2

Magnets and forces in magnetic field

Magnetic reversal in thin films and some relevant experimental methods

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Today's plan

- Permanent magnets, electromagnets
- Special sources of magnetic field
- Forces in magnetic field

Magnetic scalar potential

In many practical applications it can be assumed that the magnetization of the body is constant (in value and direction) in weak magnetic fields [1]. We have then (from L1*)

$$\nabla\cdot\vec{B}=0 \qquad\qquad \vec{B}=\mu_0(\vec{H}+\vec{M})$$

It follows that:

$$\nabla\cdot\mu_0(\vec{H}+\vec{M})=0 \;\rightarrow\; \nabla\cdot\vec{H}=-\nabla\cdot\vec{M}$$

If there are no free currents we can define magnetic scalar potential:

$$\boxed{\vec{H}=-\nabla\varphi}$$

Substituting this into the previous equation gives:

$$-\nabla\cdot\nabla\varphi=-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\varphi=-\nabla^2\varphi=-\nabla\cdot\vec{M}$$

Poisson's equation:

$$\boxed{\nabla^2\varphi=-\frac{\rho}{\varepsilon_0}}$$

Comparing $\nabla^2\varphi=\nabla\cdot\vec{M}$ with Poisson's equation we can formally introduce magnetic charges:

$$\boxed{\rho_{magn}=-\nabla\cdot\vec{M}}$$

* Lecture no. 1

Magnetic scalar potential

We have as a solution to Poisson's equation $\nabla^2\varphi=\nabla\cdot\vec{M}$ [4]:

$$\varphi(\vec{r})=-\frac{1}{4\pi}\int_V\frac{\nabla'\cdot\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3r'$$

$$(ab)'=ab'+a'b$$

Using the expression for a derivative of a product (integrating by parts) we obtain:

$$\varphi(\vec{r})=-\frac{1}{4\pi}\int_V\nabla'\cdot\left(\frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right)d^3r'+\frac{1}{4\pi}\int_V\vec{M}(\vec{r}')\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)d^3r'$$

And by the Gauss' theorem for the first term we have:

$$\varphi(\vec{r})=\frac{1}{4\pi}\int_{far\ surface}\frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}d\boldsymbol{S}'-\frac{1}{4\pi}\int_V\vec{M}(\vec{r}')\cdot\nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)d^3r'$$

We change sign of the second term using:

$$\nabla\left(\frac{1}{|r-r'|}\right)=-\nabla'\left(\frac{1}{|r-r'|}\right)$$

Since the magnetization vanishes at infinity (we seek the potential of bounded magnetization) the first integral vanishes. In the second integral we move ***M*** under nabla as ***M*** does not depend on ***r***. Finally we get:

$$\boxed{\varphi(\vec{r})=-\frac{1}{4\pi}\nabla\cdot\int_V\frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3r'}$$

Magnetic charges

We have (from L1*) the induction of magnetic dipole. We are looking for the magnetic scalar potential that gives that field $\mu_0 H$ [2].

$$\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\frac{3\hat{r}(\vec{m}\cdot\hat{r})-\vec{m}}{|\vec{r}|^3}$$

Setting:

$$\phi(\vec{r})=\boxed{\frac{\mu_0}{4\pi}\vec{m}\cdot\nabla'\frac{1}{|\vec{r}|}}=\frac{\mu_0}{4\pi}\vec{m}\cdot\nabla'\left(\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\right)=$$

$$\frac{\mu_0}{4\pi}\vec{m}\cdot\left(\frac{\vec{r}}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}}\right)=\frac{\mu_0}{4\pi}\left(\frac{(x-x')m_x+(y-y')m_y+(z-z')m_z}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}}\right)$$

and differentiating (with respect to unprimed coordinates) we get:

$$\mu_0\vec{H}=-\mu_0\nabla\phi(\vec{r})=\frac{\mu_0}{4\pi}\frac{3\hat{r}(\vec{m}\cdot\hat{r})-\vec{m}}{|\vec{r}|^3}$$

Integrating over volume containing magnetic moments we get the potential of the dipole distribution:

$\phi_m(\vec{r})=\int\vec{M}\cdot\nabla'\frac{1}{|\vec{r}|}d^3r'$, where $\vec{M}\,d^3r'$ is the moment of infinitesimal volume

Magnetic charges

Using now the identity $\nabla\cdot(f\vec{a})=\vec{a}\cdot\nabla f+f\nabla\cdot\vec{a}$ [11] we get:

$$\vec{M}\cdot\nabla'\frac{1}{|\vec{r}|}=\nabla'\cdot\left(\frac{1}{|\vec{r}|}\vec{M}\right)-\frac{1}{|\vec{r}|}\nabla'\cdot\vec{M}$$

Inserting this into the integral from the previous page we can rewrite:

$$\phi_m(\vec{r})=\int\vec{M}\cdot\nabla'\frac{1}{|\vec{r}|}d^3r'=\int\nabla'\cdot\left(\frac{1}{|\vec{r}|}\vec{M}\right)d^3r'-\int\frac{1}{|\vec{r}|}\nabla'\cdot\vec{M}\,d^3r'$$

From Gauss' theorem we get: $\int\nabla'\cdot\left(\frac{1}{|\vec{r}|}\vec{M}\right)d^3r'=\oint\frac{\vec{M}}{|\vec{r}|}\vec{ds}$

Finally, for the magnetic scalar potential of the bounded dipole distribution, we obtain:

$$\phi_m(\vec{r})=\oint_S\frac{\vec{M}\cdot\vec{ds}}{|\vec{r}|}-\int_V\frac{\nabla\cdot\vec{M}}{|\vec{r}|}d^3r'$$

Magnetic charges

Using now the identity $\nabla\cdot(f\vec{a})=\vec{a}\cdot\nabla f+f\nabla\cdot\vec{a}$ we get:

$$\vec{M}\nabla'\frac{1}{|\vec{r}|}=\nabla\cdot\left(\frac{1}{|\vec{r}|}\vec{M}\right)-\frac{1}{|\vec{r}|}\nabla\cdot\vec{M}$$

Inserting this into the previous integral we can rewrite:

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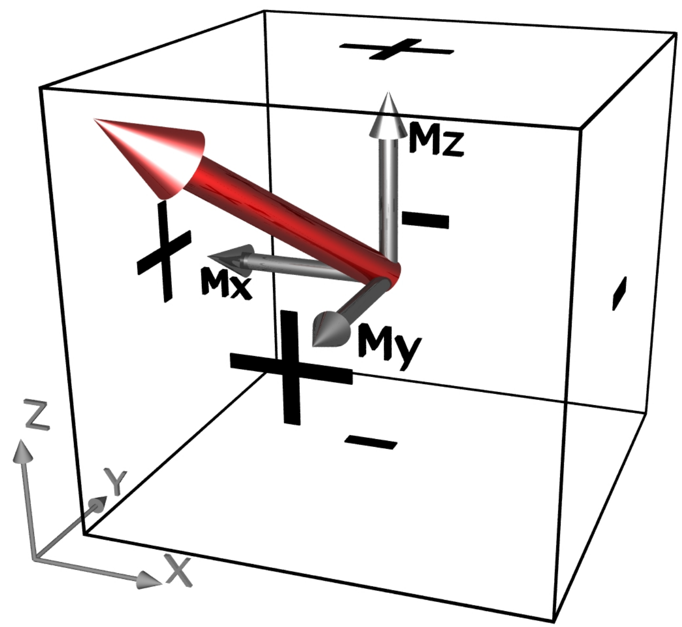
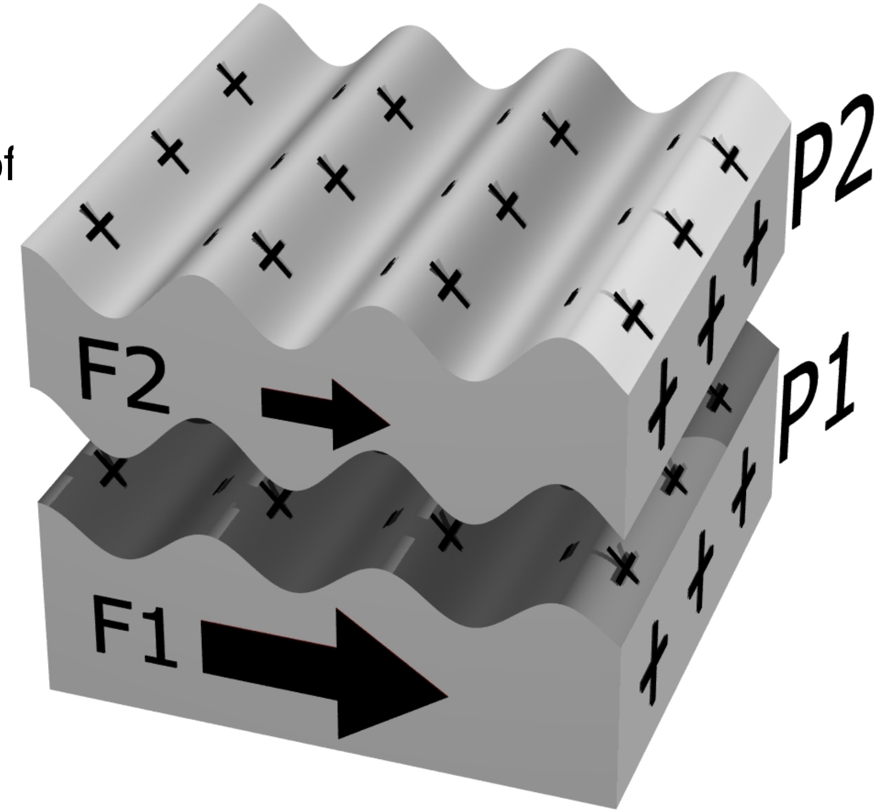
$\nabla\cdot\vec{M}$ volume magnetic charges

$\vec{M}\cdot\vec{ds}$ surface magnetic charges – magnetic poles



Magnetic charges

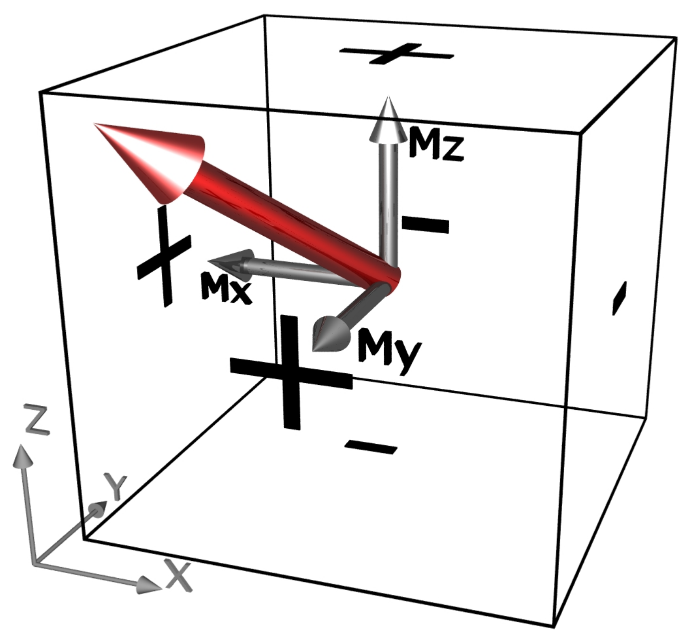
Example from the magnetostatic coupling of thin magnetic films (Neél's coupling):



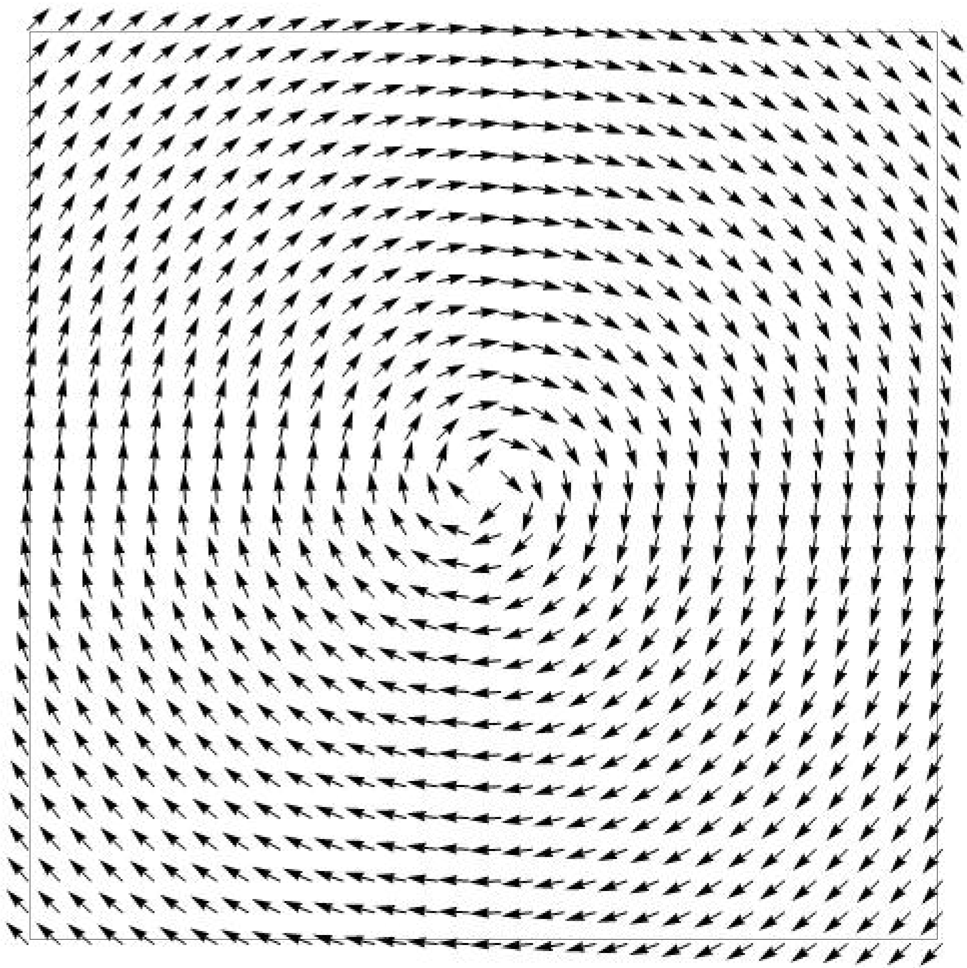
$$\phi_m(\vec{r})=\oint_S \frac{\vec{M}\cdot d\vec{s}}{|\vec{r}|}-\int_V \frac{\nabla\cdot\vec{M}}{|\vec{r}|}\, d^3 r'$$

