermi level
- density of states of d-electrons in ferromagnetic metal is different for spin-up and spin-down electrons
- the conductivity of given type of carriers depends on the number of free states available as final states of scattering events
- probability of spin-flip scattering is much lower than the probability of scattering without the change of spin

s-band

d-band

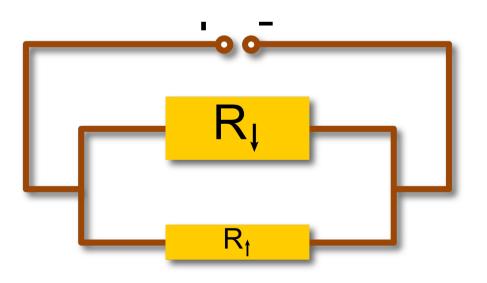


Mott – two current model, 1936.

- the conductivity of a metal is a sum of independent conductivities of up and down-spins channels [3] $\sigma = \sigma_{\uparrow} + \sigma_{\downarrow}$
- the relaxation time τ is given by Fermi golden rule (mean free path: $\lambda = \tau v_F$, v_F Fermi velocity, of the order of 0.5×10⁶m/s) [3,9]:

$$au^{-1} \propto \left\langle V_{scatt}^2 \right\rangle D(E_{\it F})$$
 average value of scattering potential

• the scattering potential needs not to be spin-dependent for resistivities of spin channels to be different – different DOS-es for opposite spins are enough



If spin-flip events are negligible current can be considered as carried in-parallel by two spin channels with spins parallel and antiparallel to quantization axis [12]

Mott – two current model, 1936.

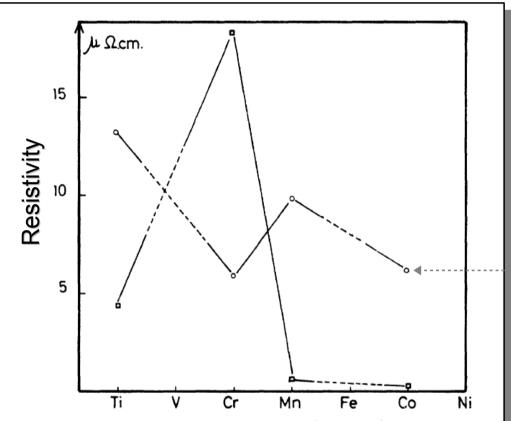


FIG. 3 Residual resistivities ρ_{\downarrow}^{0} and ρ_{\downarrow}^{0} per percent concentration of Ti, Cr, Mn, and Co impurities in Ni. Squares: ρ_{\downarrow}^{0} . Circles: ρ_{\downarrow}^{0} .

image from: A. Fert and I. A. Campbell, Phys. Rev. Lett. 21, 1190 (1968)

$$\alpha = \rho_{\downarrow}^0 / \rho_{\uparrow}^0$$

- Ni with impurities at low temperatures
- the spin-down channel conductivity can be many times lower than the resistivity of the up-spin channel

$$\alpha_{Co} = 30$$
 $\alpha_{Mn} = 16$

 the maximum for Cr impurity is associated with the bound state crossing the spin-up Fermi level [10]

spin-down channel

Giant Magnetoresistance (GMR) – Nobel Prize 2007 (A. Fert, P. Grünberg)

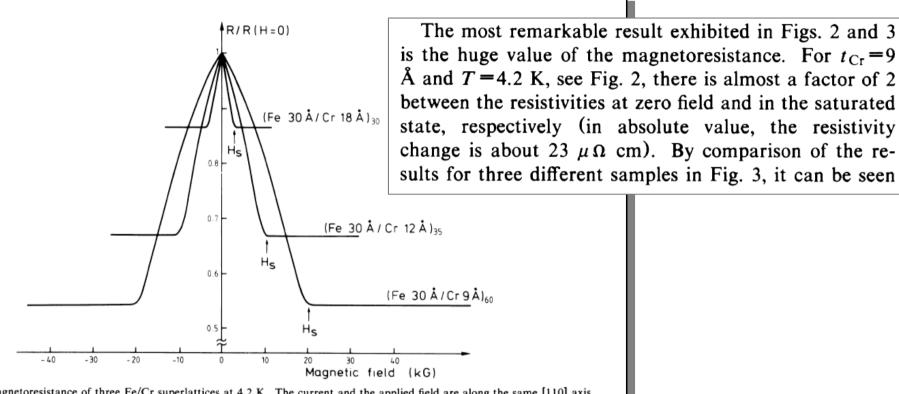
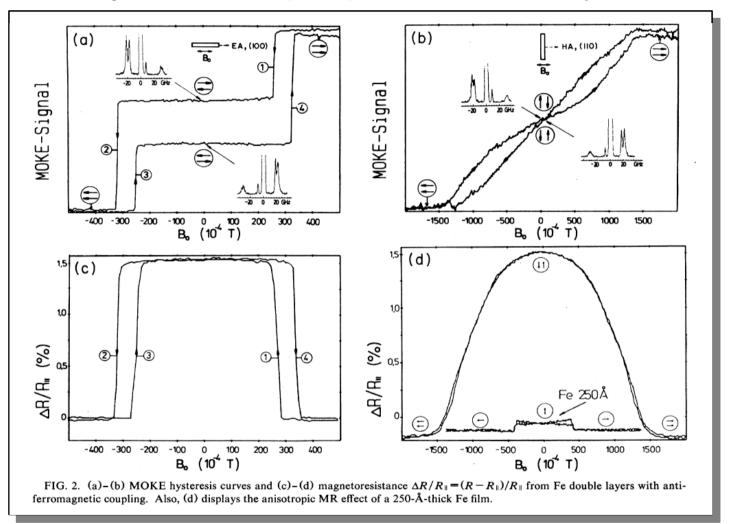


FIG. 3 Magnetoresistance of three Fe/Cr superlattices at 4.2 K. The current and the applied field are along the same [110] axis in the plane of the layers.

"In conclusion, we have found a giant magnetoresistance in (001)Fe/(001)/Cr superlattices when, for thin Cr layers (9, 12, and 18 Å), there is **an antiparallel coupling of the neighbor Fe layers** at zero field."- M.N. Baibich *et al*.

image from M.N. Baibich, J.M. Broto, **A. Fert**, F. Nguyen Van Dau, F. Petroff, P. Eitenne, G. Creuzet, A. Friederich, J. Chazelas, Phys. Rev. Lett. **61**, 2472 (1988)

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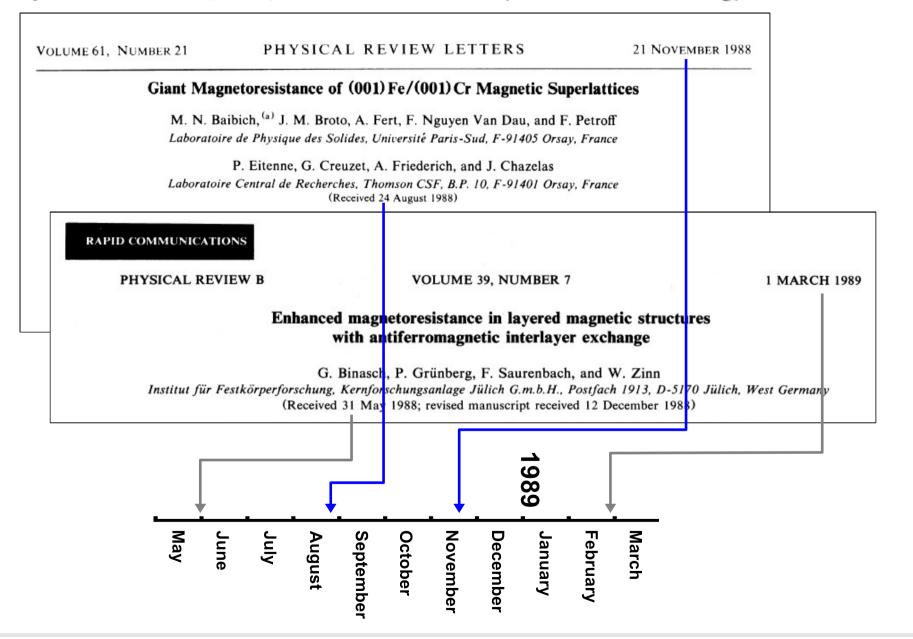


The experimental results reported here show that the antiparallel alignment of the magnetizations in our double layers produces an appreciable increase of the electrical resistivity. We propose that this is caused by spin-flip scattering. Electrons in one...

