TRANSPORT CHARACTERISTICS
OF FERROMAGNETIC SINGLE-ELECTRON TRANSISTORS

JUSTYNA WISNIEWSKA\(^1\), IRENEUSZ WEYMANN\(^1\), AND JÓZEF BARNAS\(^1,2\)

\(^1\)Department of Physics, Adam Mickiewicz University, 61-614 Poznań, Poland
\(^2\)Institute of Molecular Physics, Polish Academy of Sciences, 60-179 Poznań, Poland

Abstract: Transport of spin-polarized electrons in a single-electron transistor is analyzed theoretically in
the sequential tunneling regime. The central electrode (island) and the two external leads of the device
are ferromagnetic. The analysis is based on the master equation method, with the respective transition
rates determined from the Fermi golden rule. It is shown that electric current depends on magnetic
configuration of the system, and the resulting tunnel magnetoresistance is modulated periodically with
increasing bias voltage. Numerical results are in qualitative agreement with recent experimental obser-

1. INTRODUCTION

Electronic transport through single-electron mesoscopic devices was extensively studied
during the last decade, but mainly for nonmagnetic metallic islands or quantum dots (based on
two-dimensional electron gas) coupled to nonmagnetic reservoirs \[1\]. Transport properties of
metallic islands or quantum dots coupled to ferromagnetic leads were investigated only very
recently \[2-5\] – mainly theoretically, although some experimental results are already available,
too. In the latter case certain new effects arise from the interplay of charge discreteness,
energy levels quantization, and magnetism \[6-10\].

In this paper, we present results of our theoretical analysis of spin polarized electronic
transport in a single-electron transistor, whose two external electrodes and the central part
(island) are ferromagnetic. Transport characteristics of such a device depend on its magnetic
configuration, and this dependence stems from asymmetry between the spin-majority and
spin-minority electron bands in the corresponding ferromagnetic metals. The main objective
of the paper is to explain recent experimental data by Ernult \textit{et al.} \[11\], who observed periodi-
cally modulated tunnel magnetoresistance with increasing bias voltage \(V\).

In order to analyze transport characteristics of the device, we have employed the per-
turbation theory and limited considerations to the first order (sequential) tunneling processes.
The corresponding tunneling probabilities have been derived from the Fermi golden rule,
whereas the relevant probabilities of different charge states have been determined from the ap-
propriate master equations. We have analyzed numerically electric current flowing through the
system in both parallel and antiparallel configurations, as well as the resulting tunnel
magnetoresistance. The magnetoresistance is shown to vary periodically with increasing bias
voltage, in agreement with Ref. \[11\].
In the following two sections we describe the system and outline the corresponding theoretical description. Apart from this, we present and discuss some of the relevant numerical results.

2. MODEL AND THEORETICAL DESCRIPTION

The system under consideration consists of three ferromagnetic electrodes. The central electrode, referred to as an island in the following, is assumed to be extremely small, which results in its very low capacitance. Consequently, the energy needed to put an additional electron on the island becomes large enough to establish a new relevant energy scale - so-called charging energy. This, in turn, leads to some charging effects, which may be observed in current-voltage transport characteristics. Moreover, by applying capacitively a gate voltage to the island, it is possible to control functionality of the system. The schematic of a ferromagnetic single-electron transistor is displayed in Fig. 1, where two possible magnetic configurations are also indicated.

![Fig. 1. A schematic of the ferromagnetic single-electron transistor. The system can be in two magnetic configurations, as indicated. The barrier resistances $R_1$ and $R_2$ are spin dependent.](image)

The central electrode (island) is coupled to external leads by tunnel barriers. Thus, the current can flow through the system due to consecutive tunneling processes. Generally, one can distinguish between the first-order and second-order tunnel events. The first-order (sequential) tunneling takes place when the applied bias voltage exceeds a certain threshold voltage. If this is the case, the electrons tunnel one by one through the system. However, if the transport voltage is lower than the threshold voltage, the sequential tunneling is exponentially suppressed and the system is in the Coulomb blockade regime. Although the first-order processes are then suppressed, the current can still be mediated by the higher-order tunneling events, such as cotunneling. Cotunneling is a correlated in time tunneling process of two electrons, that takes place via some virtual intermediate states of the system. These processes do not change charge state of the island, and therefore are not blocked by the charging energy.
In our considerations, however, we take into account only sequential processes. This is justified if the resistances of both barriers are much larger than the quantum resistance, \( R_i \gg R_Q = h/e^2 \ (i = 1, 2) \), and out of the Coulomb blockade regime. Further, we consider only spin-conserving tunneling processes. Additionally, the island is assumed to be relatively large, which implies that the quantization effects of the corresponding energy levels can be neglected. Finally, we assume fast spin relaxation on the island, so the effects due to spin accumulation are absent.