$\nabla\cdot\vec{M}$ volume magnetic charges

$\vec{M}\cdot d\vec{s}$ surface magnetic charges – magnetic poles



Magnetic volume charges

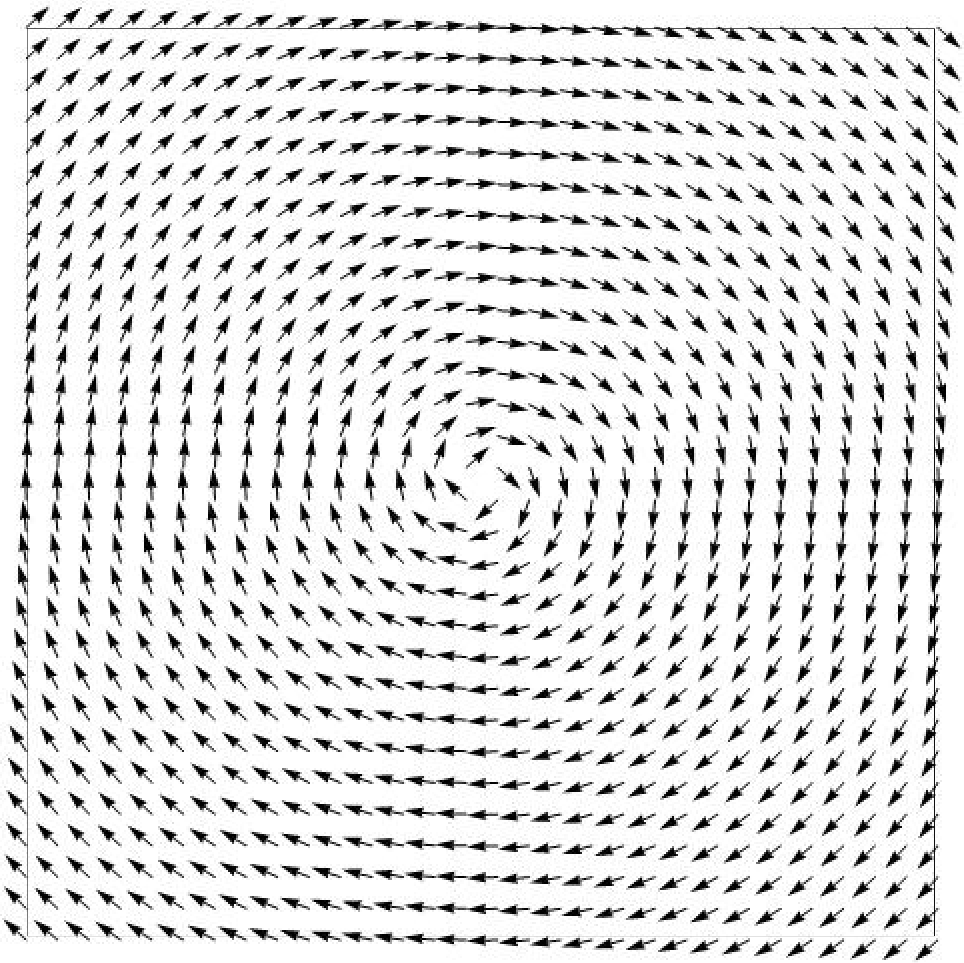


$$\vec{v} = \left(\frac{y}{\sqrt{x^2+y^2}}, -\frac{x}{\sqrt{x^2+y^2}}\right)$$

no volume (2D example) charges

Constant magnitude **divergenceless** vector field

Magnetic volume charges



$$\vec{v} = \left(\frac{1.1\ y}{\sqrt{x^2 + 1.21\ y^2}}, -\frac{x}{\sqrt{x^2 + 1.21\ y^2}} \right)$$

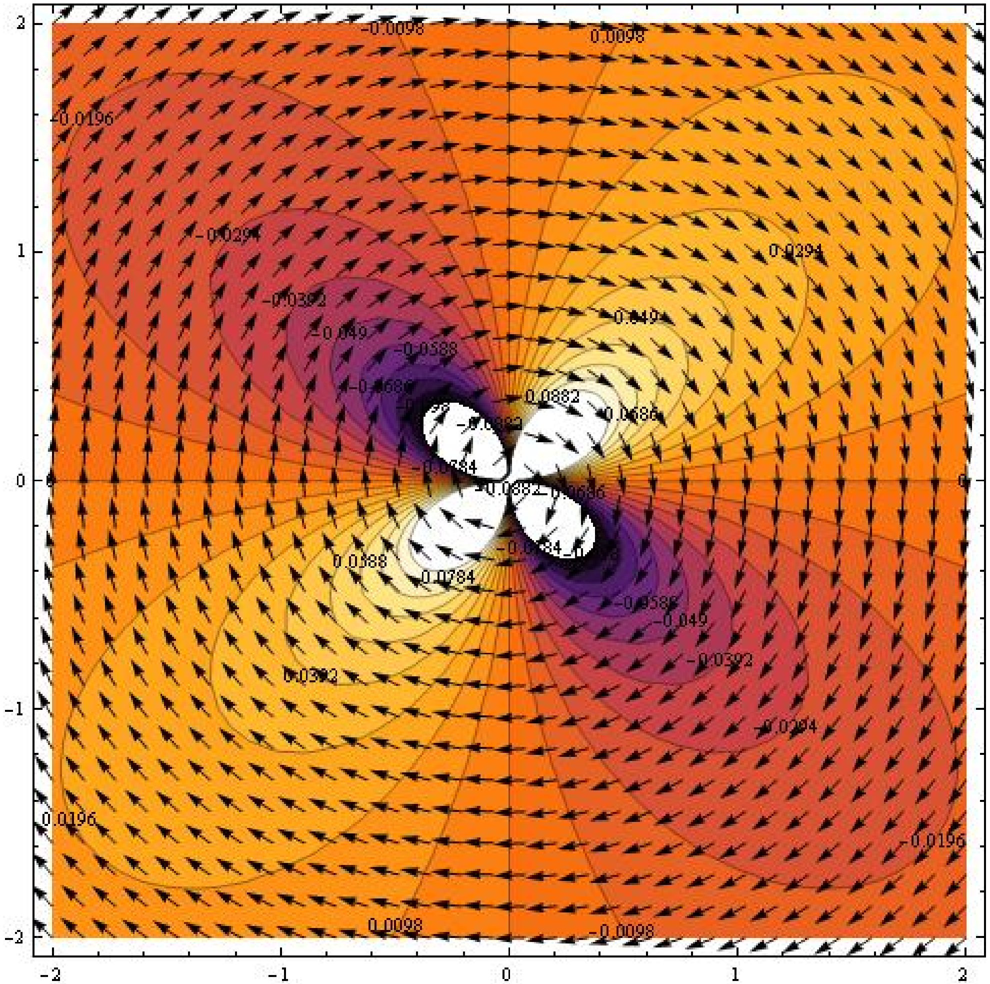
$$\nabla \cdot \vec{v} = \frac{0.11\, x\, y}{\left(x^2 + 1.21\ y^2\right)^{1.5}}$$

volume charges present

Constant magnitude vector field **with divergence**

Visual inspection of magnetic vector fields is of limited use.

Magnetic volume charges



$$\vec{v}=\left(\frac{1.1\ y}{\sqrt{x^2+y^2}},-\frac{x}{\sqrt{x^2+y^2}}\right)$$

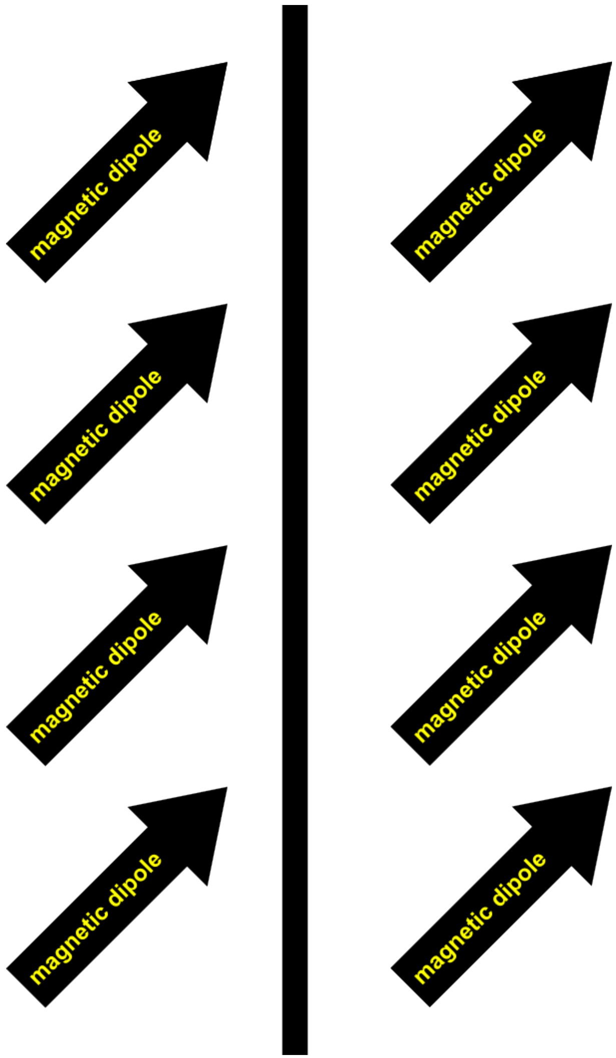
$$\nabla\cdot\vec{v}=\frac{0.11\,x\,y}{\left(x^2+1.21\,y^2\right)^{1.5}}$$

volume charges present

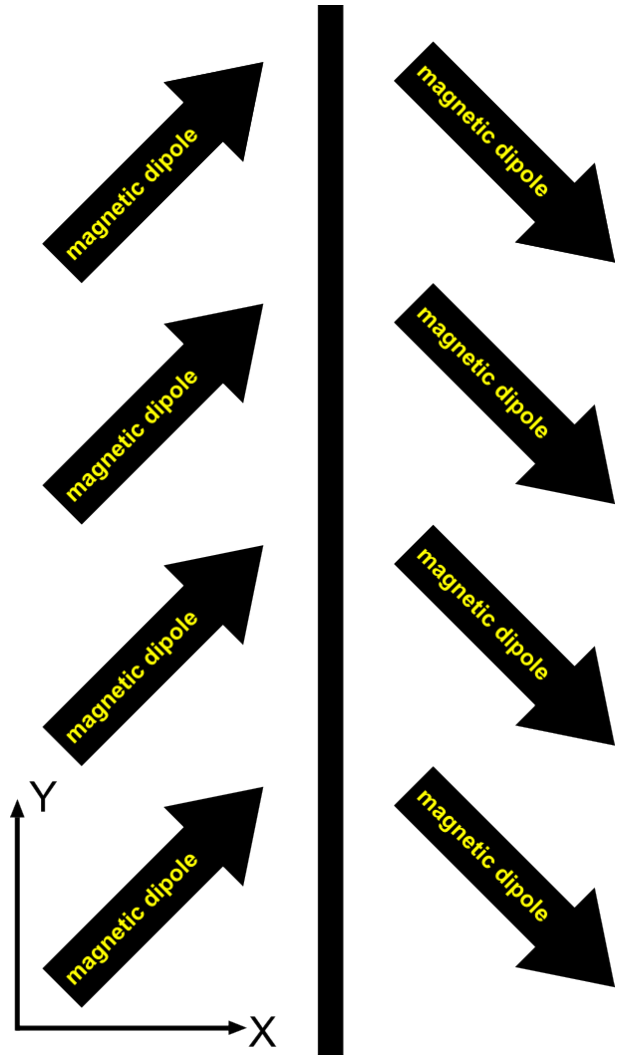
Constant magnitude vector field **with divergence**

Visual inspection of magnetic vector fields is of limited use.