image from G. Binasch, P. Grünberg, F. Saurenbach, W. Zinn, Phys. Rev. B 39, 4828 (1989)

the first interpretation was not correct – see later

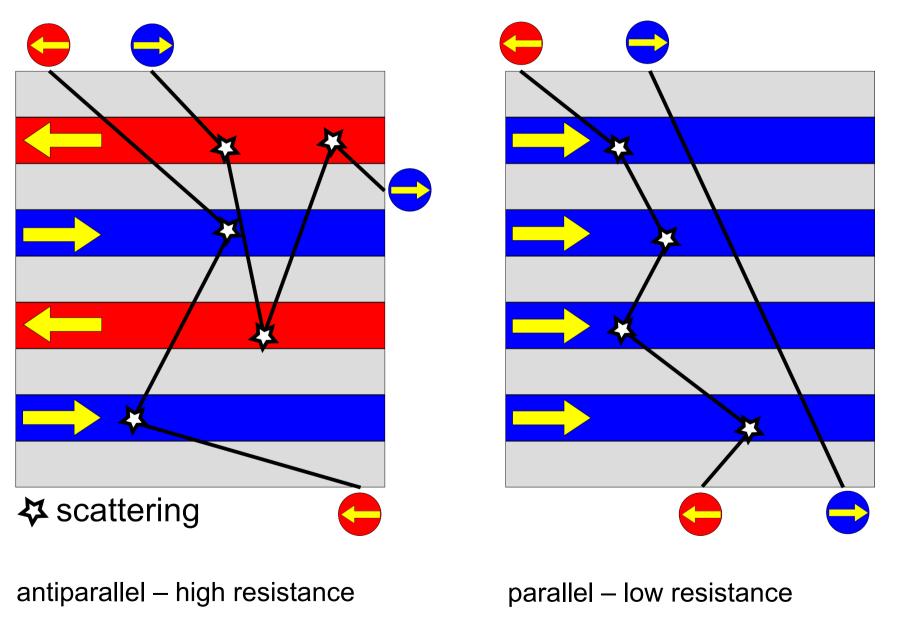
Giant Magnetoresistance (GMR) - Nobel Prize 2007 (A. Fert, P. Grünberg)



Types of magnetoresistance (incomplete list)

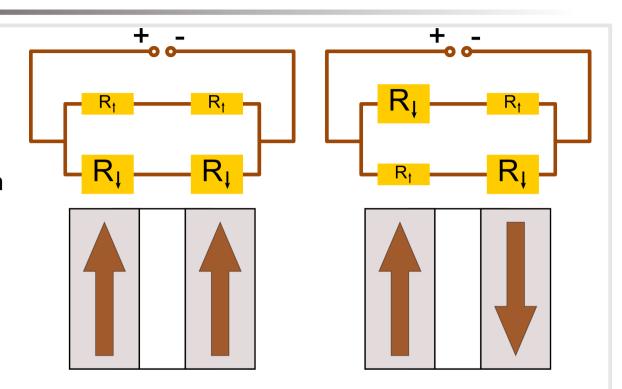
| Type of magnetoresistance | Systems | Values | Note |
|---------------------------|--------------------------|---|---|
| ordinary (OMR) | all conducting materials | approx. 10 ⁻² % in metals in RT in up to 2T | Lorentz force changes orbits of carriers |
| | | in high fields (30T) 47% in Cu (at 78K) and 380% in Bi [13] | De Haas-Shubnikov effect – high field oscillations of resistance |
| anisotropic (AMR) | ferromagnetic materials | several percent at RT (NiCo alloys [14]) in small fields | depends on orientation of current relative to magnetization; high field sensitivity (NiFe) |
| giant (GMR) | ferromagnetic materials | several dozens percent in small fields [3] | depends on relative orientation of magnetic moments |
| tunneling (TMR) | ferromagnetic materials | several hundred percent in small fields | depends on the orientation of magnetic moments of electrodes separated by insulating film |
| colossal (CMR) | transition metal oxides | several hundred percent in several Tesla | phase transition paramagnet-ferromagnet |

GMR - the simplistic explanation



GMR - resistor network model [17]

- Mott two spin-channels two conducting channels do not mix
- resistance of parallel configuration is lower



Resistance of parallel configuration

$$\frac{1}{R_P} = \frac{1}{R_{\uparrow} + R_{\uparrow}} + \frac{1}{R_{\downarrow} + R_{\downarrow}}$$

$$R_{P} = \frac{2R_{\uparrow}R_{\downarrow}}{R_{\uparrow} + R_{\downarrow}}$$

Resistance of antiparallel configuration

$$\frac{1}{R_{AP}} = \frac{1}{R_{\uparrow} + R_{\downarrow}} + \frac{1}{R_{\uparrow} + R_{\downarrow}}$$

$$R_{AP} = \frac{R_{\uparrow} + R_{\downarrow}}{2}$$

$$R_{\downarrow} = (1 + a^2) R_{\uparrow} \rightarrow \frac{R_{AP}}{R_{P}} - 1 = \frac{a^4}{4 + 4a^2} \ge 0 \rightarrow R_{AP} \ge R_{P}$$

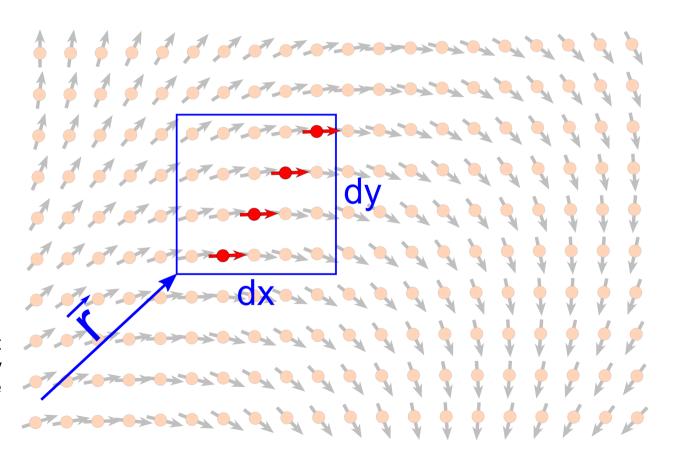
image based on Fig.2 from [17]

Boltzmann transport equation [9,15,16]

The classical theory of transport is based on a statistical distribution function *f* that specifies the probability of finding a particle with its position and momentum within a small range [15] or, which is equivalent*, concentration of carriers with a given momentum in the neighborhood of the given point in space [9].

$$n(r_x, r_x + dx, r_y, r_y + dy, r_z, r_z + dz, v_x, v_x + dv_x, v_y, v_y + dv_y, v_z, v_z + dv_z) = f(\vec{r}, \vec{v}) dx dy dz dv_x dv_y dv_z$$

number of particles in drxdv volume at r,v position in sixdimensional space



in the 2-D example to the right four particles in the dxxdy volume have v_y velocities in the range from -dv_y to +dv_y

Boltzmann transport equation [9,15,16]

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- In equilibrium the transition rates between any two states exactly balance [15]
- In the presence of external fields the equilibrium state is disturbed and the scattering tends to return the system to equilibrium
- Some steady state is attained in which the effect of external fields is balanced by scattering events
- Scattering "has the important effect of limiting the extent of the response" A.C. Smith et al. [15]

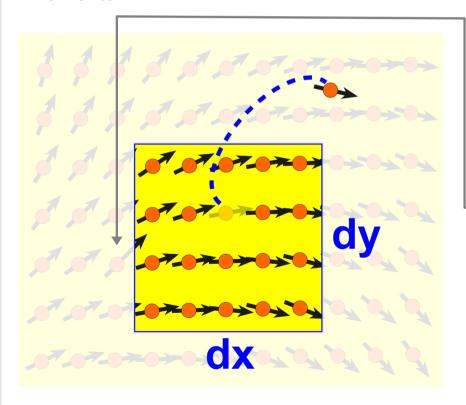
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Magnetoresistance

Rate of change of the distribution function $f(r,k,t)^*$ [from A.C. Smith *et al.*, 15]:

- the density described by f may change because of scattering of phase points into or out
 of the volume cell drxdv
- the phase points may flow in or out of the cell due to their spatial velocity and because the velocity changes under the influence of external field (streaming or drift terms)

The carrier (electron) is scattered out of the dxdy volume and it may change its momentum



The carrier (electron) is about to enter dxdy volume but his momentum is different than that of carriers in cell; it will not be in the same dx dy dv_xdv_y volume as neighboring (in real space) electrons

view in real space

Rate of change of the distribution function $f(r,k,t)^*$ [from A.C. Smith *et al.*, 15]:

- the density described by f may change because of scattering of phase points into or out of the volume cell drxdv
- the phase points may flow in or out of the cell due to their spatial velocity and because
 the velocity changes under the influence of external field (streaming or drift terms)
 Consider the 1-D movement of a phase points along x-direction. The number of points
 crossing a dy dz area normal to x is given by:

$$\delta n(x) = \int f(x,k) dk \, \dot{x} \, dy \, dz = f(x) \dot{x} \, dy \, dz$$

k disappears from here because we integrate over all velocities

note the dot

The net flow into the region between x and x+dx is given by:

$$\delta n = [\delta n(x) - \delta n(x + dx)] dy dz$$

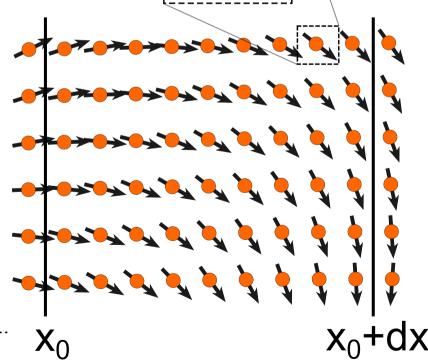
= $[f(x)\dot{x}(x) - f(x + dx)\dot{x}(x + dx)] dy dz$

Which for $dx \rightarrow 0$ is:

$$\delta n = -\left[\frac{\partial}{\partial x} \left(f(x)\dot{x} \right) \right]_{x_0} dx dy dz$$

In 3-D (change of the number of phase points in dx dy dz volume) we have:

$$\delta n = \left[\frac{\partial}{\partial x} \left(f(x) \dot{x} \right) \right]_{x_0} dx dy dz + \left[\frac{\partial}{\partial y} \left(f(y) \dot{y} \right) \right]_{y_0} dy dx dz + \dots$$



