A parametric model for global thermodynamic behavior of ultrasonic attenuation in magnetic field

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Abstract

The ultrasonic attenuation and velocity variations are theoretically investigated near the Curie temperature of ferromagnetic under an application of magnetic field. The temperature, frequency and magnetic field dependence of acoustic properties of ferromagnetic near a critical point is given. A parametric representation is used to describe a crossover from critical to classical region far away from the critical point. The crossover scaling functions are determined for sound attenuation coefficient and dispersion. We compare the proposed crossover model with experimental ultrasonic data for manganese phosphide MnP and find good agreement between theory and experiment.

Theoretical background

We consider Ising-like ($n = 1$) continuous order parameter on a d-dimensional elastic solid:

$$H = H_{\rm OP} + H_{\rm el} + H_{\rm int}, \tag{1}$$

where $H_{\rm OP}(S)$ is the Ginzburg-Landau part for the order parameter $S(\mathbf{x})$:

$$H_{\rm OP} = \int d^dx \left[\frac{1}{2}r_0 S(\mathbf{x})^2 + \frac{1}{2}(\nabla S)^2 + \frac{u}{4}S(\mathbf{x})^6 + \frac{v}{6}S(\mathbf{x})^4 - hS(\mathbf{x})\right] \tag{2}$$

$$H_{\rm el} = \frac{1}{2}\int d^dx \left\{ Bu_{ii}(\mathbf{x})^2 + 2\mu\left[u_{ij}(\mathbf{x}) - \frac{1}{d}\delta_{ij}u_{ll}(\mathbf{x})\right]^2 \right\} \tag{3}$$

is the elastic contribution in the harmonic approximation, with $u_{ij}(\mathbf{x})$ denoting the strain tensor. The interaction Hamiltonian is given by

$$H_{\rm int} = g\int d^dx u_{ii}(\mathbf{x})S(\mathbf{x})^2, \tag{4}$$

which describes the volume magnetostriction with the coupling constant g.

- **Landau-Ginzburg theory:**

  Only the relaxational term remains

$$\alpha(\omega, t, h) \sim g^2\omega^2 \frac{M^2(t,h)\chi_{GL}(t,h)^2}{1 + y_{GL}^2(\omega, t, h)} \tag{5}$$

h - magnetic field ω - sound frequency t - reduced temperature

$$\chi_{GL} = (at + 3uM^2 + 5vM^4)^{-1} \qquad \text{- susceptibility}$$

The product $M^2\chi_{GL}^2$ determines the sound attenuation exponent $\alpha \sim |t|^{-x_\pm}$ in the hydrodynamic regime where $\omega\tau_{GL} \ll 1$. The index $+$ refers to $T > T_C$ end $-$ to $T > T_C$. It is clear by a simple inspection of Eq. (5) that $x_- = 1$ for the mean-field behavior and $x_- = 3/2$ for the tricritical behavior. So, in the Landau-Ginzburg theory the effective sound attenuation exponent x_-^{eff} in the low-temperature phase may take an intermediate value between 1 and 3/2 depending on the value of the ratio $Q \equiv v/u$ and the range of the reduced temperature [1]

- **Critical theory:**

$$\alpha(t, \omega, h) = \omega^2|t|^{-x}F_\pm(\omega|t|^{-z\nu}, h|t|^{-\Delta})$$

$\Delta = \gamma + \beta$ - the gap exponent; x -the sound attenuation exponent ($x = z\nu \pm \alpha$ for metals and $x = 2\alpha$ for isolators); z - dynamic critical exponent; $F_\pm$ - scaling functions.

Linear Parametric Representation

As regards the explicit representation of the critical behavior it is frequently more convenient to express everything in the parametric representation of an asymptotic equation of state. The reason for that is that many scaling functions are given by various approximations of renormalization group theory and are sometimes very cumbersome and inconvenient for practical use [2]. Usually the starting point is a simple linear model originally introduced by Schofield [3]:

$$h = h_0 r^{\beta\delta}\theta(1-\theta^2) \equiv h_0 r^{\beta\delta}h(\theta),$$

$$t = r(1 - b^2\theta^2) \equiv rk(\theta),$$

$$M = M_0 r^\beta \theta$$

The parameter r is non-negative and measures a "radial" distance from the critical point and θ (pseudoangle) specifies the location on the contour of constant r. In this representation all critical singularities are incorporated as power laws in the variable r, while the dependence on θ is kept analytic. The great advantage of the linear parametric representation is that it generates a closed form expressions for all thermodynamic functions. For example the susceptibility is given by

$$\chi = \frac{M_0}{h_0}r^{-\gamma}\frac{1 - (1-2\beta)b^2\theta^2}{1 - 3\theta^2 + b^2\theta^2(3\theta^2 - 1 + 2\beta\delta(1-\theta^2))}$$

β, δ and γ are usual exponents and the linear model constant b^2 is sometimes identified by "minimal-isation" condition

$$b^2 = \frac{\delta - 3}{(\delta-1)(2\beta-1)}.$$

proposed in Ref. [4].