Magnetic volume charges



no magnetic charges



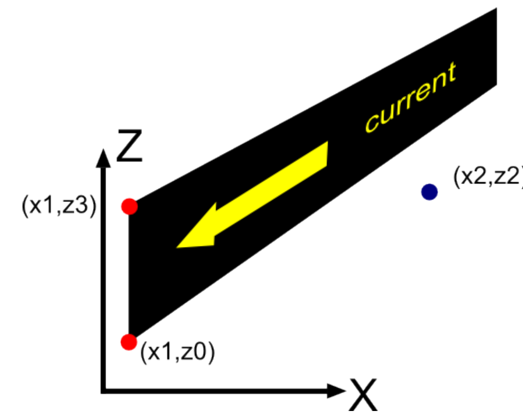
no magnetic charges

$\frac{\partial M_x}{\partial x}=0 \qquad \frac{\partial M_y}{\partial y}=0 \rightarrow \nabla \cdot \vec{M}=0 \qquad (M_z=const)$

Magnetic field of current sheet

Current sheet is an extension of current line (i.e. wire). It is a set of current lines bunched together to form a conducting stripe. We have from L.1 for the induction of straight wire carrying current *I*:

$$\vec{B}(R)=\frac{\mu_0 I}{2\pi R}$$



Integrating (the sheet extends from -∞ to +∞ along y-axis) we get:

$$B_z=\frac{\mu_0 M_S^{Co}}{2\pi}\left[\arctan\left(\frac{z_2-z}{x_2-x_1}\right)\right]_{z0}^{z3}$$

$$B_x=\frac{\mu_0 M_S^{Co}}{2\pi}\left[\ln\left[(x_2-x_1)^2-(z_2-z)^2\right]\right]_{z0}^{z3}$$

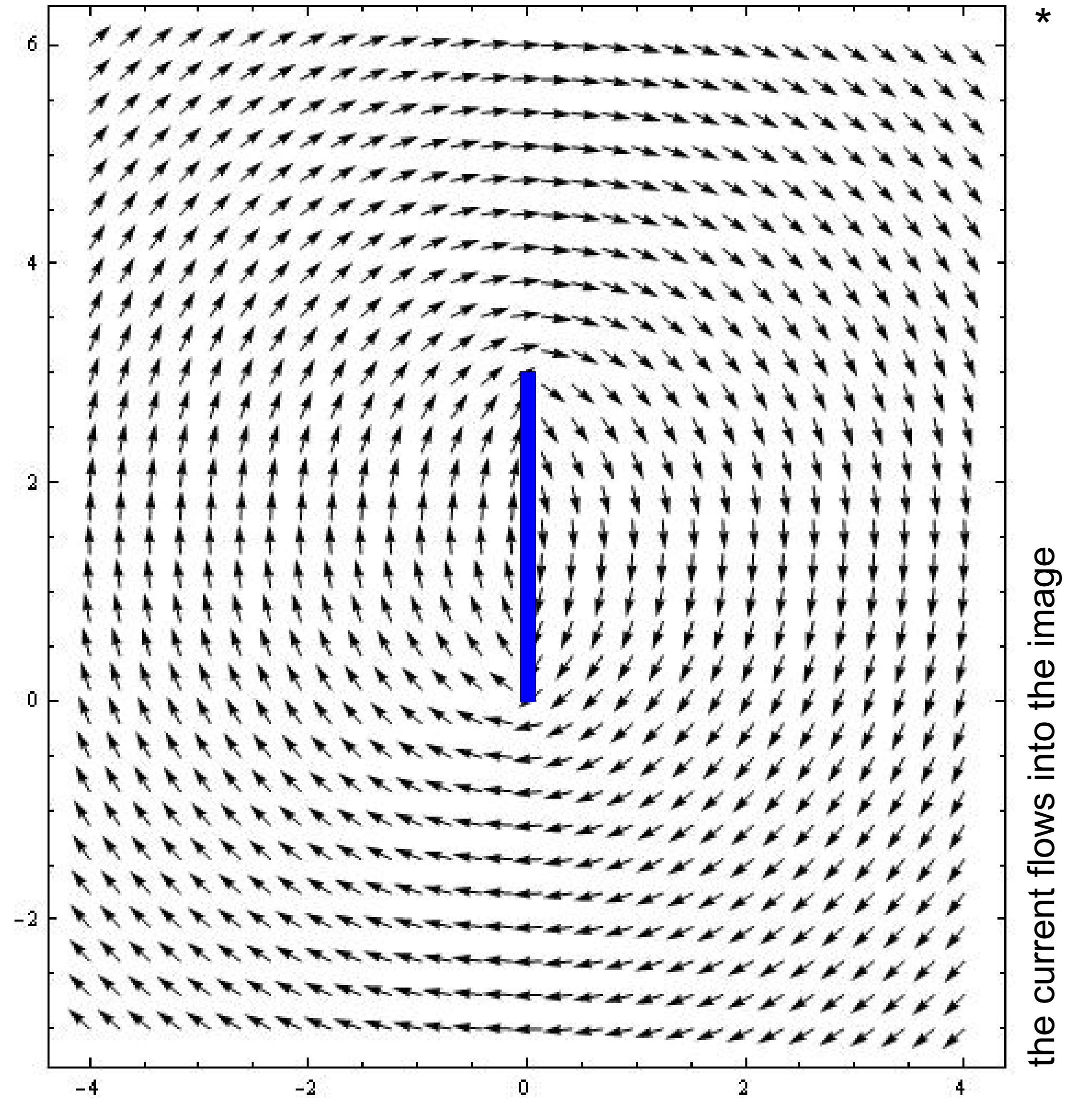
Magnetic field of current sheet

For the current sheet we have:

$$B_z = \frac{\mu_0 M_S^{Co}}{2\pi}\left[\arctan\left(\frac{z_2 - z}{x_2 - x_1}\right)\right]_{z0}^{z3}$$

$$B_x = \frac{\mu_0 M_S^{Co}}{2\pi}\left[\ln\left[(x_2 - x_1)^2 - (z_2 - z)^2\right]\right]_{z0}^{z3}$$

From far out the magnetic field of the current sheet resembles that of the current line



*for better viewing the plot shows only the direction of the field (vectors' length is constant)

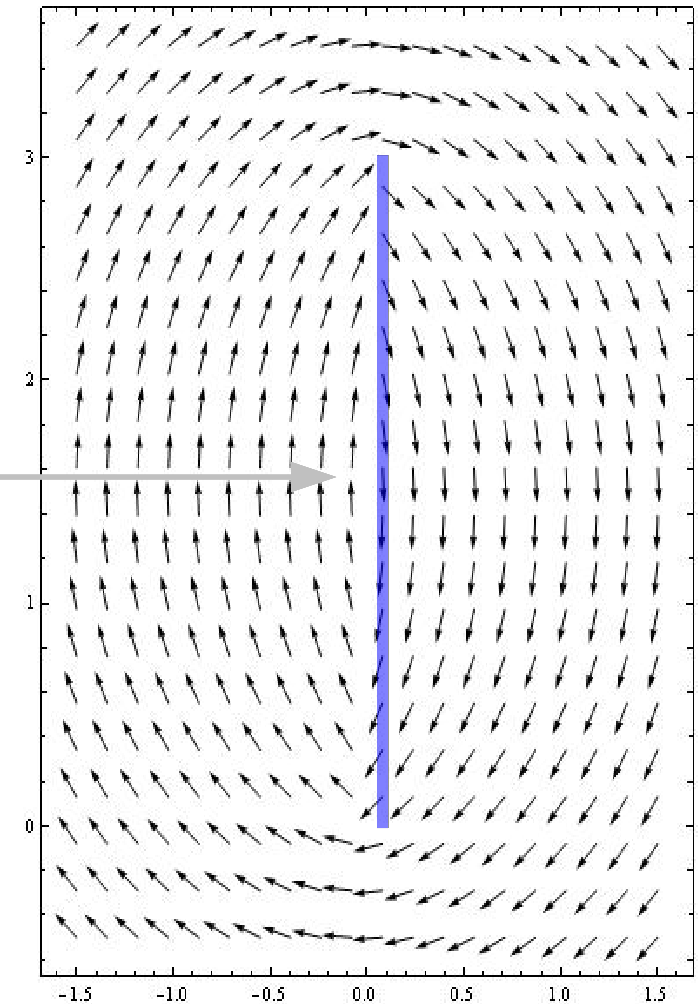
x1=0, z1=0, z3=3

Magnetic field of current sheet

In the vicinity of the sheet there is a discontinuity of a tangential component of the magnetic induction **B.**

$$B_z = \frac{\mu_0 M_S}{2\pi}\left[\arctan\left(\frac{z_2 - z}{x_2 - x_1}\right)\right]_{z0}^{z3}$$

$$B_x = \frac{\mu_0 M_S}{2\pi}\left[\ln\left[\left(x_2 - x_1\right)^2 - \left(z_2 - z\right)^2\right]\right]_{z0}^{z3}$$



*

the current flows into the image

*for better viewing the plot shows only the direction of the field (vectors' length is constant)

x1=0,z1=0,z3=3

Ampere's law

We have from L1: $\nabla \times \vec{B} = \mu_0 \vec{J}(\vec{r})$*. Using Stoke's theorem we obtain [3]:

$$\oint_{\text{closed curve}} \vec{B}(\vec{r}) \cdot d\mathbf{s} = \int_{\text{bounded surface}} \nabla \times \vec{B}(\vec{r}) \cdot d\mathbf{S} = \int_{\text{bounded surface}} \mu_0 \vec{J}(\vec{r}) \cdot d\mathbf{S} = \mu_0 I$$

Rewriting we find that:

$$\oint_{\text{closed curve}} \vec{B}(\vec{r}) \cdot d\mathbf{s} = \mu_0 I$$

which is *Ampere's Law*

The law can be used for calculating magnetic fields for symmetric current distributions

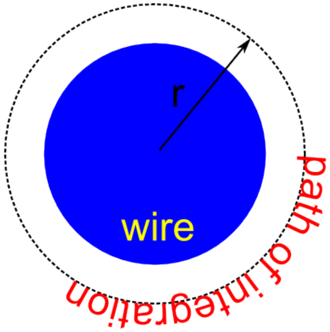
Example: Field within and outside the wire of radius R

a) outside the wire:

$$\oint \vec{B}(\vec{r}) \cdot d\mathbf{s} = 2\pi r B \;\rightarrow\; B = \frac{\mu_0 I_{total}}{2\pi r}$$

b) inside the wire:

$$I = I_{total} \frac{r^2}{R^2} \rightarrow B = \left(I_{total} \frac{r^2}{R^2} \right) \frac{\mu_0}{2\pi r} = \frac{\mu_0 I_{total}}{2\pi R^2} \; r$$



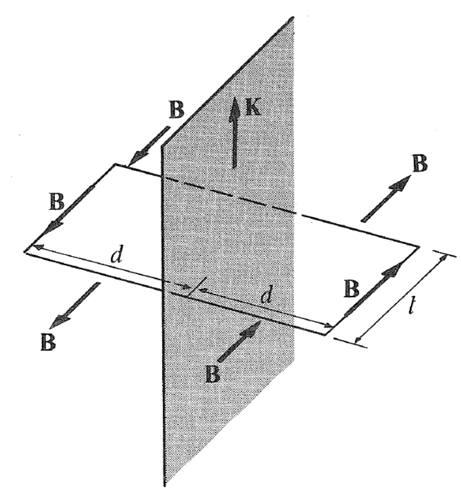
* this is called Ampere's law in differential form [4].

Discontinuity of *B* due surface current

We use Amperes's law to find B discontinuity [3].

The contour of integration:

- is planar
- is perpendicular to the current sheet
- is symmetrically placed with respect to the sheet
- has two of its sides parallel to ***B***



We have then:

$$\oint_{closed\ curve} \vec{B}(\vec{r})\cdot d\boldsymbol{s} = 2|\vec{B}|l \qquad\qquad \mu_0 I = \mu_0|\vec{K}|l$$

, where ***K*** is a surface current density (parallel to the boundary at every point).

Finally we get:

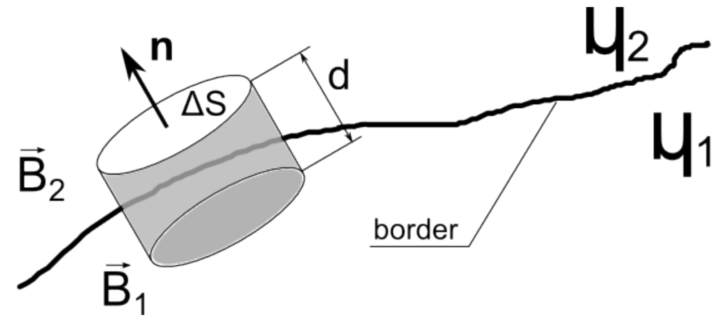
$|\vec{B}| = \frac{1}{2}\mu_0|\vec{K}|$ and, since ***B*** is symmetric with respect to current, for discontinuity we obtain:

$$\boxed{\Delta B = \mu_0|\vec{K}|}$$

or vectorially: $\vec{n}_2 \times (\vec{B}_2 - \vec{B}_1) = \mu_0 \vec{K}$

this is applicable to any surface current

Continuity conditions for magnetic field



We use the fact that ***B*** is divergenceless [4]:

- the surface integral of B over the cylinder surface should be zero

$$\int_{cylinder\ surface} \vec{B}(\vec{r})\cdot d\mathbf{s} = (\vec{B}_2\cdot\vec{n} - \vec{B}_1\cdot\vec{n})\Delta S + [side\ surface\ integral\ \propto\ d] = 0$$

Going d→0 we get:

$$\boxed{\vec{n}_2\cdot(\vec{B}_2 - \vec{B}_1) = 0}$$

In magnetostatics tangential components of magnetic induction ***B*** experience discontinuity due to the presence of surface currents. The normal components of B are always continuous.