Further we have:

$$\nabla \cdot (f \ \vec{v}) = (i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}) \cdot (i f \ \vec{v}_x + j f \ \vec{v}_y + k f \ \vec{v}_z) = \frac{\partial}{\partial x} f \ \vec{v}_x + \frac{\partial}{\partial y} f \ \vec{v}_y + \frac{\partial}{\partial z} f \ \vec{v}_z$$

...and for the change of *f*:

the last expression of the previous page divided by $dx\ dy\ dz$

0 0

$$\delta n = -\nabla \cdot (f(\vec{r})\vec{v}) d^3 r$$

To get the complete expression (drift of position and momentum/velocity points into 6-dimensional phase space volume d³r d³k) we add together six terms [15]:

$$\delta f(\vec{r}, \vec{v}, t) = -\left[\nabla_r \cdot \left(f(\vec{r}, \vec{k}, t)\vec{v}\right) + \nabla_k \cdot \left(f(\vec{r}, \vec{k}, t)\dot{\vec{k}}\right)\right]d^3rd^3k$$

The continuity equation for f^* reads (we cancel d^3rd^3k everywhere and work with density of phase points instead of their numbers):

The phrase points instead of their numbers).
$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{scatt} - \nabla_r \cdot (f \vec{v}) - \nabla_k \cdot (f \vec{k})$$

$$\nabla \cdot (a\vec{b}) = \vec{b} \cdot \nabla a + a \nabla \cdot \vec{b}$$

$$\nabla_r \cdot (f \vec{v}) + \nabla_k \cdot (f \vec{k}) = \vec{v} \cdot \nabla_r f + f \nabla_r \vec{v} + \vec{k} \cdot \nabla_k f + f \nabla_k \cdot \vec{k}$$

$$\nabla_r \cdot (f \vec{v}) + \nabla_k \cdot (f \vec{k}) = \vec{v} \cdot \nabla_r f + f \nabla_r \vec{v} + \vec{k} \cdot \nabla_k f + f \nabla_k \cdot \vec{k}$$

$$\nabla_r \cdot \vec{v} = \hbar^{-1} \nabla_r \cdot [\nabla_k E_n(k)] = 0$$

$$\nabla_k \cdot \vec{k} = \nabla_k \cdot \frac{q}{\hbar} \left[\vec{k} + \vec{v} \cdot \vec{v} \cdot \vec{b} \right] = \text{electric field } \vec{E} \text{ does not depend on position } \vec{r}$$

$$\frac{q}{\hbar} \nabla_k \cdot (\hat{i} \cdot (B_z v_y - B_y v_z) + \hat{j} \cdot (-B_z v_x + B_z v_x + B_z v_z) + \hat{k} \cdot (B_y v_x - B_x v_y)) =$$

$$\frac{q}{\hbar^2} \left(\hat{i} \cdot \frac{\partial}{\partial k_x} + \hat{j} \cdot \frac{\partial}{\partial k_y} + \hat{k} \cdot \frac{\partial}{\partial k_z} \right)$$

$$\left(\hat{i} \cdot (B_z \cdot \frac{\partial E_n(k)}{\partial k_x} - B_y \cdot \frac{\partial E_n(k)}{\partial k_x} + B_x \cdot \frac{\partial E_n(k)}{\partial k_x} + B_x \cdot \frac{\partial E_n(k)}{\partial k_x} + B_x \cdot \frac{\partial E_n(k)}{\partial k_x} - B_z \cdot \frac{\partial E_n(k)}{\partial k_x} -$$

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{scatt} - \vec{v} \cdot \nabla_r f - \dot{\vec{k}} \cdot \nabla_k f$$

Boltzmann transport equation

- "derived under the condition that the fictitious particle representing electron executes a classical motion" [15]
- · not valid for large external fields and when band-to-band transitions occur

The scattering term is due to: lattice vibrations, impurities, electron-electron scattering, electron-magnon scattering, vacancies, grain boundaries dislocations etc.

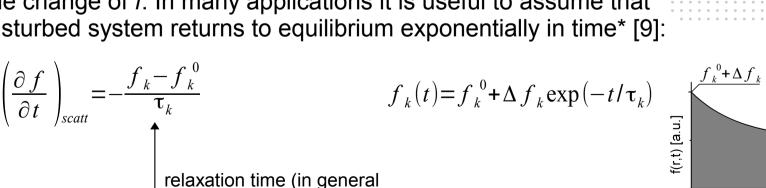
In steady state *f* is constant and we have:

$$\left(\frac{\partial f}{\partial t}\right)_{scatt} - \frac{\vec{v} \cdot \nabla_r f}{\vec{v} \cdot \nabla_r f} - \frac{\dot{\vec{k}} \cdot \nabla_k f}{\vec{k} \cdot \nabla_k f} = 0 \qquad \text{effect of fields}$$

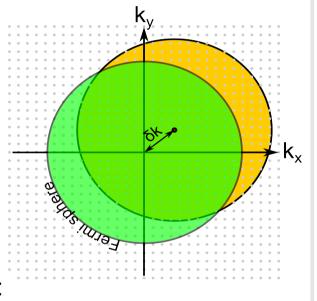
$$\frac{\text{diffusion}}{\vec{k} \cdot \nabla_k f} = 0$$

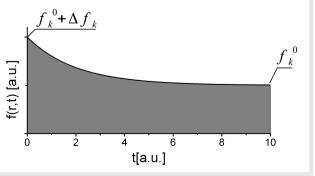
The relaxation time approximation:

In general it is not possible to calculate scattering contribution to the change of f. In many applications it is useful to assume that disturbed system returns to equilibrium exponentially in time* [9]:



different for each Bloch state)





^{*}this approximation is valid for pure metals for temperatures exceeding Debye temperature and for contaminated metals (or those with defects) for all temperatures [9]

Linearized Boltzmann equation

The distribution function can in general be expanded in powers of the driving field [18]:

$$f = f^{0} + \left(\frac{\partial f}{\partial E}\right) \vec{E} + \left(\frac{\partial^{2} f}{\partial E^{2}}\right) \vec{E}^{2} + \dots$$

, with f^0 given for electrons by Fermi-Dirac statistics [9]:

$$f_k^0 = \frac{1}{\exp(E_k - E_F)/k_b T + 1}$$

"When the electric field is small, only a small amount of current flows. The system is only slightly out of equilibrium." [19]. The distribution function can be written as:

$$f=f^0+f^1$$

 f^{-1} is a small change

Giant magnetoresistance from Boltzmann equation*
Investigating electron transport in thin films one can assume that the system is infinitely extended in xy-plane so that the distribution function depends only on z-coordinate (perpendicular to the film plane).

Using two-channel model of Mott the distribution function is decomposed into two parts:

-equilibrium distribution function $f_0(z,v)$ – in zero electric field

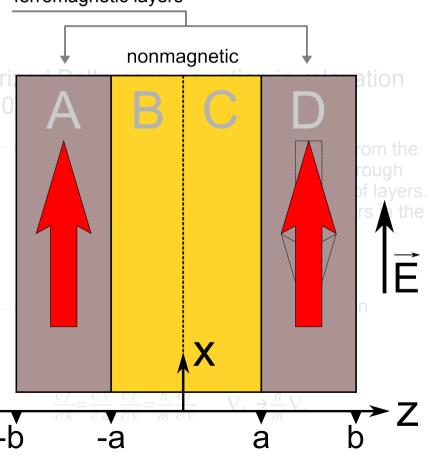
$$f^{\uparrow(\downarrow)}(z,\vec{v}) = f_0(\vec{v}) + g^{\uparrow(\downarrow)}(z,\vec{v})$$

Substituting the above distribution function into linearize time approximation we obtain the expression for g [20]

$$\frac{\partial g^{\uparrow(\downarrow)}(z,\vec{v})}{\partial z} + \frac{g^{\uparrow(\downarrow)}(z,\vec{v})}{\tau^{\uparrow(\downarrow)}v_z} = \frac{e\vec{E}}{mv_z} \frac{\partial f_o(\vec{v})}{\partial v_x}$$

m, e - electron effective mass and charge

- relaxation times are spin dependent (spin channels)
- g is divided into two parts depending on the sign of v_{_} component of velocity



Giant magnetoresistance from Boltzmann equation*

Investigating electron transport in thin films one can assume that the system is infinitely extended in xy-plane so that the distribution function depends only on z-coordinate (perpendicular to the film plane).