Crossover Parametric Representation

The asymptotic parametric models are valid only in the vicinity of the critical point. In order to describe a crossover from critical to mean-field behavior a simple crossover parametric model is proposed:

$$h = h_0 g^{\beta\delta - 3/2} r^{3/2}\theta[Y^{(2\beta\delta-3)/2\Delta_s}(1-\theta^2) + vr\theta^4],$$

$$M = g^{\beta - 1/2}M_0 r^{1/2} Y^{(\beta-1/2)/\Delta_s}\theta$$

while the dependence of t on r and θ is left unchanged. g is a crossover parameter proportional to the Ginzburg number and Y is a crossover function given by the relation [2]:

$$1 - (1-\overline{u})Y(r) = \overline{u}(1 + \frac{\Lambda^2}{arY(r)^{(2\nu-1)/\Delta_s}})^{1/2}Y(r)^{\nu/\Delta_s}.$$

In the asymptotic critical limit ($r \to 0$) $Y \to (r/g)^{\Delta_s}$ the term θ^4 is very small and can be neglected so the linear model equations are recovered whereas far away from criticality ($r \to \infty$) $Y \to 1$ and above equation reduce to Ginzburg-Landau expression for the equation of state. Similar classical expressions are obtained for susceptibility and other thermodynamic quantities (see Appendix).

Crossover parametric sound attenuation coefficient

We obtained the crossover parametric attenuation coefficient and dispersion

$$\alpha(\omega, r, \theta) = A\omega g^{-\alpha}Y(r)^{-\alpha/\Delta_s} Im\left[f_{rel}(y,\theta,r) + \frac{A_{fluc}}{A_{rel}}f_{fluc}(y)\right], \tag{6}$$

$$c^2(\omega) - c^2(0) = 2Ac_0^3 g^{-\alpha}Y(r)^{-\alpha/\Delta_s} Re\left[f_{rel}(y,\theta,r) - f_{rel}(0,\theta,r) + \frac{A_{fluc}}{A_{rel}}(f_{fluc}(y) - f_{fluc}(0))\right] \tag{7}$$

As an illustration, we show here how the crossover parametric model can be applied to ultrasonic attenuation data obtained by Komatsubara et al. [4] for MnP. We have fitted the experimental data to Eqs. (6,7) using the critical temperature T_C and the attenuation (A), time (τ_0), temperature (a) and magnetic field (h_0) scales as well as $\frac{A_{fluc}}{A_{rel}}$, v, u and g as adjustable parameters.