Continuity conditions for magnetic field

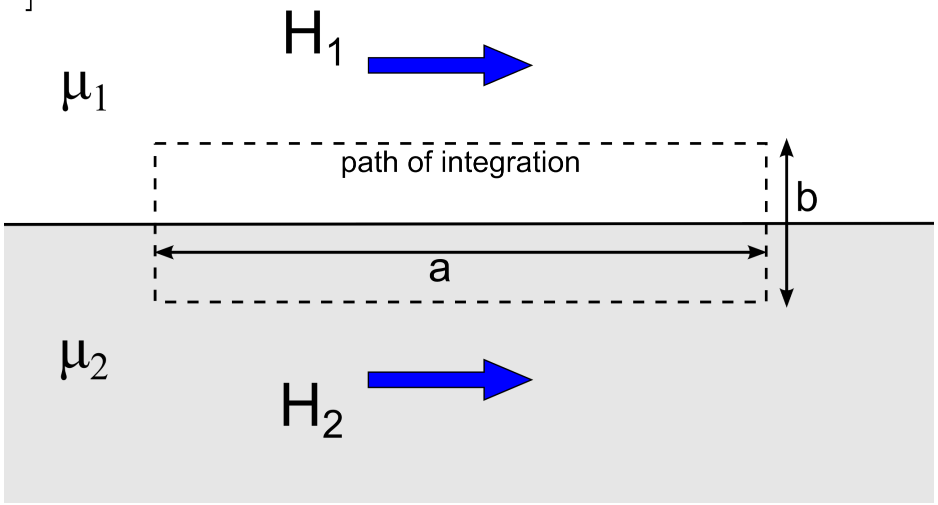
We use the fact that curl of ***H*** is null in the absence of surface currents [10]:

- the path integral of H over the rectangle edges should be zero

$$\oint_{rectangle\ edges} \vec{H}(\vec{r}) \cdot d\boldsymbol{l} = (H_1 a - H_2 a) + [side\ integral \propto b] = 0$$

Going b→0 we get:

$$\vec{n}_2 \times (\vec{H}_2 - \vec{H}_1) = 0$$



Continuity conditions for magnetic field

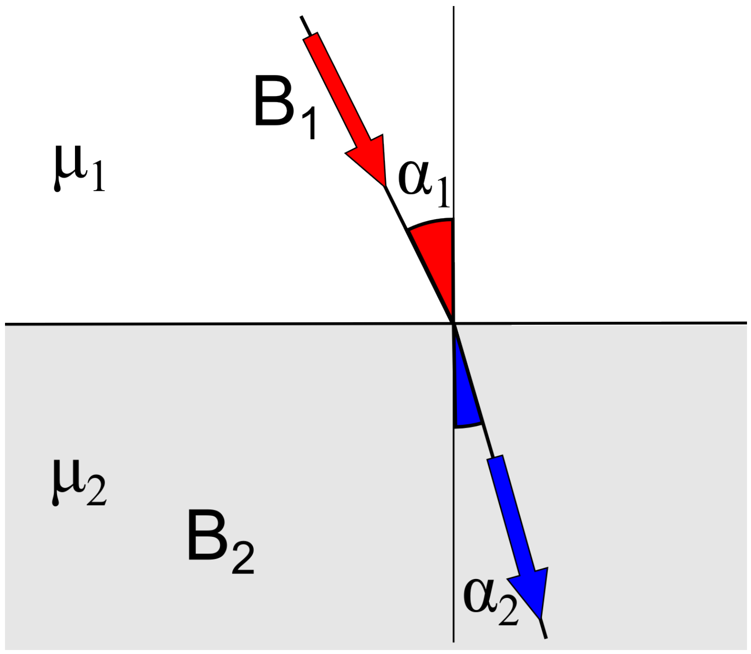
$$\vec{n}\cdot\vec{B}_2=\vec{n}\cdot\vec{B}_1 \qquad \vec{B}_2\times\vec{n}=\frac{\mu_2}{\mu_1}\vec{B}_1\times\vec{n}$$

$$\vec{n}\cdot\vec{H}_2=\frac{\mu_1}{\mu_2}\vec{n}\cdot\vec{H}_1 \qquad \vec{H}_2\times\vec{n}=\vec{H}_1\times\vec{n}$$

$$\vec{B}=\mu\ \vec{H}$$

•Law of refraction for magnetic lines [10]:

$$\frac{\tan\alpha_1}{\tan\alpha_2}=\frac{\mu_1}{\mu_2}$$



Uniqueness of solutions of magnetostatic boundary problems

We have four equations for the scalar magnetic potential ([5] A. Aharoni):

$$\nabla^2 \varphi_{inside} = \nabla \cdot \vec{M} \qquad \nabla^2 \varphi_{outside} = 0 \qquad \boxed{\varphi_{inside} = \varphi_{outside} \qquad \frac{\partial \varphi_{inside}}{\partial n} - \frac{\partial \varphi_{outside}}{\partial n} = \vec{M} \cdot \vec{n}}$$

at the boundary

Let us suppose there are two regular* functions φ_1 and φ_2 that fulfill the above equations. Then the function $\varphi_3 = \varphi_1 - \varphi_2$ must be continuous everywhere. Let us integrate:

$$\int_V (\nabla \varphi_3)^2 d^3 r = \int_V \left[\nabla \cdot (\varphi_3 \nabla \varphi_3) - \varphi_3 \underbrace{\nabla^2 \varphi_3}_{0} \right] d^3 r = \int_S \varphi_3 \frac{\partial \varphi_3}{\partial n} d\mathbf{S}$$

Regularity condition requires that $\frac{\partial \varphi_3}{\partial n}$ decreases as r^{-2} and φ_3 as r^{-1}. So if one extends the surface of integration to infinity (d**S** increases as r^2) the above integral vanishes. Since integrand is a square (i.e. >=0) the divergence of φ_3 vanishes. Φ_3 must be constant, but non-zero constant is not regular at infinity. φ_3 =0 everywhere. We have then:

$$\boxed{\varphi_1 = \varphi_2}$$

"*There is thus only one possible solution to the potential problem of any geometry and any distribution of the magnetization. Therefore, it is never necessary to give the intermediate steps, or to justify in any other way a solution to a potential problem.*" (A. Aharoni,[5] p.111)

*A function is termed regular if and only if it is analytic and single-valued throughout a region R (mathworld.wolfram.com).

Uniqueness of solutions of magnetostatic boundary problems

"*There is thus only one possible solution to the potential problem of any geometry and any distribution of the magnetization. Therefore, it is never necessary to give the intermediate steps, or to justify in any other way a solution to a potential problem.*" (A. Aharoni,[5] p.111)

"*It should be noted , however, that while a magnetization distribution determines a unique field outside the ferromagnet, the reverse is not true. A measurement of the field outside a ferromagnetic body* ***is not sufficient*** *to determine a unique magnetization distribution that creates this field*" (A. Aharoni,[5] p.112)

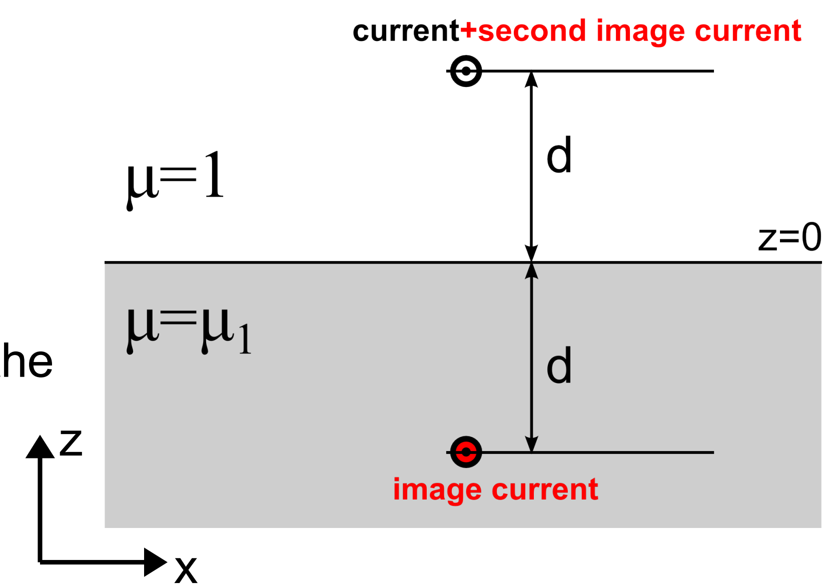
Boundary conditions – an example from Jackson

- Current line parallel to the planar boundary (z=0) separating two regions of different permeabilities
- For z>0 the the permeability is one and for z<0 it is equal to μ
- The current density is present in z>0 region
- Assume that the current flows in a line with (0,0,z) coordinates and that we are interested in the field at the boundary. We postulate that the magnetic induction can be calculated as a superposition of the real current and two image currents.
- For the field at (x1,y1,z1) of the current element (Jx,Jy,Jz) placed at (x,y,z) we have (Biot-Savart law):

$$\vec{B} \propto \frac{\vec{I}\times\vec{r}}{|\vec{r}|^3} = \left(\frac{Jz\,y - Jz\,y1 - Jy\,z + Jy\,z1}{\left((x1-x)^{2,}\;(y1-y)^{2,}\;(z1-z)^2\right)^{3/2}},\; \frac{-Jz\,x + Jz\,x1 + Jx\,z - Jx\,z1}{\left((x1-x)^2+(y1-y)^2+(z1-z)^2\right)^{3/2}},\; \frac{Jy\,x - Jy\,x1 - Jx\,y + Jx\,y1}{\left((x1-x)^2+(y1-y)^2+(z1-z)^2\right)^{3/2}} \right)$$

- In our case (y=0, y1=0, Jx=0, Jz=0, z1=0, x=0) this simplifies to:

$$\vec{B} \propto \frac{\vec{I}\times\vec{r}}{|\vec{r}|^3} = \left(\frac{-Jy\,z}{\left(x1^2+z^2\right)^{3/2}},\; 0,\; \frac{-Jy\,x1}{\left(x1^2+z^2\right)^{3/2}} \right)$$



- We assume that the image currents flow in the same direction as the existent current.
- We assume that the first image current is *mn1* times the real current and the second *mn2* times.

Boundary conditions – an example from Jackson

•For z>0 the field is the superposition of fields from the existent current and the first image current:

$$\vec{B}_{z>0} = \left(\frac{-Jy\,z + Jy\,mn1\;z}{(x1^2+z^2)^{3/2}},\ 0,\ \frac{-Jy\,x1 - Jy\,mn1\,x1}{(x1^2+z^2)^{3/2}} \right)$$

•For z<0 we guess that the field originates from the second image current:

$$\vec{B}_{z<0} = mn2 \left(\frac{-Jy\,z}{(x1^2+z^2)^{3/2}},\ 0,\ \frac{-Jy\,x1}{(x1^2+z^2)^{3/2}} \right)$$

•From boundary conditions $\vec{n}\cdot\vec{B}_2 = \vec{n}\cdot\vec{B}_1$ and $\vec{B}_2\times\vec{n} = \frac{\mu_2}{\mu_1}\vec{B}_1\times\vec{n}$ we have:

$$\left(\begin{array}{l} B_z^{z<0} = B_z^{z>0} \\ B_x^{z<0} = \mu_1 B_x^{z>0} \end{array} \right)_{z1=0}$$

solving a set of equations* we get

$$mn1 = \frac{\mu_1 - 1}{\mu_1 + 1}$$

$$mn2 = \frac{2\,\mu_1}{\mu_1 + 1}$$

Multiplying the image currents by the appropriate multipliers ensures the fulfillment of magnetic boundary conditions

•Because of the uniqueness of the solutions of the magnetostatic boundary problems this is the only solution.