Using two-channel model of Mott the distribution function is decomposed into two parts:

- -equilibrium distribution function $f_0(z,v)$ in zero electric field
- -small change g, induced by external field (electric), that depends on electrons spin

$$f^{\uparrow(\downarrow)}(z,\vec{v})=f_0(\vec{v})+g^{\uparrow(\downarrow)}(z,\vec{v})$$

Substituting the above distribution function into linearized Boltzmann equation in relaxation time approximation we obtain the expression for g [20]:

$$\frac{\partial g^{\uparrow(\downarrow)}(z,\vec{v})}{\partial z} + \frac{g^{\uparrow(\downarrow)}(z,\vec{v})}{\tau^{\uparrow(\downarrow)}v_z} = \frac{e\vec{E}}{mv_z} \frac{\partial f_o(\vec{v})}{\partial v_x}$$

m, e – electron effective mass and charge

- Note that magnetoresistance results from the presence of external magnetic field through the orientation of magnetic moments of layers. Magnetic field does not explicitly occur in the model.
- Note that previously we had Boltzmann equation in r,k-space relaxation times are spin dependent (spin $p=m v=\hbar k$
- g is divided into two parts depending on the sign $\frac{\partial f}{\partial k} = \frac{\partial v}{\partial k} \frac{\partial f}{\partial v} = \frac{\hbar}{m} \frac{\partial f}{\partial v} \qquad \nabla_k \rightarrow \frac{\hbar}{m} \nabla_v$ of v_{z} component of velocity

$$\frac{\partial f}{\partial k} = \frac{\partial v}{\partial k} \frac{\partial f}{\partial v} = \frac{\hbar}{m} \frac{\partial f}{\partial v} \qquad \nabla_k \rightarrow \frac{\hbar}{m} \nabla$$

channels)

^{*}theory developed by J. Barnaś and coworkers [20]

The general solution can be written as [20]:

$$g_{\pm}^{\uparrow(\downarrow)}(z,\vec{v}) = \frac{e\vec{E}\,\tau^{\uparrow(\downarrow)}}{m} \frac{\partial f_o(\vec{v})}{\partial v_x} \times \left[1 + F_{\pm}^{\uparrow(\downarrow)}(v) \exp\left(\frac{\mp z}{\tau^{\uparrow(\downarrow)}|v_z|}\right)\right]$$

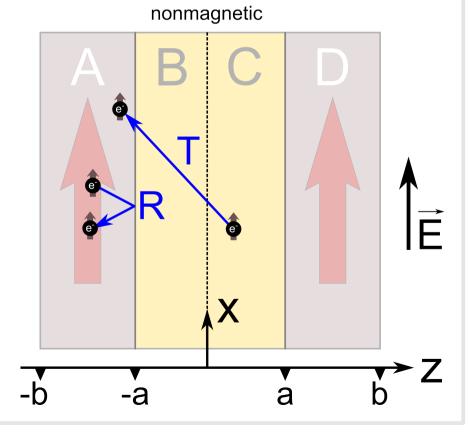
4 arbitrary functions to be determined from boundary conditions

Boundary conditions (BCs) – in steady state the current flowing in the conductor is continuous At the interface at z=-a between the ferromagnetic film and the spacer BC can be written in the form:

$$g_{A-}^{\uparrow(\downarrow)}(-a,\vec{v}) = T^{\uparrow(\downarrow)}g_{B-}^{\uparrow(\downarrow)}(-a,\vec{v}) + R^{\uparrow(\downarrow)}g_{A+}^{\uparrow(\downarrow)}(-a,\vec{v}),$$

where *T* and *R* are coefficients of a non-diffusive transmission and a reflection (conserving momentum, specular) of electrons.

- the above condition states that in the vicinity of interface the current of electrons flowing in -z direction consists of electrons that came from region B and those which were heading in +z direction and were reflected back to region B
- fraction (1-T) of electrons is scattered diffusely



The general solution can be written as [20]:

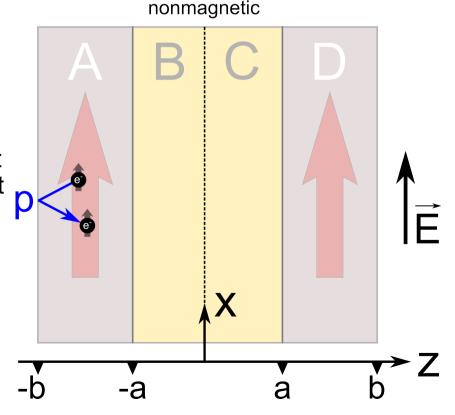
$$g_{\pm}^{\uparrow(\downarrow)}(z,\vec{v}) = \frac{e\vec{E}\,\tau^{\uparrow(\downarrow)}}{m} \frac{\partial f_o(\vec{v})}{\partial v_x} \times \left[1 + F_{\pm}^{\uparrow(\downarrow)}(v) \exp\left(\frac{\mp z}{\tau^{\uparrow(\downarrow)}|v_z|}\right)\right]$$

4 arbitrary functions to be determined from boundary conditions

Similar Fuchs* BCs are introduced for outer interfaces with specularity factors depending generally on spin:

$$g_{A+}^{\uparrow(\downarrow)}(z,\vec{v}) = p_A^{\uparrow(\downarrow)}g_{A-}^{\uparrow(\downarrow)}(z,\vec{v})$$
 $z = -b$

- the condition states that in the vicinity of the outer interface electrons traveling into the multilayer are those reflected from the interface
- the electrons which are diffusely reflected do not contribute to conductivity along the film (their net contribution is zero)
- any angle dependence of the specularity is neglected for simplicity [20]



At the fictitious interface at z=0 one can write [20]:

$$\begin{split} g_{C+}^{\uparrow(\downarrow)}(z,\vec{v}) &= \cos^2(\theta/2) g_{B+}^{\uparrow(\downarrow)}(z,\vec{v}) + \sin^2(\theta/2) g_{B+}^{\downarrow(\uparrow)}(z,\vec{v}) \\ g_{B-}^{\uparrow(\downarrow)}(z,\vec{v}) &= \cos^2(\theta/2) g_{C-}^{\uparrow(\downarrow)}(z,\vec{v}) + \sin^2(\theta/2) g_{C\mp}^{\downarrow(\uparrow)}(z,\vec{v}), \end{split}$$

Note that all terms are proportional to $cos(\theta)$

$$2\cos^{2}(\theta/2)-1=\cos(\theta)$$
$$1-2\sin^{2}(\theta/2)=\cos(\theta)$$

which assures an agreement with observed resistance changes versus θ . The total current (per unit length along the y axis; electric field is along x axis) is given by:

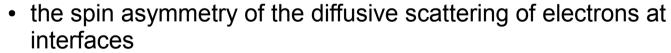
$$I=e\int dz\int d^3v\left[g^{\uparrow}(\vec{v},z)+g^{\downarrow}(\vec{v},z)\right]$$
 a sum of two spin-channels currents

The equations were solved numerically

• the amplitude of magnetoresistance is given by

$$GMR = \frac{\rho^{\uparrow \downarrow} - \rho^{\uparrow \uparrow}}{\rho^{\uparrow \uparrow}}$$

To better analyze the results additional parameters are introduced to describe:

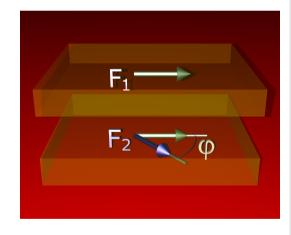


$$N_s = (1 - T^{\bullet})/(1 - T^{\bullet})$$

spin asymmetry of the bulk scattering rate in the ferromagnetic material

$$N_b = \lambda^{\uparrow}/\lambda^{\downarrow}$$

 λ - electron mean free paths



The main results of the analysis of Barnaś *et al.* [20] can be summarized as follows [21]:

- GMR increases monotonically with mean free path λ if the interface scattering dominates
- GMR displays clear maximum versus λ if the bulk scattering dominates

Assumptions:

- $p_A^{\uparrow} = p_A^{\downarrow} = p_D^{\uparrow} = p_A^{\downarrow} = p$ -Fuchs specularity factors
- r = s = t = 1 conductivities of all layers are equal
- $D^{\uparrow(\downarrow)} = (1 T^{\uparrow(\downarrow)}) = 0$ "the omission of specular

reflection at the interfaces."

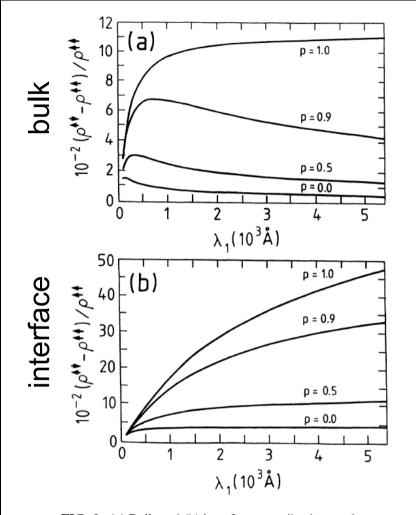


FIG. 2. (a) Bulk and (b) interface contribution to the magnetoresistivity in double layers with d=100 and 10 Å for different values of the specularity factor p. The curves in (a) have been calculated for $N_b=2$, $D^{\dagger}=0$, and r=s=t=1, whereas the curves in (b) for $N_s=6$, $D^{\dagger}=0.5$, $N_b=1$, and r=s=t=1.

The main results of the analysis of Barnaś *et al.* [20] can be summarized as follows [21]:

- GMR decreases monotonically with the thickness of ferromagnetic layer if the interface scattering dominates
- GMR shows a distinct maximum versus thickness of the ferromagnetic layers if the bulk scattering dominates (if the thickness of the ferromagnetic layer exceeds λ part of it becomes inactive in GMR but still contributes to conductivity [21])

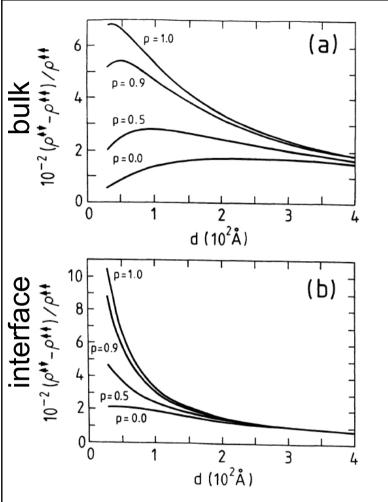


FIG. 3. (a) Bulk and (b) interface contribution to the magnetoresistivity in double layers vs thickness of the ferromagnetic layers. The curves are calculated for the specularity factors as indicated. The interlayer thickness is assumed to be constant, $d_0 = 10$ Å. The other parameters assumed in numerical calculations are (a) $N_b = 2$, $D^{\dagger} = 0$, $\lambda_1 = 200$ Å, and r = s = t = 1; (b) $N_s = 6$, $D^{\dagger} = 0.5$, $N_b = 1$, $\lambda_1 = 200$ Å, and r = s = t = 1.