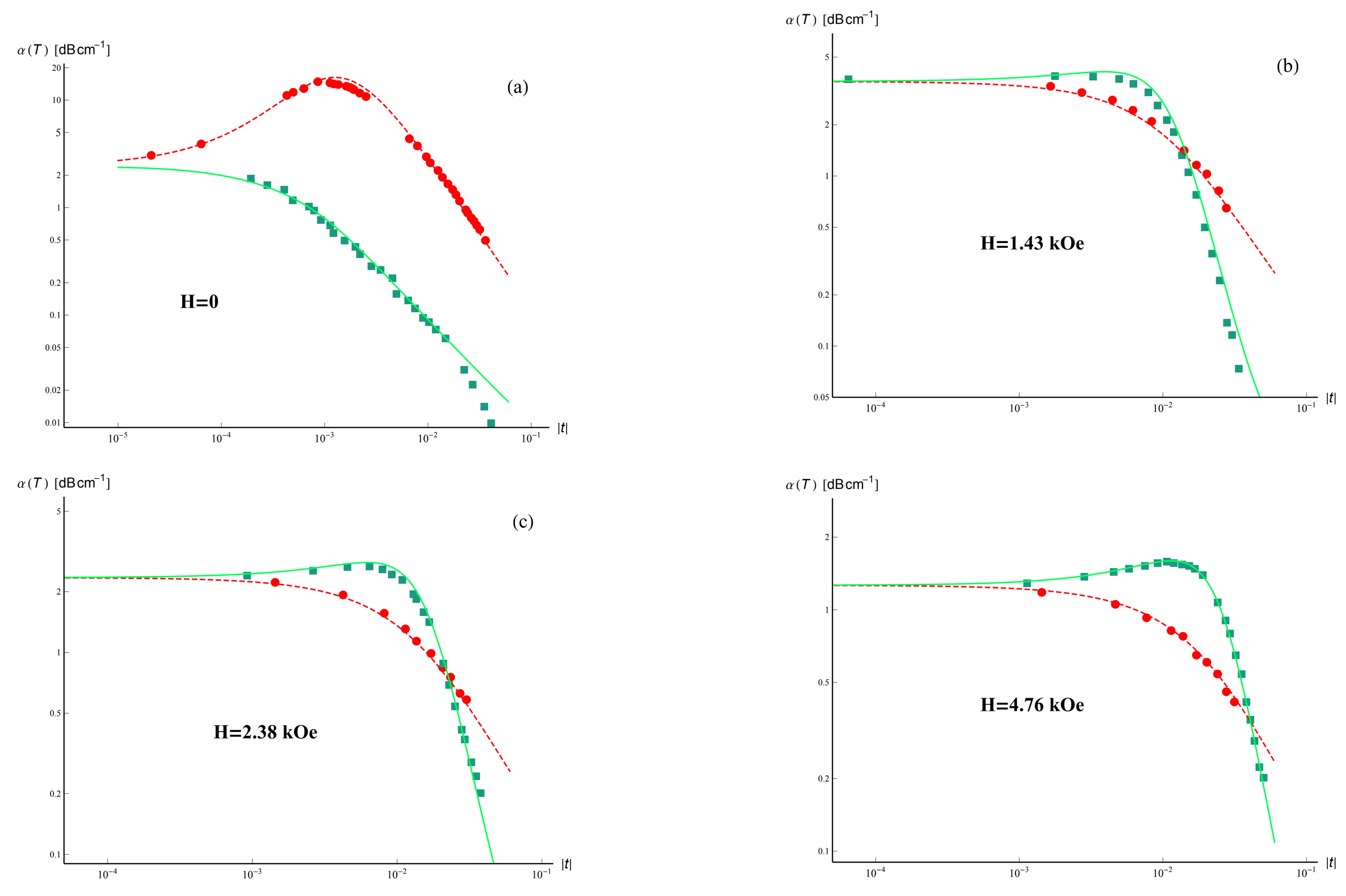

Fig1. Ultrasonic attenuation in MnP. The dashed lines correspond to $T < T_C$ and the continuous lines to $T > T_C$. The data are taken from Ref. [4].

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1 INTRODUCTION

The singular behaviour of the sound attenuation coefficient is connected with very strong fluctuations of the magnetic order parameter near the critical temperature. In such materials the maximum in the sound attenuation coefficient occurs in the ordered phase for vanishing magnetic field due to the domination of the magnetic analogue of the Landau-Khalatnikov sound damping contribution near the superfluid transition of liquid ^{4}He [1]. As was shown recently [2] away from the critical point a sixth order term in Landau-Ginzburg energy may be of importance. We show that the crossover parametric model is able to describe both the asymptotic critical behavior near the Curie temperature as well as the simple Landau-Ginzburg behaviour of attenuation with sixth-order term included to properly describe the tricritical limit [2]. We illustrate our findings with ultrasonic data in manganese phosphide MnP.

2 MODEL

2.1 Statics

$$H = H_{\mathrm{OP}} + H_{\mathrm{el}} + H_{\mathrm{int}}, \tag{1}$$

$$H_{\mathrm{OP}} = \int d^d x \left[\frac{1}{2} r_0 S(\mathbf{x})^2 + \frac{1}{2} (\nabla S)^2 + \frac{u}{4} S(\mathbf{x})^6 + \frac{v}{6} S(\mathbf{x})^4 - h S(\mathbf{x}) \right] \tag{2}$$

$$H_{\mathrm{el}} = \frac{1}{2} \int d^d x \left\{ B u_{ii}(\mathbf{x})^2 + 2\mu \left[u_{ij}(\mathbf{x}) - \frac{1}{d} \delta_{ij} u_{ll}(\mathbf{x}) \right]^2 \right\} \tag{3}$$

with $u_{ij}(\mathbf{x})$ denoting the strain tensor related to the displacement vector components $u_i(\mathbf{x})$ by

$$u_{ij}(\mathbf{x}) = \frac{1}{2} (\nabla_i u_j + \nabla_j u_i).$$

$$H_{\mathrm{int}} = g \int d^d x u_{ii}(\mathbf{x}) S(\mathbf{x})^2, \tag{4}$$

In the magnetic part $r = at$ is linear in the reduced temperature $\frac{T - T_C}{T_C}$ and the parameters u are v are coupling constants.