*Solve[{(-Jy x1-Jy mn1 x1)+Jy mn2 x1==0,(-Jy z+Jy mn1 z)/(-Jy mn2 z)-(1/mi)==0},{mn1,mn2}]

Boundary conditions – an example from Jackson

- Current line parallel to the planar boundary separating two regions of different permeability
- For z>0 the the permeability is one and for z<0 it is equal to μ.
- The current density is present in z>0 region.
- For **general current distribution** it can be shown [1] that the magnetic boundary conditions are satisfied if the following image currents are added:

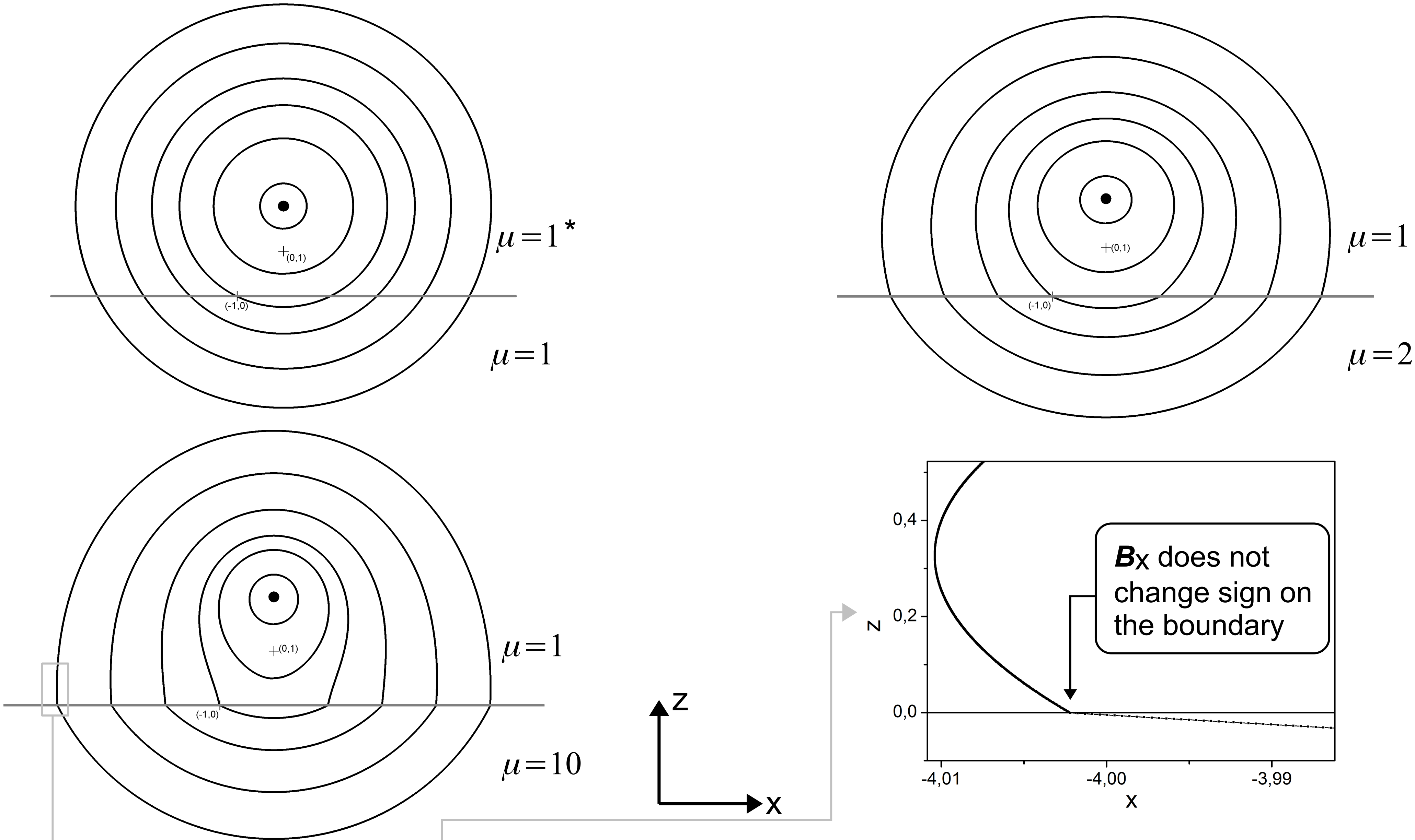
-In the region z>0 the effect of the region with permeability μ is equal to the effect of the image current **J*** of the following components (the **J*** is placed symmetrically relative to the boundary):

$$\frac{\mu-1}{\mu+1} J_x(x, y, -z), \qquad \frac{\mu-1}{\mu+1} J_y(x, y, -z), \qquad -\frac{\mu-1}{\mu+1} J_z(x, y, -z)$$

-In the region z<0 the effect of the current is that of the real current multiplied by $\frac{2\mu}{\mu+1}$

Boundary conditions – an example from Jackson

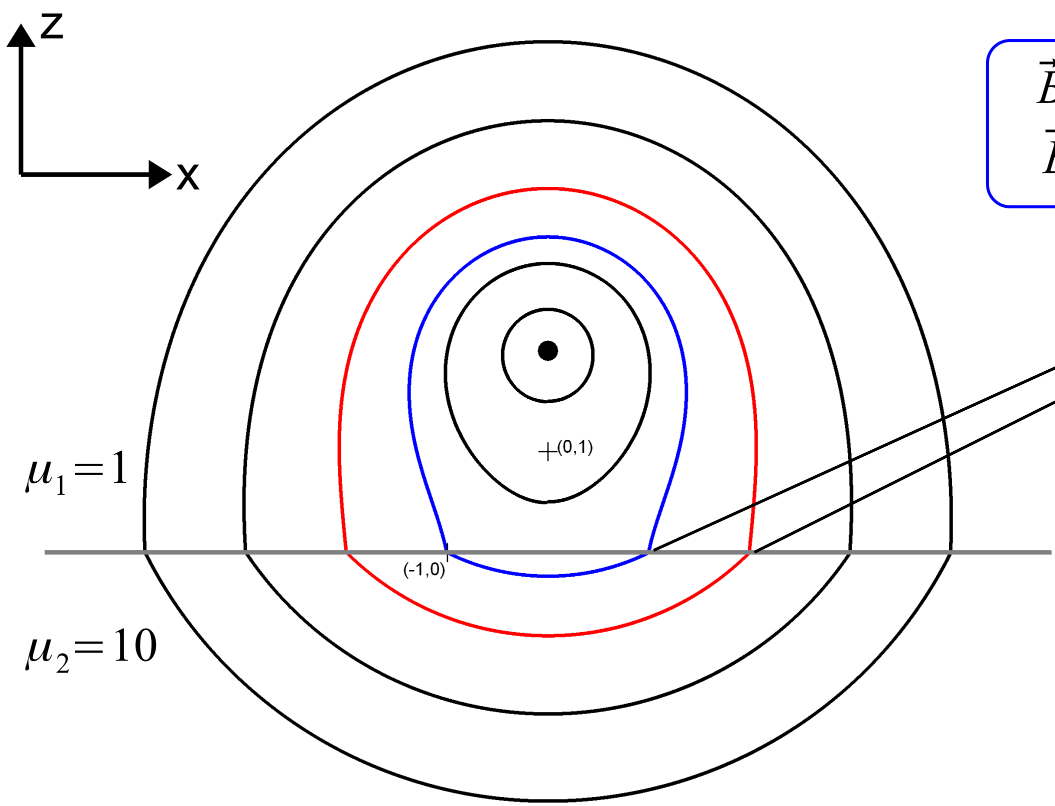
- Stream lines of ***B*** produced by a current in the vicinity of "permeability boundary":



*the image for μ=1 in both regions is slightly scaled down (grey boundary lines in all images are from x=-5 to x=+5)

Boundary conditions – an example from Jackson

•Stream lines of ***B*** produced by a current in the vicinity of “permeability boundary”:



$$\vec{B}_{\mu=1}(1,0) = (0.0115749\,,\ 0.0578745)$$
$$\vec{B}_{\mu=10}(1,0) = (0.115749\,,\ 0.0578745)$$

$$\vec{B}_{\mu=1}(2,0) = (0.0072343\,,\ 0.072343)$$
$$\vec{B}_{\mu=1}(2,0) = (0.072343\,,\ 0.072343)$$

$$\vec{n}_2 \cdot (\vec{B}_2 - \vec{B}_1) = 0$$
$$\vec{B}_2 \times \vec{n} = \frac{\mu_2}{\mu_1} \vec{B}_1 \times \vec{n}$$

$$\frac{B^z_{\mu=10}}{B^z_{\mu=1}} = 1$$

$$\frac{B^x_{\mu=10}}{B^x_{\mu=1}} = 10$$

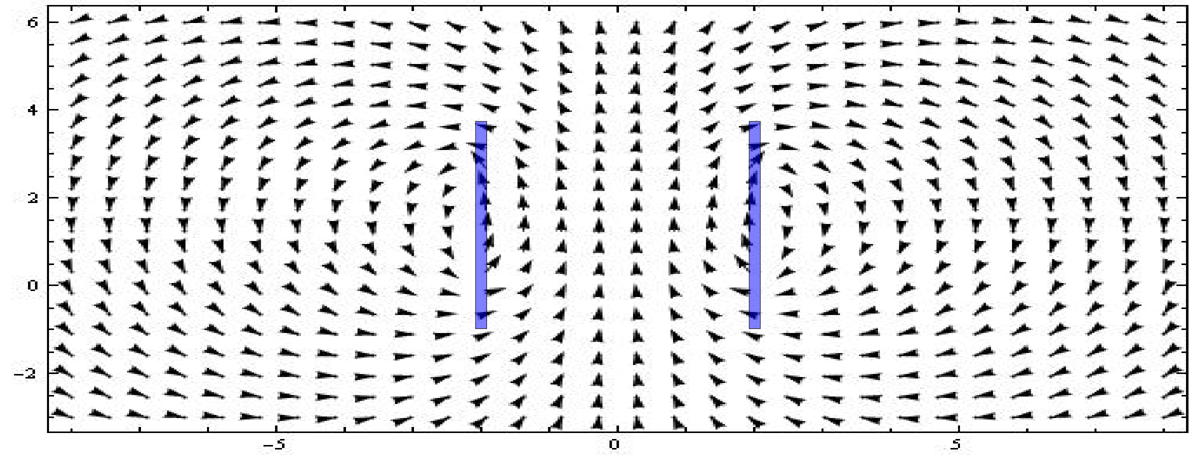
Magnetic field of two current sheets

We use the same expressions for the induction of magnetic field of current sheet as previously:

$$B_z = \frac{\mu_0 M_S}{2\pi}\left[\arctan\left(\frac{z_2 - z}{x_2 - x_1}\right)\right]_{z0}^{z3}$$

$$B_x = \frac{\mu_0 M_S}{2\pi}\left[\ln\left[(x_2 - x_1)^2 - (z_2 - z)^2\right]\right]_{z0}^{z3}$$

, but this time for two shifted sheets with the opposite current flow direction.



x1=-2, z1=0, z3=3
x1=2, z1=0, z3=3

Magnetic field of two current sheets

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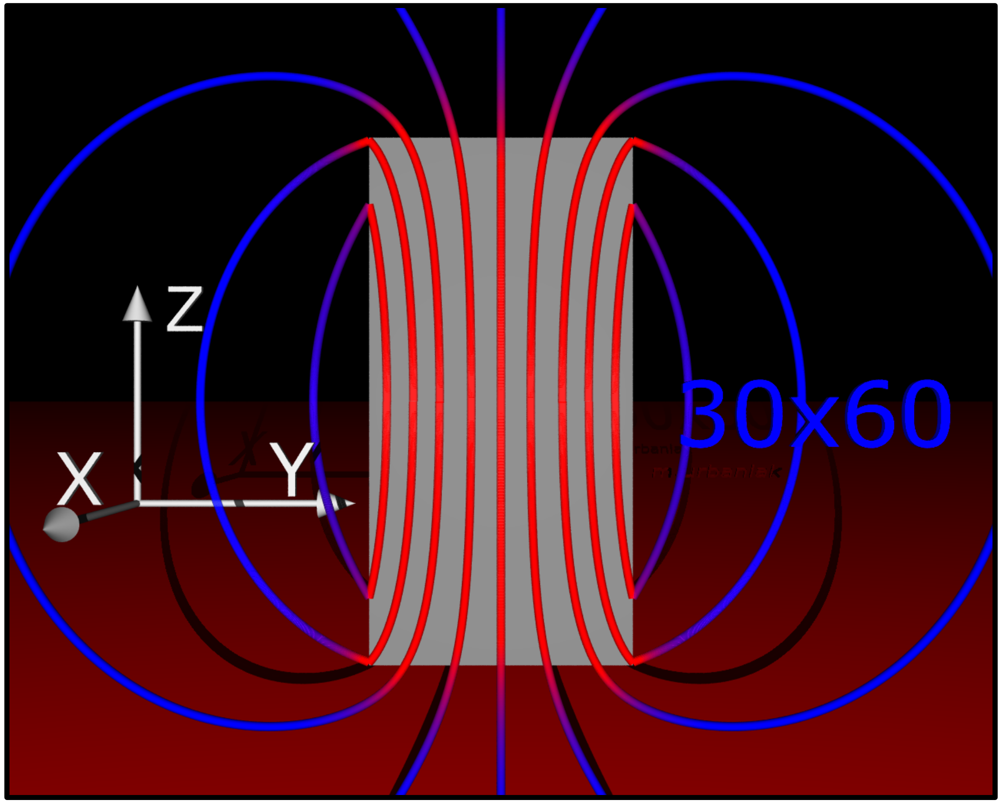
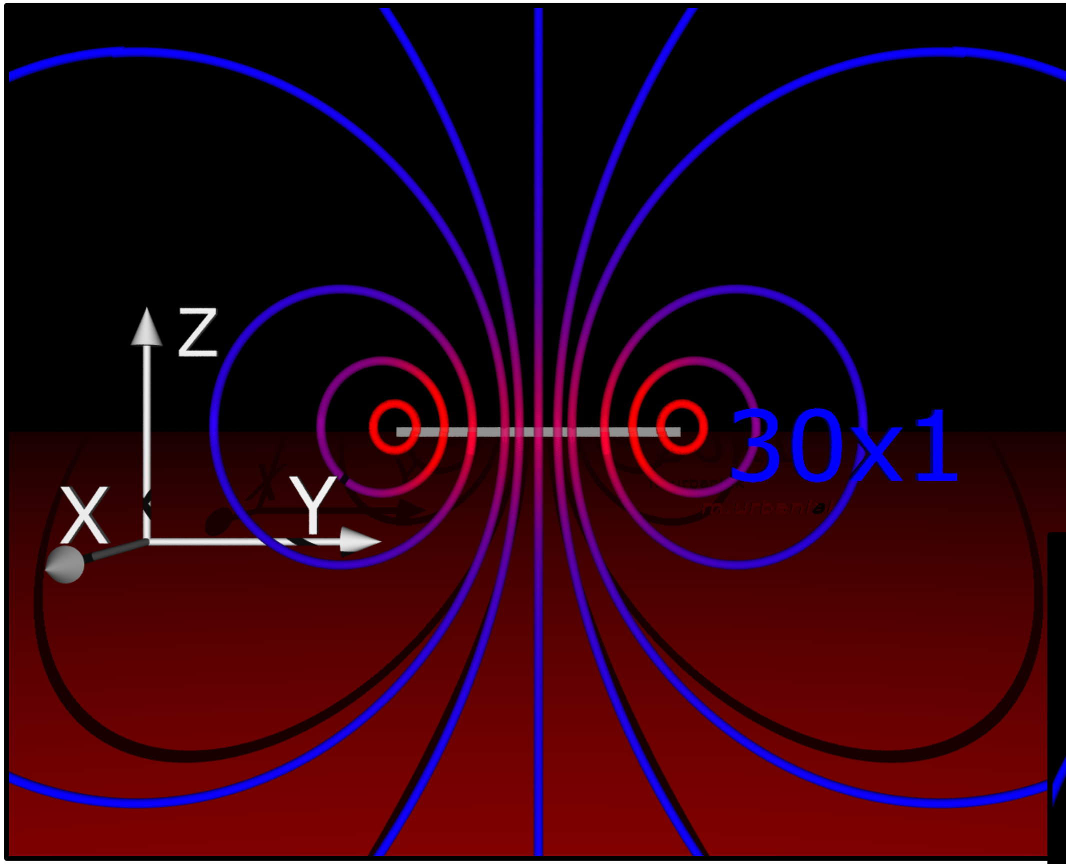
, but this time for two shifted sheets with the opposite current flow direction.