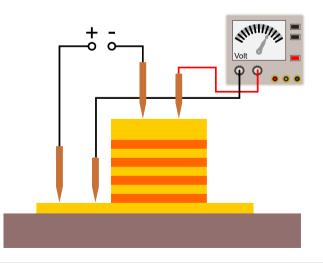
The main results of the analysis of Barnaś et al. [20] can be summarized as follows [21]:

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- GMR decreases monotonically with the thickness of ferromagnetic layer if the interface scattering dominates
- GMR shows a distinct maximum versus thickness of the ferromagnetic layers if the bulk scattering dominates (if the thickness of the ferromagnetic layer exceeds λ part of it becomes inactive in GMR but still contributes to conductivity [21])
- GMR increases with the increase of repetition number of basic bilayers
 (ferromagnet/nonmagnetic spacer) number of GMR active interfaces within λ increases
 (additionally in thick multilayers the influence of outer surfaces of the system decreases)

Angular dependence of GMR

$$\Delta R \propto \cos(\theta)$$

- in the limit of quantum transport (QT) deviations from the dependence occur due to interference of electron waves reflected from interfaces and/or surfaces [23]
- in QT limit the dependence is proportional to cosine if the structure is symmetrical and the crystal potential is independent of spin
- in the case of current perpendicular to plane geometry (CPP) the significant deviations were observed too [23]



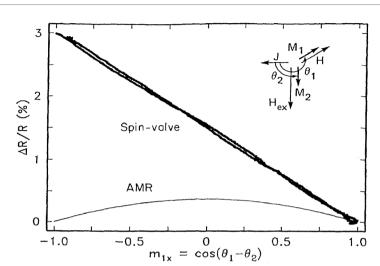


FIG. 2. Relative change in resistance vs the cosine of the relative angle between the magnetizations of the two NiFe layers of Si/(60-Å NiFe)/(26-Å Cu)/(30-Å NiFe)/(60-Å FeMn)/(20-Å Ag). Inset shows the orientation of the current J, exchange field $H_{\rm ex}$, applied field H, and magnetizations M_1 and M_2 .

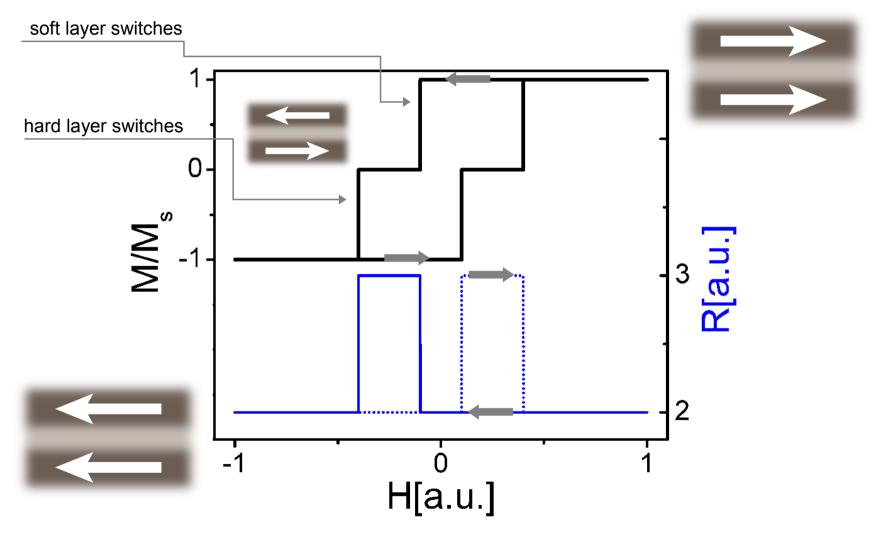
image from B. Dieny, V.S. Speriosu, S.S.P. Parkin, B.A. Gurney, D.R. Wilhoit, D. Mauri, Phys. Rev. B **43**, 1297 (1991)

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Magnetoresistance

Angular dependence of GMR

• knowing field dependence of magnetic moments configuration one can approximately predict the shape (not the amplitude!) of R(H) dependence



example: GMR of spin valve with two layers of different switching fields

Angular dependence of GMR

- Co(10nm)/Au(6nm)/Co(10nm)nagnetic moments confipredict the shape (not the amplitude!) of R(H) depend
- note the increase of GMR amplitude with decreasing temperature (increase of mean free path and a decreased phononic contribution) from 1.2 to 1.7%
- magnetic layers have different magnetic moments so after switching of the soft layer the net magnetization is different from zero

A digression:

"All these features can be used for verification of the theoretical predictions with the experimental results. However, the most reliable one seems to be the temperature dependence of the effect. This follows from the fact that the relevant experiments are performed on one single sample. In the case of other features one has to compare data obtained on different samples." J. Barnaś et al. [20]

H[a.u.]

image from: J. Barnaś, A. Fuss, R.E. Camley, example: GMR of spin \P. Grunberg, W. Zinn, Phys. Rev. B 42, 8110 (1990)

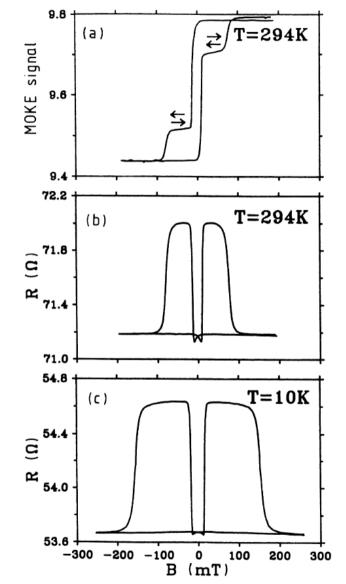


FIG. 9. (a) MOKE-signal and (b) and (c) resistance of the Co/Au/Co structure with the Co-film thickness d = 100 Å and the Au-interlayer thickness $d_0 = 60 \text{ Å}$.

RKKY-like interlayer coupling

- two Fe layers separated by a Cr wedge-shaped spacer; scanning electron microscopy with polarization analysis (SEMPA)
- measurement on a single specimen!
- up to six oscillations in coupling were observed

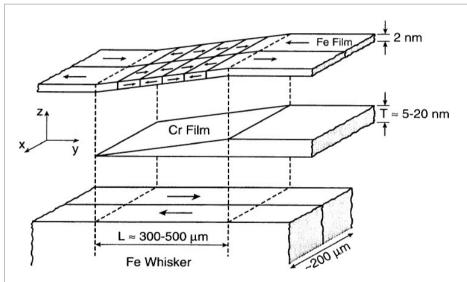


FIG. 1. A schematic exploded view of the sample structure showing the Fe(100) single-crystal whisker substrate, the evaporated Cr wedge, and the Fe overlayer. The arrows in the Fe show the direction of the magnetization in each domain. The z scale is expanded approximately 5000 times; the actual wedge angle is of order 10^{-3} deg.

Obtaining wedge-shaped films:

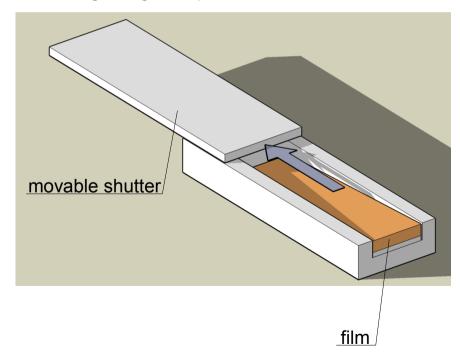


image from J. Unguris, R. J. Celotta, and D. T. Pierce Phys. Rev. Lett. 67, 140 (1991)

RKKY-like interlayer coupling

• two Fe layers separated by a Cr wedge-shaped spacer; scanning electron microscopy with

polarization analysis (SEMPA)

- measurement on a single specimen!
- up to six oscillations in coupling were observed
- different periods of coupling depending on temperature of the substrate during the film growth: samples grown at elevated temperature are of better quality and the magnetization of the upper Fe layer changes with each atomic-layer change in Cr thickness
- "lower quality" samples display only RKKY-like coupling

grown at elevated temperatures (200-300°C)