2.2 Dynamics

In an isotropic solid, which is considered here for simplicity, the transverse sound decouple from the order parameter and will be neglected. We consider only the longitudinal sound and order-parameter modes which are mutually coupled to each other.

$$\dot{S}_{\mathbf{k}} = -\Gamma \frac{\delta H}{\delta S_{-\mathbf{k}}} + \xi_{\mathbf{k}}, \qquad \ddot{u}_{\mathbf{k}} = -\frac{\delta H}{\delta u_{-\mathbf{k}}} - \Theta k^2 \dot{u}_{\mathbf{k}} + \eta_{\mathbf{k}}, \tag{5}$$

where the index 'longitudinal' for the elastic mode has been omitted. The Fourier components of the Gaussian white noises ξ nd η have variances related to the bare damping terms $\Gamma$and Θk^2 through the usual Einstein relations. Here, Θk^2 is responsible for the noncritical sound dumping and Γ is a relaxation coefficient of the order parameter.

2.3 Critical attenuation

In the framework of dynamic renormalization group we use here the direct perturbational method [4, 5] with sharp cutoff $\Lambda = 1$ and coupling constants $u = u^*$, $v = 0$ equal its fixed point values. The one-loop diagrams contributing to Π are shown in Fig. 1. Exponentiation of logarithms gives the acoustic

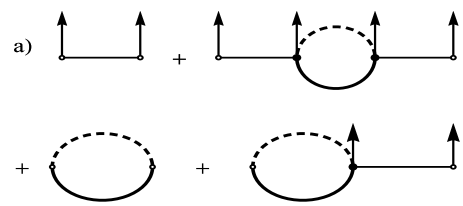

Figure 1: Fig. 1. One-loops diagrams for the acoustic self-energy. Full lines represent the spin response function and the dashed lines represent the correlation function. The arrows represent the static magnetization M and the black points the coupling constant u.

self-energy:

$$\Sigma_{\rm ac} = \chi^{\alpha/\gamma} f(y,s), \tag{Phi}$$

where $\chi(t,h)$ is the static susceptibility and f is a scaling function of reduced frequency $y = \omega/(\Gamma b \chi^{-z\nu/\gamma})$ where $s = 2tm^{-1/\beta}$ and $m^2 = 2u^* M^2$, and $z = 2 + c\eta$ is a dynamic exponent for universality class A and ν, γ and η are usual static critical exponents. For calculating the attenuation scaling function we used an approximation: $f = A_{\rm rel} f_{\rm rel} + A_{\rm fluc} f_{\rm fluc}$ where $A_{\rm rel}$ and $A_{\rm fluc}$ are critical amplitudes and the relaxational and fluctuation scaling function are found to first order in ϵ:

$$f_{\rm rel}(y,s) = P(s)^{2\beta/\nu} \frac{1 - 6u^*\,{\rm K}_d\,\pi_1(y)}{1 - iy - 9u^*\,{\rm K}_d\,\pi_2(y,s)}, \tag{6}$$

$$f_{\rm fluc}(y) = \frac{iz\nu}{2\alpha y}\,{\rm K}_d \left[\left(1 - \frac{iy}{2}\right)^{1-\frac{\alpha}{z\nu}} - 1 \right] + O(\epsilon^2), \tag{7}$$

where $\pi_1(y) = \frac{i}{y}(1 - \frac{iy}{2}) \ln(1 - \frac{iy}{2})$ and $\pi_2(y,s) = (\frac{2}{s+3})[-\frac{1}{2} + \frac{iy}{8} + \frac{i}{y}(1 - \frac{iy}{2}) \ln(1 - \frac{iy}{2})$ with ${\rm K}_d = 2^{-d+1}\pi^{-d/2}/\Gamma(d/2)$.