Magnetic induction ***B*** produced by two infinite current sheets corresponds to the field of infinite permanent magnet

red – high field, blue – weak field

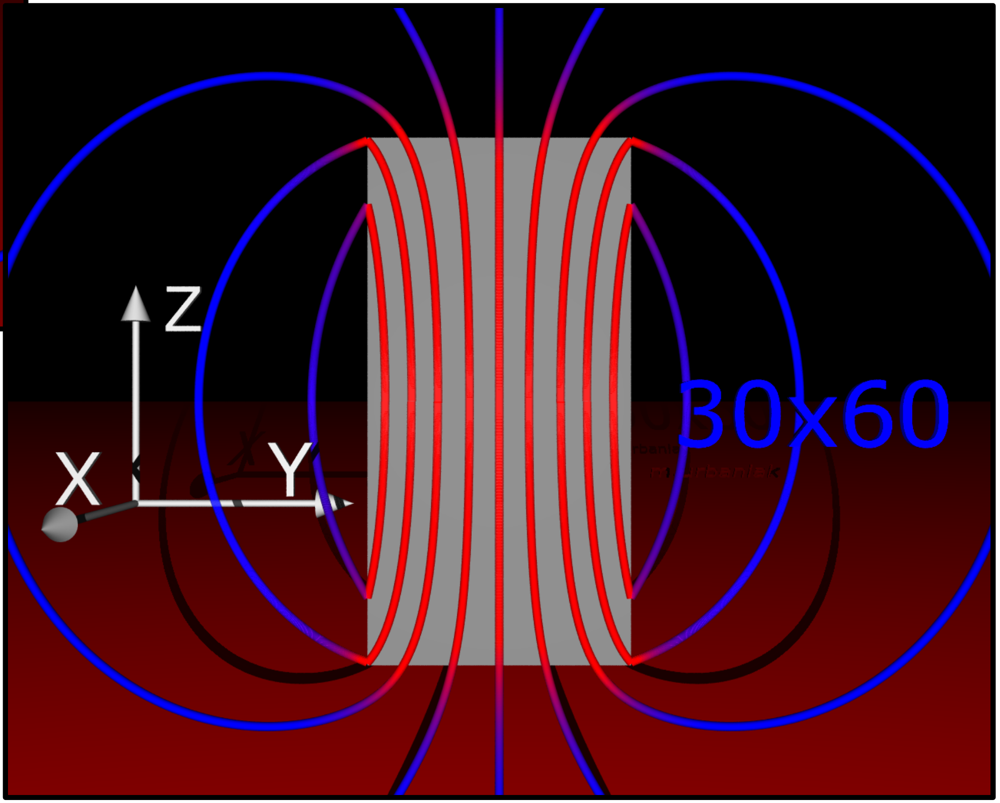


Magnetic field of thin magnets*



*magnetic moments of both magnets point upward

red – high field, blue – weak field

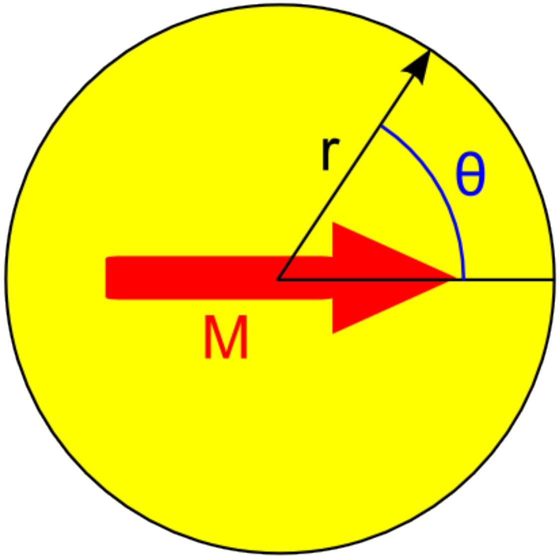


To note is that, contrary to thick magnets, thin magnets produce higher fields in its outer regions

Magnetic field of uniformly magnetized sphere - example

We assume that the sphere is homogeneously magnetized in *z* direction and copying from Bartelmann [7] we seek the magnetic potential.
We have: $\vec{M} = M_0 \hat{z}$

The problem is axially symmetric: we use a spherical coordinates (r, θ, φ) . The potential does not depend on azimuthal angle φ.



Magnetic field of uniformly magnetized sphere - example

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The problem is axially symmetric: we use a spherical coordinates (r, θ, φ) . The potential does not depend on azimuthal angle φ. We try to expand the potential into Legendre polynominals $P_l(\cos(\theta))$:

$$\varphi(r,\theta) = \sum_{l=0}^{\infty} \frac{\alpha_l P_l(\cos(\theta))}{r^{l+1}}$$

The general solution of Laplace's equation when the potential does not depend on azimuthal angle is:

$$\varphi(r,\theta) = \sum_{l=0}^{\infty} \left(\beta r^l + \alpha r^{-(l+1)}\right) P_l(\cos(\theta))$$

Assuming (guessing) that **i**nside the sphere induction ***B*** is parallel to *z*-axis we have:

$$\vec{H}_i = \frac{1}{\mu_0} \vec{B}_i - \vec{M} = \left(\frac{1}{\mu_0} B_0 - M_0\right) \hat{z}$$

i-inside
out-outside

The continuity conditions for ***B*** on the surface of the sphere give:

$$\vec{B}_i \cdot \hat{r} = \vec{B}_{out} \cdot \hat{r} \qquad \vec{H}_i \cdot \hat{\theta} = \vec{H}_{out} \cdot \hat{\theta}$$

Since it was assumed that ***B***$_{\text{inside}}$||***z*** we obtain:

$$P_1(x) = x$$

$$\vec{B}_i \cdot \hat{r} = B_0 \cos(\theta) = -\mu_0 \frac{\partial}{\partial r} \varphi(r,\theta)\Big|_R = \mu_0 \sum_{l=0}^{\infty} \alpha_l (l+1) \frac{P_l(\cos(\theta))}{R^{l+2}} = \mu_0 \alpha_1 \frac{2}{R^3} P_1(\cos(\theta)) = \mu_0 \alpha_1 \frac{2}{R^3} \cos(\theta)$$

surface of the sphere

Magnetic field of uniformly magnetized sphere - example

From the second continuity condition follows [7]:

$$\vec{H}_i \cdot \hat{\theta} = \left(\frac{1}{\mu_0} B_0 - M_0\right) \sin(\theta) = -\frac{1}{R} \frac{\partial}{\partial \theta} \varphi(r, \theta) = -\sum_{l=0}^{\infty} \frac{\alpha_l}{R^{l+2}} \frac{d P_l(\cos(\theta))}{d\theta}$$

Because of $d P_1(\cos(\theta))/d\theta = \sin(\theta)$ we have:

$\left(\frac{1}{\mu_0} B_0 - M_0\right) \sin(\theta) = -\frac{\alpha_1}{R^3} \sin(\theta)$ and from previous page $B_0 \cos(\theta) = \mu_0 \alpha_1 \frac{2}{R^3} \cos(\theta)$

Comparing coefficients of sine and cosine we get:

$$\left(\frac{1}{\mu_0} B_0 - M_0\right) = -\frac{\alpha_1}{R^3}, \quad B_0 = \mu_0 \alpha_1 \frac{2}{R^3}$$

Solving for α_1 we get:

$$\alpha_1 = \frac{1}{3} R^3 M_0$$

The scalar magnetic potential of the sphere is then:

$$\varphi(r, \theta) = \frac{\alpha_1 P_1(\cos(\theta))}{r^2} = \frac{1}{3} R^3 M_0 \frac{\cos(\theta)}{r^2}$$

Magnetic field of uniformly magnetized sphere - example

Taking the gradient of scalar potential we get magnetic induction [7]:

$$\vec{B} = -\mu_0 \nabla_{r,\theta,\varphi} \varphi(r,\theta) = \frac{2}{3}\mu_0 R^3 M_0 \frac{\cos(\theta)}{r^3}\hat{r} + \frac{1}{3}\mu_0 R^3 M_0 \frac{\sin(\theta)}{r^3}\hat{\theta}$$

Which corresponds to the dipole field.

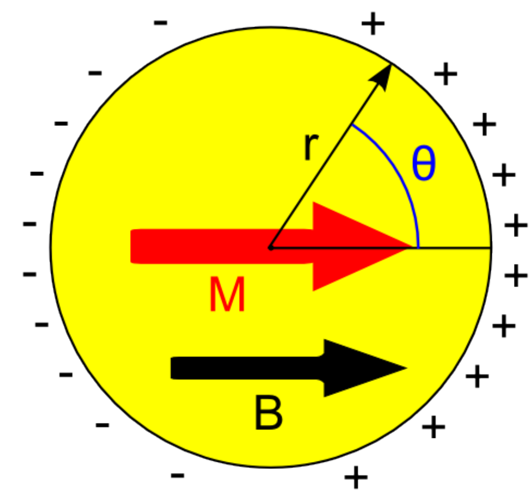
The magnetic field of uniformly magnetized sphere has dipolar character in the whole space outside the sphere.

We have a set of equations:

$$\vec{H}_i = \left(\frac{1}{\mu_0} B_0 - M_0\right)\hat{z} \qquad B_0 = \mu_0 \alpha_1 \frac{2}{R^3} \qquad \alpha_1 = \frac{1}{3} R^3 M_0$$

It follows from them that:

$$\vec{H}_i = \left(\frac{1}{\mu_0} B_0 - M_0\right)\hat{z} = -\frac{1}{3} M_0 \qquad \vec{B}_i = \frac{2}{3}\mu_0 M_0$$



Inside uniformly magnetized sphere magnetic induction is parallel to magnetization

Magnetizable sphere in magnetic field

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu \vec{H}$$

$$\vec{M} = \chi \vec{H}$$

volume susceptibility

$$(M_i = \chi_{ij} H_j)$$

usually tensor

$$\vec{B} = \mu_0 (\vec{H} + \chi \vec{H}) = \mu_0 (1 + \chi) \vec{H} = \mu_o \mu_r \vec{H}$$

relative permeability

The sphere of permeability μ is placed in an external field $\boldsymbol{B_0}$ [1,8]. From previous page we have:

$$\mu_0 \vec{H}_i = -\frac{1}{3} \mu_0 M \qquad \vec{B}_i = \frac{2}{3} \mu_0 M$$

1 $\quad +\vec{B}_0$

$$\mu_0 \vec{H}_i = \vec{B}_0 - \frac{1}{3} \mu_0 M \qquad \vec{B}_i = \vec{B}_0 + \frac{2}{3} \mu_0 M \equiv \mu \vec{H}_i$$

2

3

$$\mu \left(\frac{1}{\mu_0} \vec{B}_0 - \frac{1}{3} \vec{M} \right) = \vec{B}_0 + \frac{2}{3} \mu_0 \vec{M}$$

Solving for ***M*** we get:

$$\vec{M} = \vec{B}_0 \frac{3}{\mu_0} \left(\frac{\mu - \mu_0}{\mu + 2\mu_0} \right)$$

4

5

$$\boxed{\vec{B}_i = \vec{B}_0 \left(\frac{3\mu}{\mu + 2\mu_0} \right)}$$

For high permeability materials:

$\vec{B}_i = 3 \vec{B}_0$ **- field amplification**

Magnetizable sphere in magnetic field

Since permeability is field dependent (i.e. at high fields M saturates) we have: $\mu=\mu_0+\frac{\vec{M}}{\vec{H}}$

That is for high external fields: $\mu\approx\mu_0$

$\vec{B}_i=\vec{B}_0\left(\frac{3\mu}{\mu+2\mu_0}\right)\approx\vec{B}_0$ **- no field amplification**

- The amplification factor 3 is specific to a magnetizable sphere
- For other geometries (elongated rod) it can be considerably larger and is limited by the intrinsic properties of the material (magnetocrystalline anisotropy etc.)