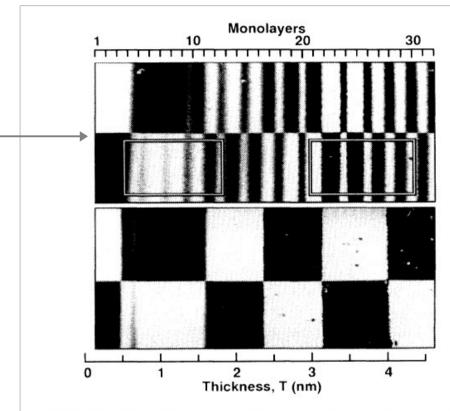


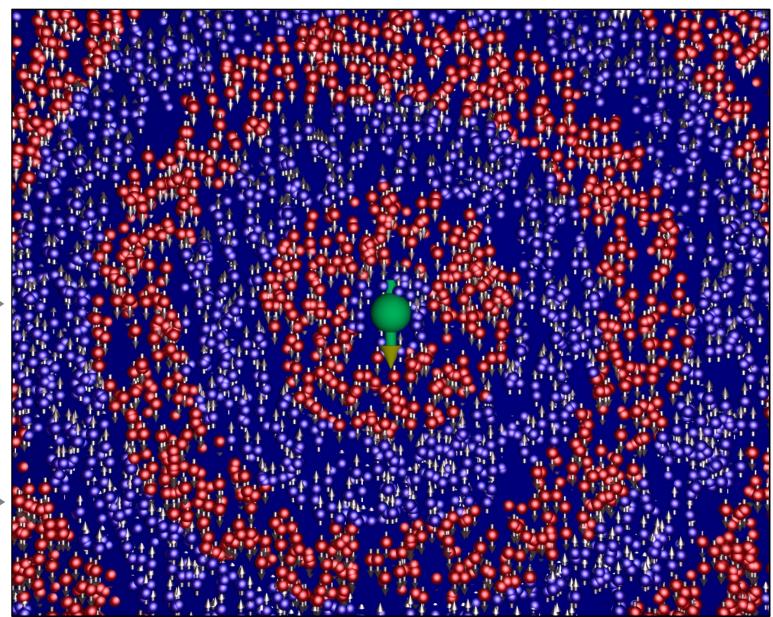
FIG. 3. The difference in the magnetic coupling of the Fe layers in the Fe/Cr/Fe sandwich for the Cr wedge grown (2.7 ML/min) on a substrate at room temperature (lower panel) and grown (7.2 ML/min) on a substrate at elevated temperature (upper panel) is clear in these SEMPA M_y images. The images in the upper and lower panels represent areas 300×280 and $350 \times 290 \ \mu m$, respectively.

RKKY-like interlayer coupling

Magnetic impurity in a conducting medium induces spatial fluctuations of spin polarization of selectrons about the impurity [9]

 the oscillatory term of wave number 2 k_F falls off like r⁻³ at large distances

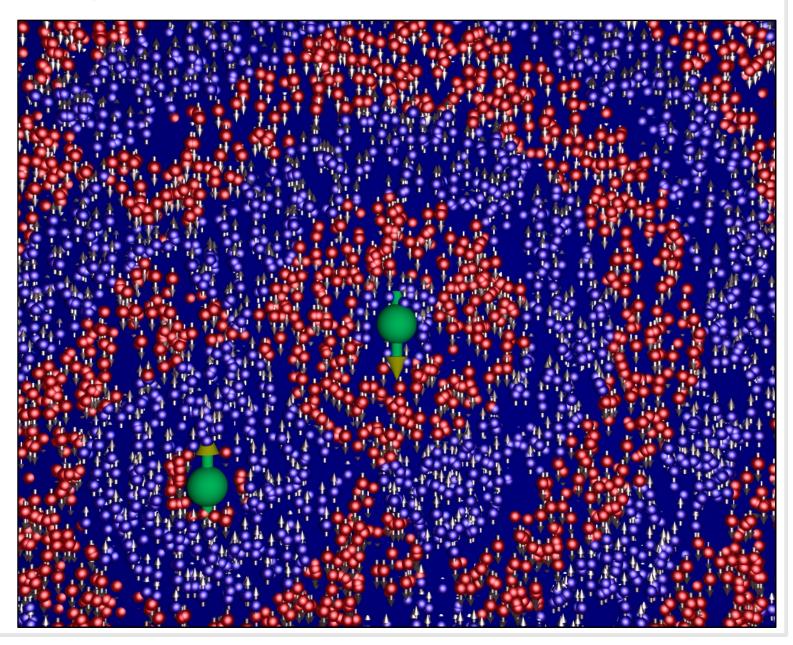




RKKY-like interlayer coupling

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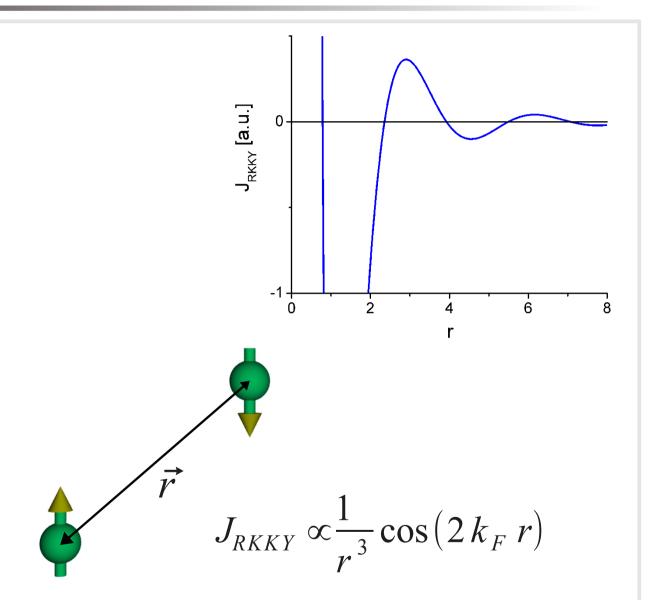
- the oscillatory term of wave number 2 k_F* falls off like r⁻³ at large distances
- the second impurity placed in the vicinity experiences interaction with the first impurity
- depending on the distance between impurities the interactions may be ferromagnetic or antiferromagnetic



RKKY-like interlayer coupling

Magnetic impurity in a conducting medium induces spatial fluctuations of spin polarization of selectrons about the impurity [9]

- the oscillatory term of wave number 2 k_F falls off like r⁻³ at large distances
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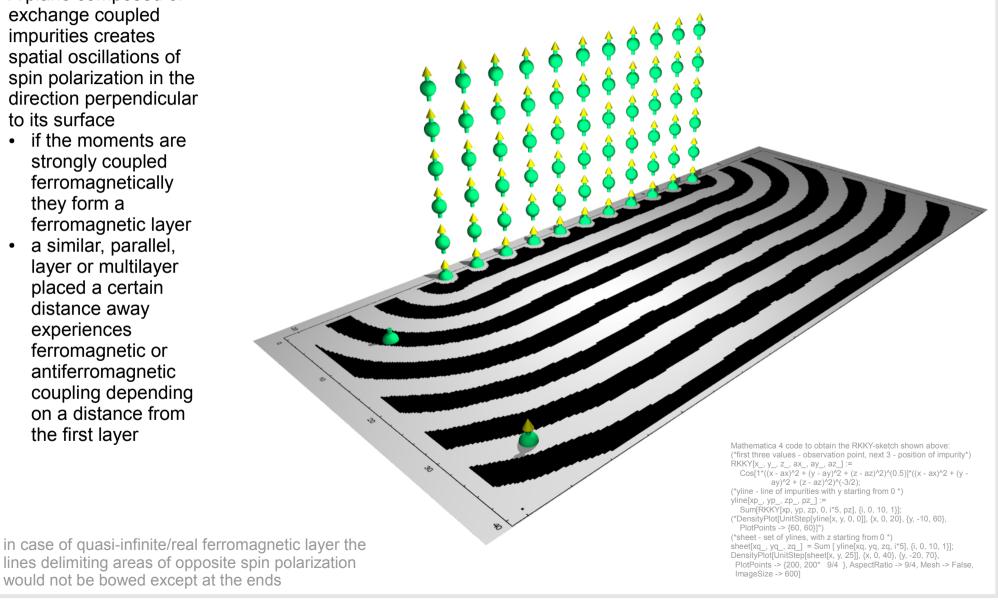


RKKY-like interlayer coupling

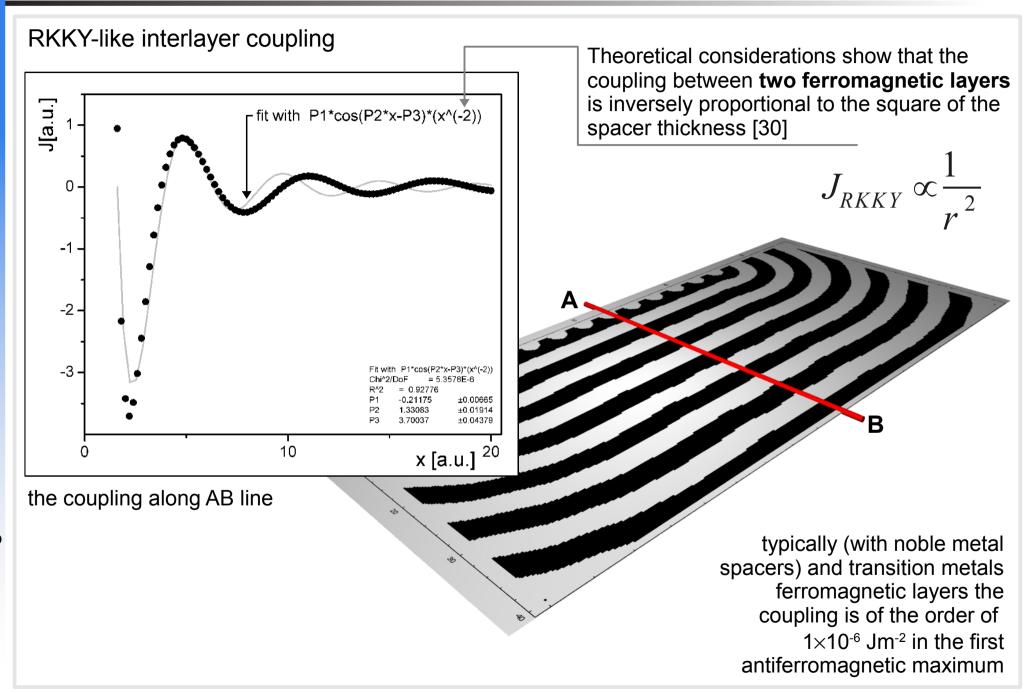
A plane composed of exchange coupled impurities creates spatial oscillations of spin polarization in the direction perpendicular to its surface