3 Crossover parametric model

Various improvements of the linear parametric model has been proposed [3]. Many of them concentrate on the critical region with inclusion of the Wegner correction–to-scaling contributions and the others on the crossover from the asymptotic critical region to the regular classical (mean-field) behavior far away from the critical point. According to non-asymptotic renormalization group procedure and implementing a matching point proposed by Nicol and co-workers [6] Chen et al. [7] put forward the following crossover expression for the singular part of Helmholtz free energy in Ising system:

$$\Delta A_s = \frac{1}{2} t M^2 \mathcal{T}\mathcal{D} + \frac{\overline{u} u^* \Lambda}{4!} M^4 \mathcal{D}^2 \mathcal{U} - \frac{1}{2} t^2 \mathcal{K}, \tag{8}$$

where $\overline{u} = u/u^*$ and $\mathcal{T}$,$\mathcal{D}$,$\mathcal{U}$ and $\mathcal{K}$ are rescaling functions defined by

$$\mathcal{T} = Y^{(2\nu-1)/\Delta_s},\ \mathcal{D} = Y^{-\eta\nu/\Delta_s}, \mathcal{U} = Y^{\nu/\Delta_s}, \mathcal{K} = \frac{\nu}{\alpha \overline{u} \Lambda}(Y^{-\alpha/\Delta_s} - 1). \tag{9}$$

The parameter Λ is to be interpreted as a dimensionless wave number related to the actual cutoff wave number [6, 3]. The crossover function Y is given by the relation [3]:

$$1 - (1 - \overline{u})Y = \overline{u}(1 + \frac{\Lambda^2}{\kappa^2})^{1/2} Y^{\nu/\Delta_s}, \tag{10}$$

with κ (a parameter proportional to the inverse of correlation length and is a measure of the distance to the critical point) defined by: $\kappa^2 = t\mathcal{T} + \frac{1}{2}\overline{u} u^* \Lambda M^2 \mathcal{D}\mathcal{U}$.

In order to built a crossover parametric model able to describe the crossover from critical to mean-field behavior we relate the distance parameter r to the inverse of correlation length by equation

$$\kappa^2(r) = arY^{(2\nu-1)/\Delta_s}, \tag{11}$$

which reflects the observation that the parameter κ^2 plays a similar role to the distance variable r ($\kappa \to 0$ near the critical point and $\kappa \to \infty$ far away from the critical point). It is easy to see from Eq. (11) that $\kappa \sim r^\nu$ near the critical point and $\kappa \sim r^{1/2}$ far away the critical point. The crossover function is again given in implicit form by

$$1 - (1 - \overline{u})Y(r) = \overline{u}(1 + \frac{\Lambda^2}{arY(r)^{(2\nu-1)/\Delta_s}})^{1/2} Y(r)^{\nu/\Delta_s}. \tag{12}$$

The crossover function is only a function of r and is independent from the angle variable θ. It also depends on two parameters $\overline{u}$ and Λ^2/a called also crossover variables [3] which may be related to the Ginzburg number.

To completely define the crossover parametric model one need to specify the equations for h, t and M. For this purpose we modify the linear model equation as

$$h = h_0 g^{\beta\delta - 3/2} r^{3/2} \theta [Y^{(2\beta\delta - 3)/2\Delta_s}(1 - \theta^2) + \widetilde{v} r \theta^4], \tag{13}$$

$$M = g^{\beta-1/2} M_0 r^{1/2} Y^{(\beta-1/2)/\Delta_s} \theta \tag{14}$$

while the dependence of t on r and θ is left unchanged. Note that near the critical point $(r \to 0,\ Y \to (r/g)^{\Delta_s})$ the term $\widetilde{v} r \theta^4$ in the square brackets in Eq. (13) is very small and can be neglected so the linear model equations are recovered. The virtue of this parametrization is that in the other (classical) limit $r \to \infty$ these equation reduces to

$$h = r^{3/2}\theta[\widetilde{h}_0(1-\theta^2) + \widetilde{v} r \theta^4], \qquad M = \widetilde{M}_0 r^{1/2}\theta \tag{15}$$

which in turn leads to the very simple Ginzburg-Landau expression for the equation of state:

$$h = aM[t + \frac{u'}{a}M^2 + \frac{v'}{a}M^4]. \tag{16}$$

Similar classical expressions are obtained for susceptibility and other thermodynamic quantities.

We finally obtain the crossover parametric attenuation coefficient:

$$\alpha(\omega, r, \theta) = A\omega g^{-\alpha} Y(r)^{-\alpha/\Delta_s}\, \mathrm{Im}\left[f_{\mathrm{rel}}(y,\theta,r) + \frac{A_{\mathrm{fluc}}}{A_{\mathrm{rel}}} f_{\mathrm{fluc}}(y)\right], \tag{17}$$

As an illustration, we show here how the crossover parametric model can be applied to ultrasonic attenuation data obtained by Komatsubara et al. [8] for MnP.

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