Demagnetizing factor

If and only if the surface of uniformly magnetized body is of second order the magnetic induction inside is uniform and can be written as:

$$\vec{B}=\mu_0(-N\cdot\vec{M}+\vec{M})$$

N is called the demagnetizing tensor [5]. If magnetization is parallel to one of principle axes of the ellipsoid *N* contracts to **three numbers** called demagnetizing (or demagnetization) factors sum of which is one:

$$N_x+N_y+N_z=1$$

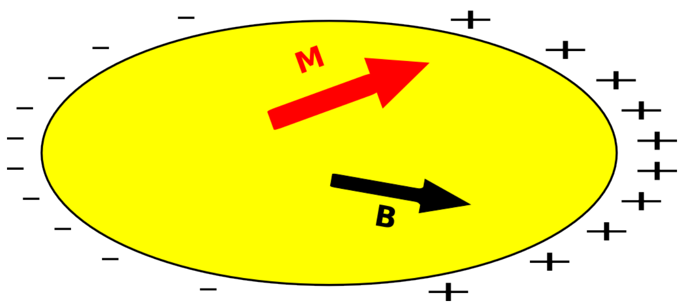
For the sphere, from symmetry considerations:

$N_x=N_y=N_z=\frac{1}{3}$ from which we obtain: $\vec{B}=\mu_0(-\frac{1}{3}\vec{M}+\vec{M})=\mu_0\frac{2}{3}\vec{M}$

For a general ellipsoid magnetization and induction are not necessarily parallel.

Demagnetization decreases the field inside ferromagnetic body.

Demagnetizing tensor describes just the influence of the body's shape on magnetic field inside it. The tensor/factor is only auxiliary quantity.



Demagnetizing factor

In some limiting cases the calculable demagnetizing factor of the ellipsoid can be used for the calculation (*approximate*) of the fields inside bodies of other shapes.

• For infinite cylinder (in *z*-direction) there is no discontinuity of magnetization along *z*-axis so:

$N_z = 0$ and because of axial symmetry: $N_x = N_y = \frac{1}{2}$

In the infinite cylinder magnetized along its long axis the induction is*: $\vec{B} = \mu_0 \vec{M}$

•For infinite planar sample perpendicular to *z*-axis we have no magnetic charges along *x* and *y* axes:

$N_z = 1$ and because of axial symmetry: $N_x = N_y = 0$

In the infinite planar sample magnetized in-plane the induction is*: $\vec{B} = \mu_0 \vec{M}$

*Plus the external field if present.

Demagnetizing factor

DEMAGNETIZATION FACTORS 355

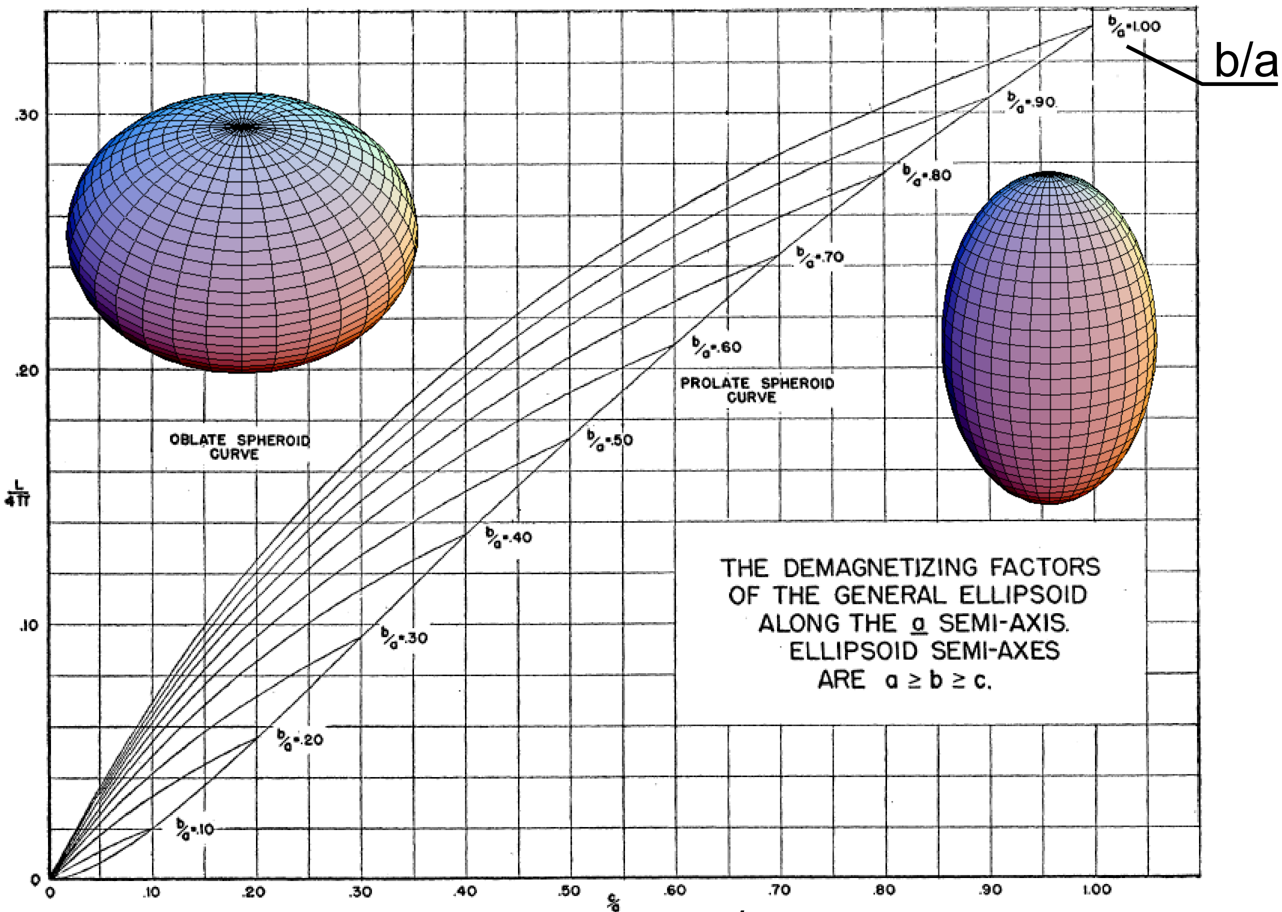


FIG. 1.

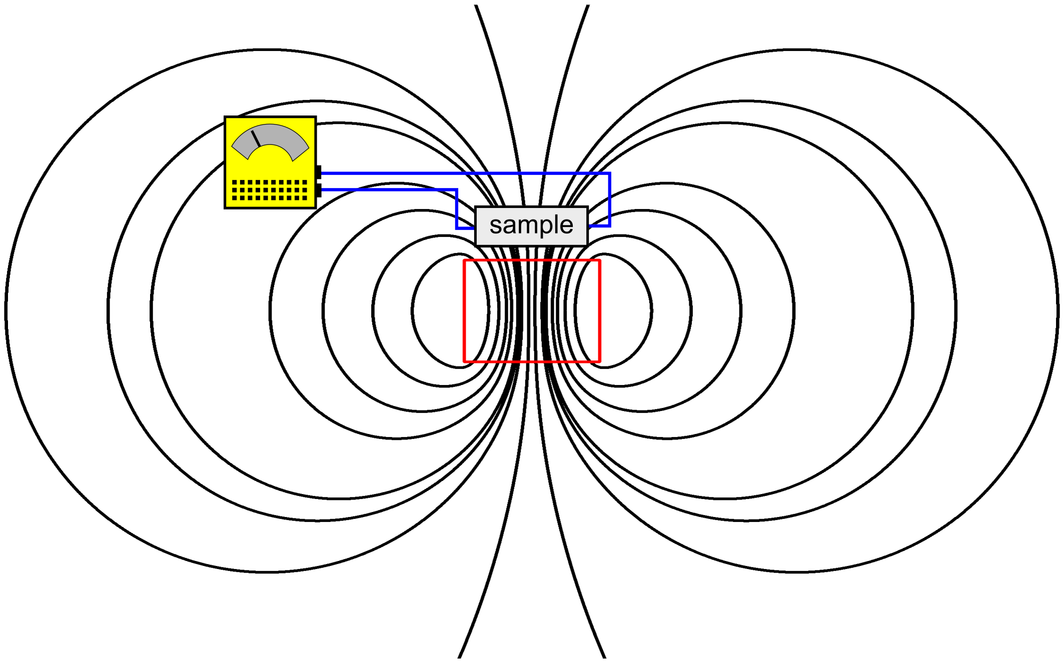
L/4π – demag. factor along the longest semi-axis (a)

J.A. Osborn, Phys.Rev. 1945, 67 (351)

images of spheroids from Wikimedia Commons; author: Cdw1952

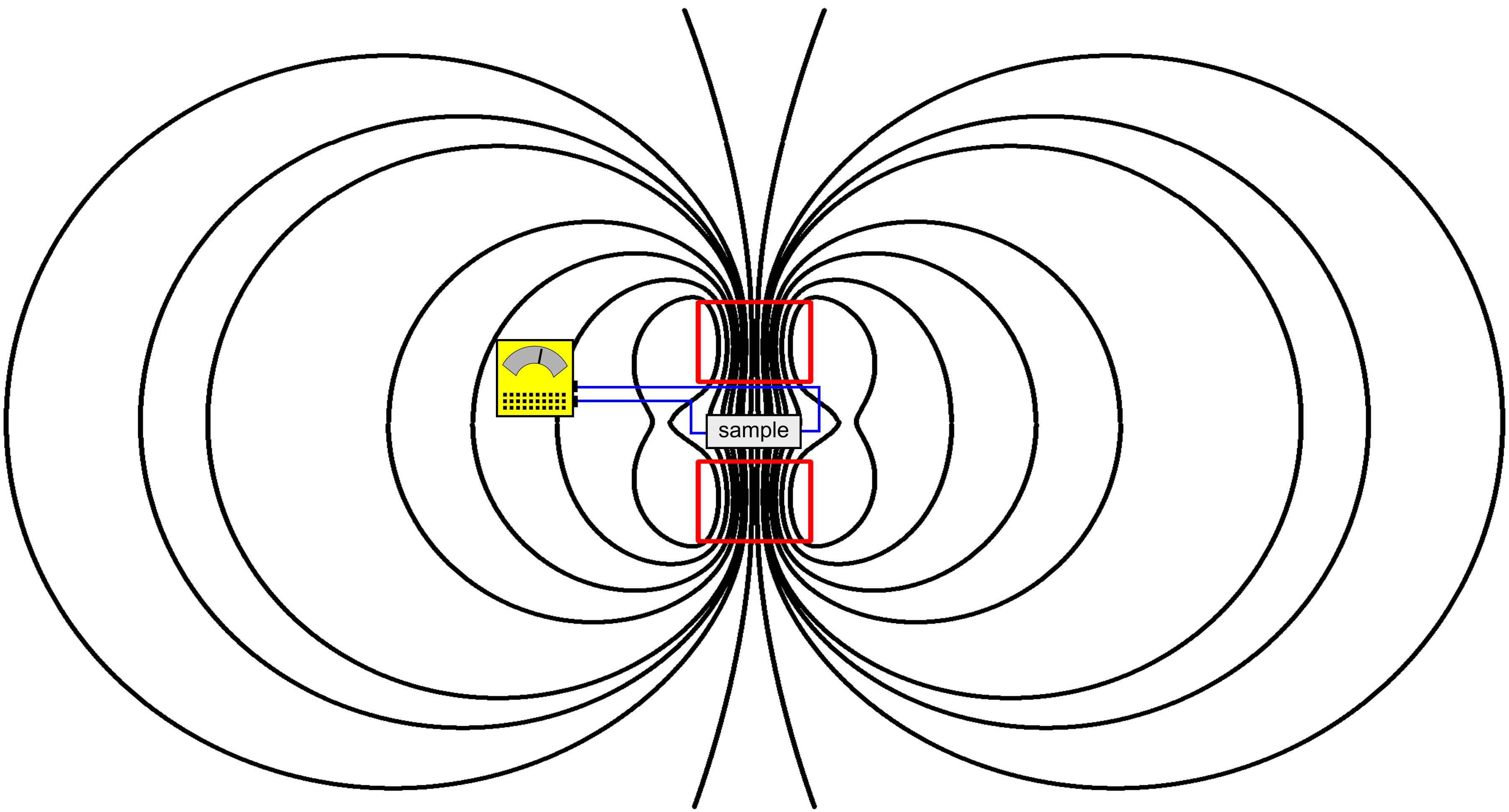
Sources of magnetic fields for measurements

- In open magnetic circuits of typical sizes the spatial variations of the intensity and direction of magnetic induction ***B*** are to high to provide enough space for experiments involving homogeneous magnetic field.
- Field of one magnet: **sample in highly inhomogeneous field**



Sources of magnetic fields for measurements

- In open magnetic circuits of typical sizes the spatial variations of the intensity and direction of magnetic induction ***B*** are to high to provide enough space for experiments involving homogeneous magnetic field.
- Field of two magnets: **sample in highly homogeneous field**, of higher strength, *if gap is narrow*



Special purpose magnets configuration - examples

from fridgemagnets-ecard.appspot.com

Refrigerator magnets (to stick things to refrigerator etc.)

- they use special configuration of magnetization to obtain **one-sided flux*** – no magnetic field is present on other side
- If the magnetization vector is constant and rotates clockwise when viewed moving from left to right:

$$M_x = M_0 \sin(kx) \qquad M_y = M_0 \cos(kx) \qquad M_z = 0$$

then the flux emerges exclusively below the structure.