- if the moments are strongly coupled ferromagnetically they form a ferromagnetic layer
- a similar, parallel, layer or multilayer placed a certain distance away experiences ferromagnetic or antiferromagnetic coupling depending on a distance from the first layer

schematic drawing of a RKKY spin polarization due to single atom thick (11×11atoms) layer of impurities*



*the drawing shows the sign of the coupling (black and gray correspond to positive and negative spin polarization)



^{*}the drawing shows the sign of the coupling (black and gray correspond to positive and negative spin polarization)

RKKY-like interlayer coupling

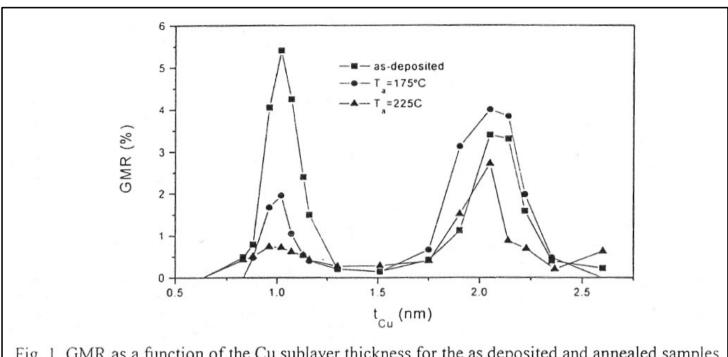
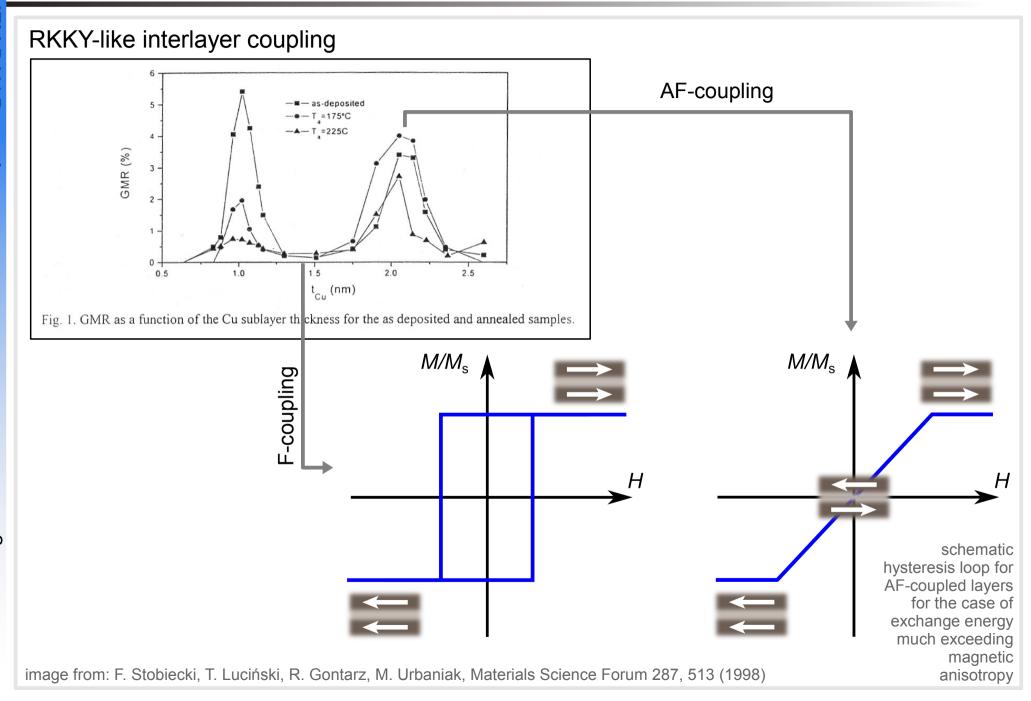


Fig. 1. GMR as a function of the Cu sublayer thickness for the as deposited and annealed samples.

- $Si(100)/Cu(20nm)[Ni_{83}Fe_{17}(2nm)/Cu(t_{Cu})]_{100}$
- GMR reflects the oscillatory character of the RKKY-like coupling between permalloy layers
- in MLs with identical magnetic layers (the same switching fields) GMR can be observed only for spacer thicknesses corresponding to antiferromagnetic coupling; otherwise the magnetic field does not change relative orientation of magnetic moments of neighboring layers



Inverse CPP GMR [31]

- "Fe doped with V gains negative spin asymmetry for bulk scattering" the up-spin channel is characterized by higher resistivity $\rho_{\uparrow(\downarrow)} = \rho_{bulk} (1 \mp \beta)$
- similarly the interface resistivity depends on spin orientation – factor γ (positive)
- in (FeV/Cu/Co/Cu)₂₀ multilayers the resistance of saturated system (all magnetizations pointing in one direction) may be higher than for the case of antiparallel orientation of magnetizations in neighboring magnetic layers
- depending on the FeV layer thickness the GMR can be either normal or inverse; at small FeV layer thicknesses the interface scattering dominates resulting in normal GMR
- the crossover thickness of FeV layers lies between 2 and 3 nm

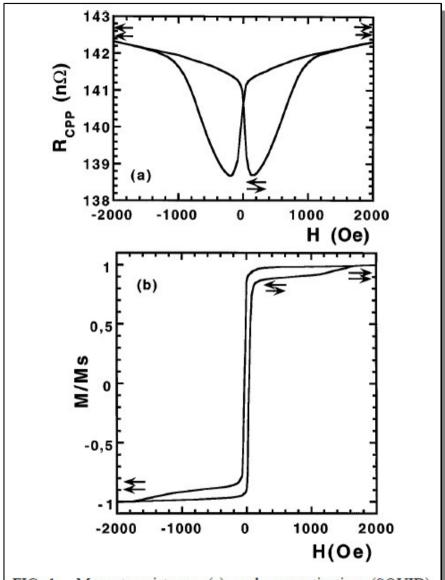
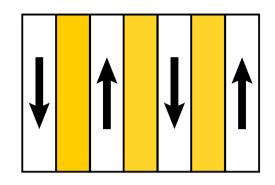
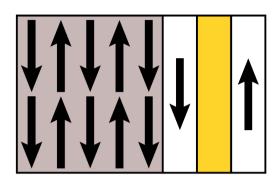
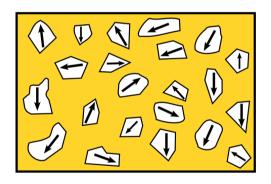


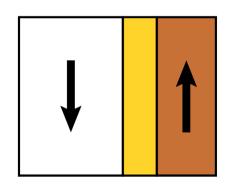
FIG. 1. Magnetoresistance (a) and magnetization (SQUID) (b) curves at 4.2 K for a (Fe₇₂V₂₈ 6 nm/Cu 2.3 nm/Co 0.4 nm/Cu 2.3 nm)₂₀ multilayer.

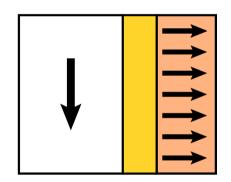
Typical GMR systems









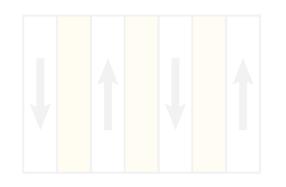


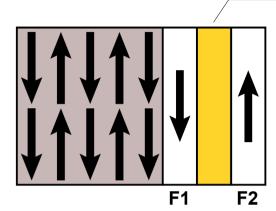
- nonmagnetic conductor
- different ferromagnetic conductors
 - antiferromagnet

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Magnetoresistance

Typical GMR systems





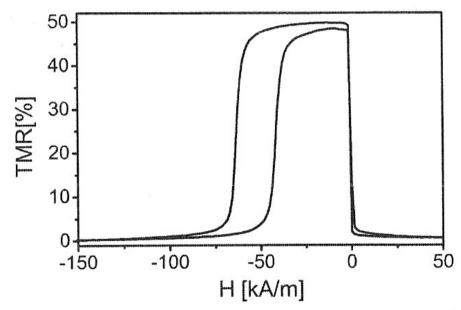


Fig. 6 The room temperature TMR curve of $Cu(30 \text{ nm})/Ni_{80}Fe_{20}(4 \text{ nm})/Mn_{83}Ir_{17}(15 \text{ nm})/Co_{70}Fe_{30}$ (2 nm)/Al(1.4 nm) + Ox/Ni₈₀Fe₂₀(4 nm)/Ta(3 nm)/ Cu(55 nm)/Au (20 nm) multilayer.