In order to calculate electric current flowing through the system, one has to calculate the rates of the corresponding tunneling processes. The first-order tunneling rates can be determined with the aid of the Fermi golden rule. The rate for tunneling of spin polarized electrons onto (out of) the island through the \( i \)-th barrier associated with a change of the island charge state from \( n \) to \( n + 1 \) (\( n \) to \( n - 1 \)) is given by

\[
\Gamma_i^{\sigma} (n \to n \pm 1, V) = \frac{1}{e^2 R_{i\sigma}} \frac{\Delta E_i(n \to n \pm 1, V)}{\exp \left( \frac{\Delta E_i(n \to n \pm 1, V)}{k_B T} \right) - 1},
\]

with \( T \) denoting the temperature, \( R_{i\sigma} \) being the spin-dependent resistance of the \( i \)-th barrier, and \( \Delta E_i(n \to n \pm 1, V) \) describing a change of the system energy caused by the respective tunneling events. This energy change is given by

\[
\Delta E_i(n \to n \pm 1) = \frac{e^2}{2C_x} \pm eU_i(n),
\]

where \( C_x = C_1 + C_2 + C_g \) is the total capacitance of the island, \( C_1 \) and \( C_2 \) are the capacitances of the two junctions, and \( C_g \) is the gate capacitance. In Eq. (2) \( U_i(n) \) denotes the voltage drop between the \( i \)-th electrode and the island, \( U_i(n) = C_x^{-1}[ne + (C_j + C_g)V_i - C_jV_j - C_gV_g] \), where \( V_i \) is the electrostatic potential of the \( i \)-th electrode and \( j \neq i \ (j = 1, 2) \), whereas \( e \) is the absolute value of the electron charge (\( e > 0 \)).

Having found the tunneling rates, one can set up the corresponding steady-state master equation, which allows us to determine the probability \( P(n, V) \) of having \( n \) additional electrons on the island when a bias voltage \( V \) is applied. This probability can be obtained using the recursion relation [12]:

\[
P(n + 1, V) \sum_{\sigma} \left[ y^{\sigma} (n + 1, V) \right] = P(n, V) \sum_{\sigma} \left[ x^{\sigma} (n, V) \right],
\]

where \( x^{\sigma} (n, V) = \sum_{i=1,2} \Gamma_i^{\sigma} (n \to n + 1, V) \) and \( y^{\sigma} (n, V) = \sum_{i=1,2} \Gamma_i^{\sigma} (n \to n - 1, V) \) corresponding to the rates for tunneling into and out of the island, respectively. Finally, the electric current flowing through the system can be calculated as

\[
I(V) = -e \sum_{\sigma} \sum_{n=-\infty}^{\infty} \left[ \Gamma_i^{\sigma} (n \to n + 1, V) - \Gamma_i^{\sigma} (n \to n - 1, V) \right] P(n, V).
\]
3. NUMERICAL RESULTS AND DISCUSSION

Now, we will present results of our numerical calculations of the corresponding transport characteristics. Tunnel magnetoresistance is characterized by the relative change of the resistance when magnetic configuration varies from antiparallel to parallel, and is described quantitatively by the factor \( TMR = \frac{(R_{ap} - R_p)}{R_p} \times 100\% \), with \( R_p \) being the total resistance of the system in the parallel (antiparallel) configuration [13, 14]. The relevant parameters are taken in such a way that the resulting tunnel magnetoresistance fits well to the experimental data obtained in Ref. [11]. In particular, the distance between the two maxima in tunnel magnetoresistance and the absolute height of tunnel magnetoresistance at the first maximum are fitted to the experimental observations.

![Fig. 2. Currents (a) flowing through the system in the parallel \( (I_p) \) and antiparallel \( (I_{ap}) \) configurations and the resulting tunnel magnetoresistance (b) as a function of the bias voltage \( V \). The parameters assumed for numerical calculations are: \( T = 4.2 \, \text{K} \), \( C_1 = 0.1 \, \text{aF} \), \( C_2 = 1 \, \text{aF} \), \( C_3 = 4.8 \, \text{aF} \), \( V_1 = V/2 \), \( V_2 = -V/2 \), and \( V_g = 0 \). The resistances in the parallel configuration are \( R_p^\uparrow = 0.415 \, \text{M}\Omega \), \( R_p^\downarrow = 0.075 \, \text{M}\Omega \), \( R_2^\uparrow = 5 \, \text{M}\Omega \), \( R_2^\downarrow = 2.5 \, \text{M}\Omega \), whereas in the antiparallel configuration \( R_{ap}^\uparrow = R_{ap}^\downarrow = \sqrt{R_2^\uparrow R_2^\downarrow} \) for \( i = 1, 2 \). These parameters correspond to the experimental data by Ernult et al. [11].](image)

The corresponding current voltage curves are shown in Fig. 2a for both parallel and antiparallel configurations, whereas the resulting tunnel magnetoresistance is shown in Fig. 2b. The Coulomb steps are well resolved for the parameters assumed for numerical calculations. Moreover, the Coulomb staircase in the parallel configuration is different from that in the antiparallel one, and this difference follows from the spin asymmetry and leads to the tunnel magnetoresistance displayed in Fig. 2b. The two local maxima seen in Fig. 2b occur at the Coulomb steps in Fig. 2a. The first maximum (at the threshold voltage) is however significantly larger than the second one.

In conclusion, we have calculated current voltage characteristics and tunnel magnetoresistance in a ferromagnetic single-electron transistor and fitted tunnel magnetoresistance curves to those observed experimentally by Ernult et al. [11]. The agreement between theory based on sequential tunneling is quite satisfactory, except the Coulomb blockade regime.
(small voltages regime), where tunnel magnetoresistance is different from that observed experimentally. The difference is a consequence of the the fact that the dominant contribution to electric current in the Coulomb blockade regime is due to cotunneling processes, which usually lead to a magnetoresistance ratio which is different from that obtained for sequential tunneling.

Acknowledgments

The work was supported by the Polish State Committee for Scientific Research through the projects PBZ/KBN/044/P03/2001 and 2 P03B 116 25.

References