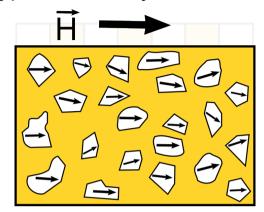
GMR (or TMR*) in systems with exchange bias

insulator

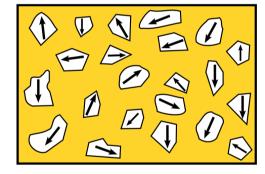
- magnetization of ferromagnetic layer F1 "fixed" by the anisotropy of the antiferromagnetic layer
- magnetization of the F2 layer is "free" to rotate in external magnetic field
- both conducting and insulating spacer may be used
- very high field sensitivities of the effect achievable

image from: M. Urbaniak, J. Schmalhorst, A. Thomas, H. Brückl, G. Reiss, T. Luciński, F. Stobiecki Phys. Stat. Sol. (a) **199**, 284 (2003)

Typical GMR systems



$$\vec{H} = 0$$



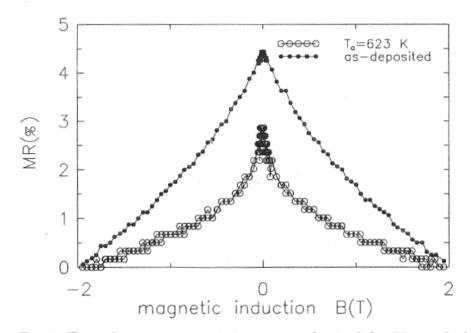


Fig. 1. Exemplary magnetoresistive curves obtained for 76 nm thick ${\rm Co_{20}Ag_{80}}$ film in as-deposited state and after annealing at 623 K

Granular GMR (G²MR)

 magnetic grains in nonmagnetic matrix (content below percolation threshold)

directions of the magnetic

moments within the grain depend on external field and on the effective magnetic anisotropy of the grain (shape, magnetocrystalline etc.) and on interactions

with other grains.

resistance saturates in high fields

- nonmagnetic conductor
- different ferromagnetic conductors
 - antiferromagnet

image from: M. Urbaniak, I. Gościańska, H. Ratajczak, Phys. Stat. Sol. (a) **160**, 121 (1997)

Typical GMR systems

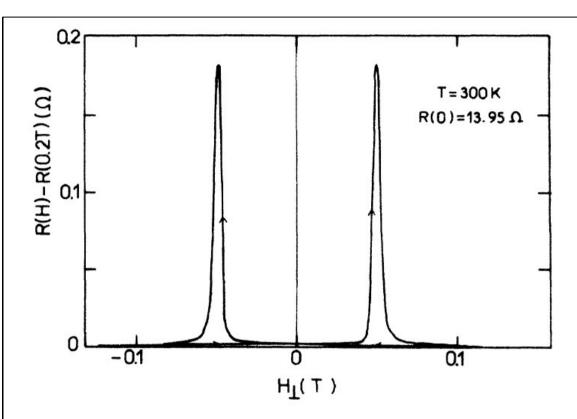
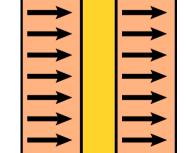


FIG. 3. Room-temperature perpendicular magnetoresistance of sample 3: Au/Co (0.76 nm)/Au (3 nm)/Co (0.76 nm)/Au. The coercive field is H_c =493 G and $\delta R_c/R$ =1.3%.

GMR in systems with perpendicular magnetic anisotropy

- two Co layers with slightly different coercive fields
- first observation of GMR –
 before "Nobel papers" by
 A. Fert and P. Grünberg; the
 explanation (three different
 mechanisms proposed) of the
 effect was not correct



different ferromagnetic conductors

antiferromagnet

image from: E. Vélu, C. Dupas, D. Renard, J.P. Renard, J. Seiden, Phys. Rev. B 37, 668 (1988)

Typical GMR systems

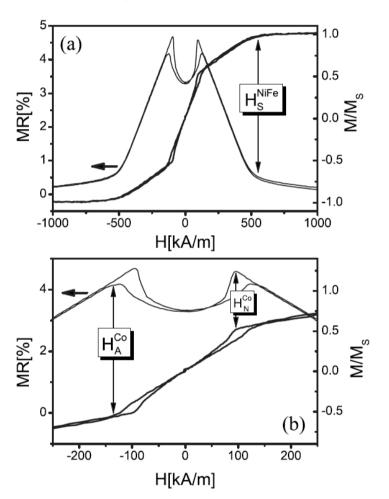
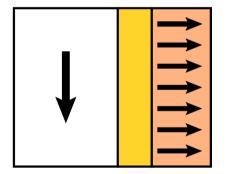


FIG. 1. The magnetization hysteresis and the current-in-plane magnetoresistance of sputtered [Ni $_{80}$ Fe $_{20}$ (2 nm)/Au(2.4 nm)/Co(1.1 nm)/Au(2.4 nm)] $_{10}$ ML measured with the magnetic field applied perpendicularly to the sample plane (source: Ref. 3). Panel (b) shows a small field range. $H_S^{\rm NiFe}$ denotes a saturation field of the NiFe layers; $H_N^{\rm Co}$ and $H_A^{\rm Co}$ denote nucleation and annihilation fields of the Co domain structure, respectively.

GMR in systems with alternating direction of magnetic anisotropy

- maximal angle between magnetic moments of neighboring magnetic layers approx. 90 deg
- in MIs the domain structure of one layer can influence the reversal of the second layer
- approx. linear* dependence of resistance on the applied perpendicular field strength – sensor applications





onductors

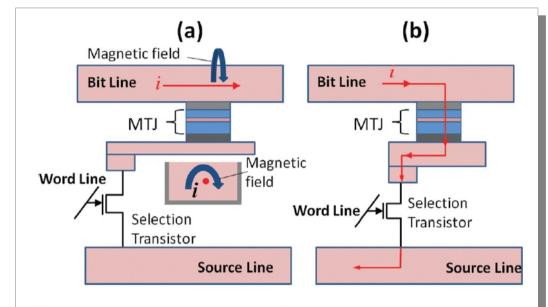
image from: M. Urbaniak, J. Appl. Phys. 104, 094909 (2008)

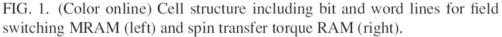
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Magnetoresistance

Switching of magnetic moments by a spin-torque [24]

- spin polarized electrons can transfer their magnetic moment to ferromagnetic layer
- the layer oscillates or switches (changes its magnetization orientation)
- the smaller memory cells require in general higher magnetic field to switch magnetization direction. "This has an implication on the length of the selection transistor, which has to be large to enable sufficient current for the reversal of the FL magnetization. As a consequence, it becomes difficult to achieve large storage capacity using field switching" Sbiaa et al. [24]
- spin torque transfer random access memories* (STT-RAM) do not require electrodes providing magnetic field – simpler fabrication





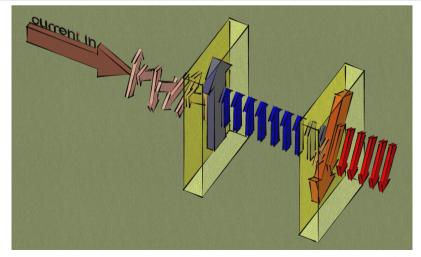


image from R. Sbiaa, S.Y.H. Lu, R. Law, H. Meng, R. Lye, H.K. Tan, J. Appl. Phys. **109**, 07C707 (2011)

Electric field-induced magnetization reversal [25]

nanometers

- Ta (5)/Ru (10)/Ta (5)/Co_{0.2}Fe_{0.6}B_{0.2} (0.9)/MgO(1.4)/Co_{0.2}Fe_{0.6}B_{0.2} (1.8)/Ta (5)/Ru(5) is deposited by rf magnetron sputtering on an Al₂O₃ substrate
- CoFeB layers possess a perpendicular magnetic anisotropy
- application of the electric field (through bias voltage) temporarily changes the effective anisotropy of CoFeB
- the magnetization reversal takes place when bias voltage pulse duration is equal to half period of magnetization precession
- it is hoped that electric field-induced switching will require less energy to write a single bit of information than conventional methods (magnetic field or STT); sub pJ energies are sufficient to switch magnetization of a TMR stack with SST [26]

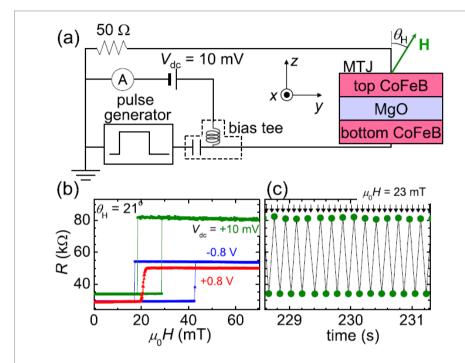


FIG. 2. (a) Schematic of experimental setup. (b) Minor resistance versus magnetic field curves as a function of dc bias voltage. The magnetic field is applied along $\theta_{\rm H}\!=\!21^\circ$. (c) Resistance measured after the application of voltage pulses of 0.9 V with 1.8 ns duration. Arrows indicate the time at which voltage pulse is applied. The applied magnetic field is 23 mT along $\theta_{\rm H}\!=\!21^\circ$ and dc bias for measurements is 10 mV.

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- OpenOffice www.openoffice.org
- Inkscape inkscape.org
- POV-Ray www.povray.org
- Blender www.blender.org
- SketchUp sketchup.com.pl

I also used "Fizyczne metody osadzania cienkich warstw i metody analizy powierzchniowej" lectures by Prof. F. Stobiecki which he held at Poznań University of Technology in 2011.