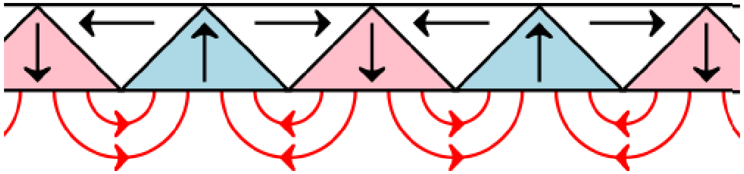


image source Wikimedia Commons; author Pieter Kuiper

- Because M_x depends on x the divergence of $\boldsymbol{M}$ is:

$$\nabla \cdot \vec{M} = M_0 k \cos(kx) \neq 0 \quad \rightarrow$$ **magnetic charges within the tape (or film)**

Within the tape the scalar potential must obey Poisson's equation (below and above Laplace's): $$\nabla^2 \varphi(x, y) = M_0 k \cos(kx)$$

The solution is on-sided flux: with regard to the upper region, surface and volume poles conceal each other exactly.

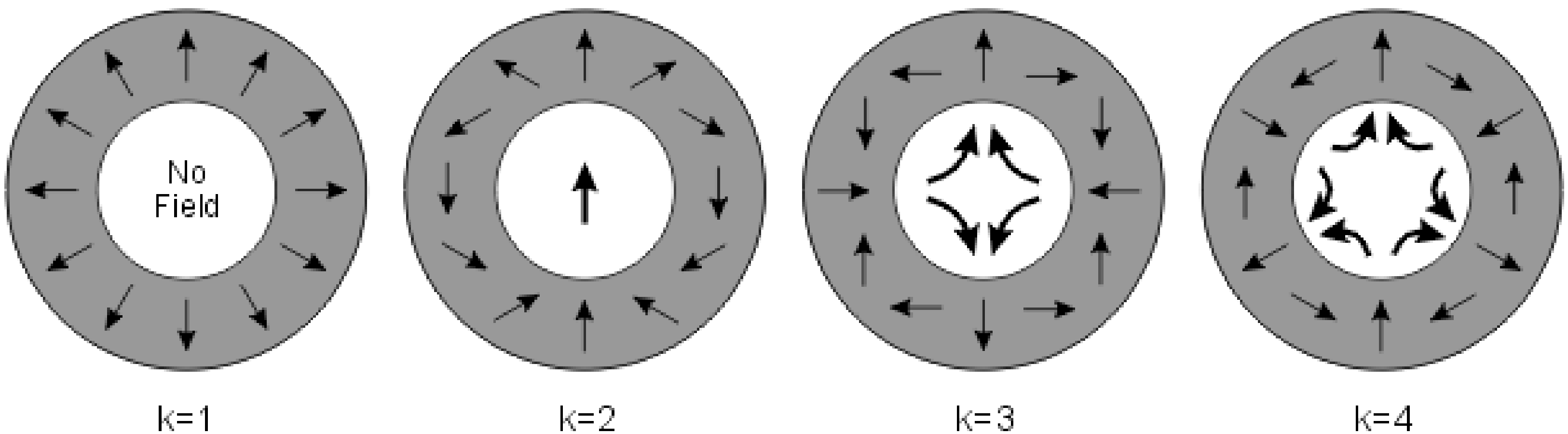
•The one-sided flux increases the holding force almost by a factor of 2.
•Note that spatially alternating magnetization increases gradient to magnetic field ***B***.

*H.A. Shute, J.C. Mallinson, D.T. Wilton, D.J. Mapps, IEEE 2000, **36** (440)

Special purpose magnets configuration - examples

Halbach cylinders:

- use the same principle as refrigerator magnets to create uniform field within spacious volume
- allow high field magnetic measurements with **very-low power consumption**: two coaxial Halbach cylinders can produce magnetic field of arbitrary direction

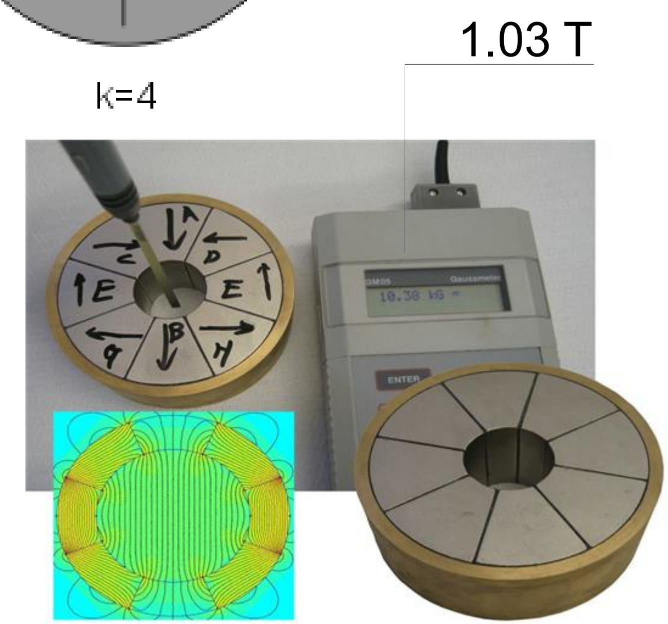


source Wikimedia Commons; author:User:Hiltonj

For k=2 the uniform field in the centre of the cylinder is*:

$$\vec{B} = \mu_0 M_0 \ln\left(\frac{r_{outer}}{r_{inner}}\right)$$

The induction may be greater than $\mu_0 M_0$.



source: e-magnetsuk.com

Fields reaching 5T were already obtained**.

*H.A. Shute, J.C. Mallinson, D.T. Wilton, D.J. Mapps, IEEE 2000, **36** (440)
**cerncourier.com/cws/article/cern/28598

Electromagnets

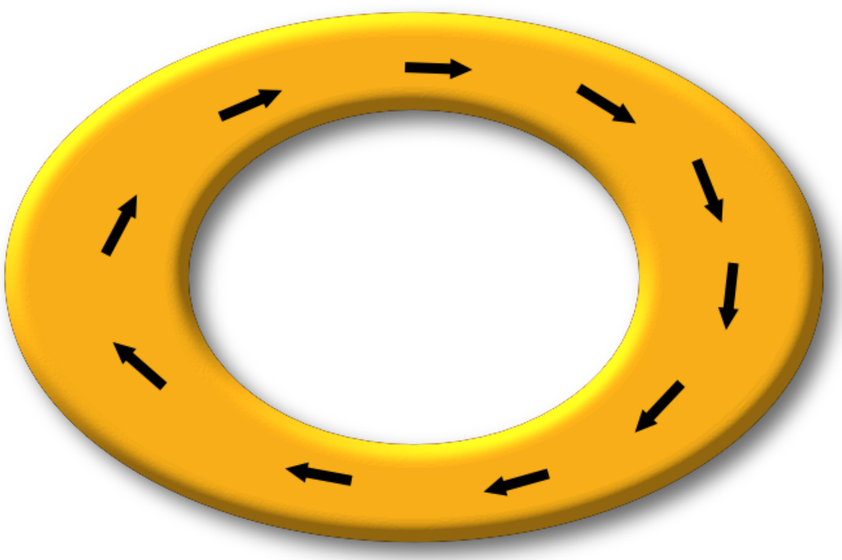
Magnetic circuits [3]:

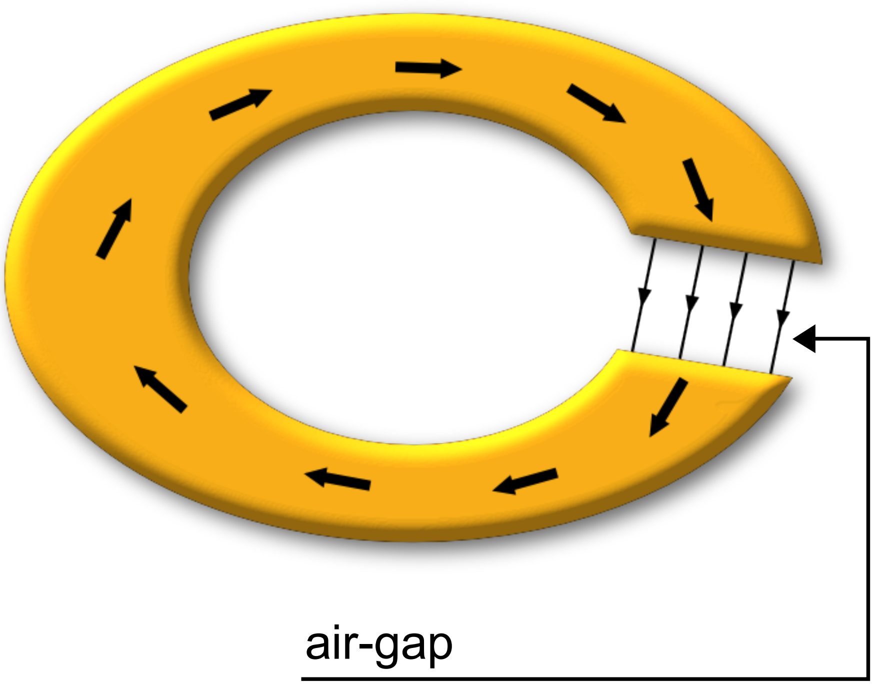
• not all magnetic boundary problems can be solved analytically

• for some problems involving arrangements of materials of high permeability sensible first order estimates of the fields within the regions of interest can be found without knowing the analytic solution

• the problems involve multiply connected tubular regions of high magnetization so that the **outside field can be neglected at first**

• the problem can be extended to systems with small air-gaps – neglecting fringing fields





Electromagnets

The high permeability material is used to produce a magnetic field in a narrow gap [8,9]:
•the material is magnetized by a current carrying wire, wound N times around the core
•for simplicity we assume that core is a torus o central radius equal *r*, the gap width is *d*.
From the assumptions of the previous slide (no flux leakage) it follows that (*C*-core, *G*-gap):

$$B_C = B_G$$

Ampere's law:

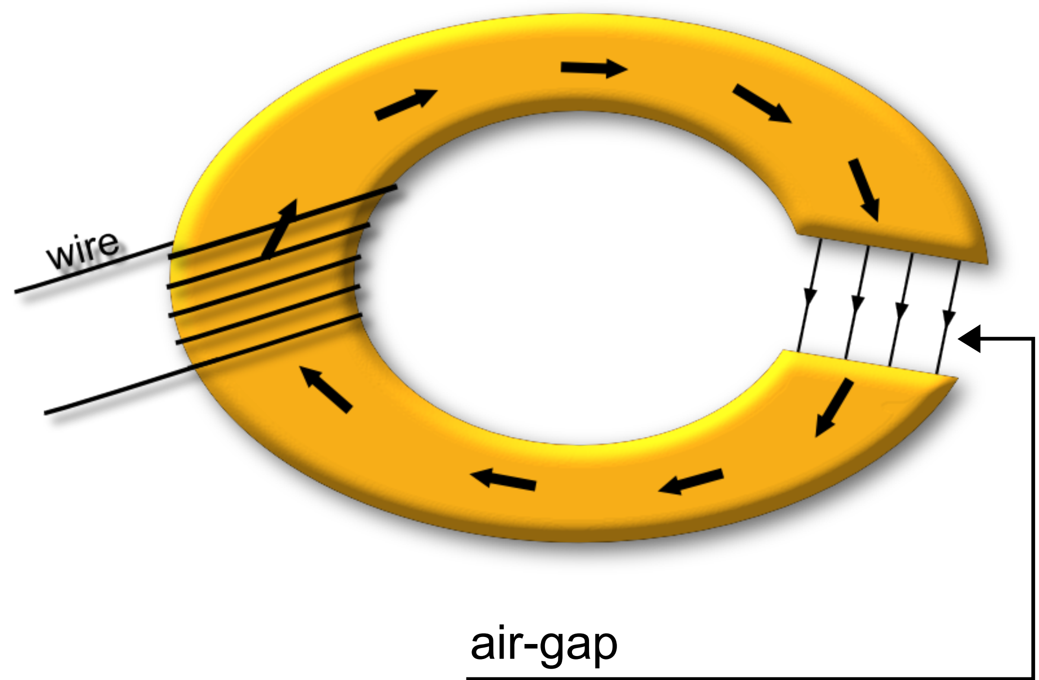
$$\oint_{\text{closed curve}} \vec{B}(\vec{r})\cdot d\boldsymbol{l} = \mu_0 I$$

From Ampere's law we have:

$$\oint \vec{H}(\vec{r})\cdot d\boldsymbol{l} = \underbrace{(2\pi r - d)H_C}_{\text{in the core}} + \underbrace{d\,H_G}_{\text{in the gap}} = N I$$

Using $\vec{H}_C = \frac{1}{\mu}\vec{B}_G, \quad \vec{H}_G = \frac{1}{\mu_0}\vec{B}_G$ we get:

$$B_g = \frac{N I}{\frac{1}{\mu}(2\pi r - d) + d\frac{1}{\mu_0}}$$



Electromagnets

- The magnetic induction of typical laboratory electromagnets is about 2 T.
- The high field magnets use no magnetic core- instead high currents produce fields (Bitter electromagnets)
- World record for the magnetic field produced by a nondestructive electromagnet is **97.4 T** (set on August 23, 2011) at Los Alamos [previous record:91.4 Tesla (Dresden, June 2011)] – three seconds span.
- The strongest man-made magnetic field* ~**2800 T** (Russia, 2003) – imploding magnets – very short duration (*Magnetic Flux Compression Generator*)

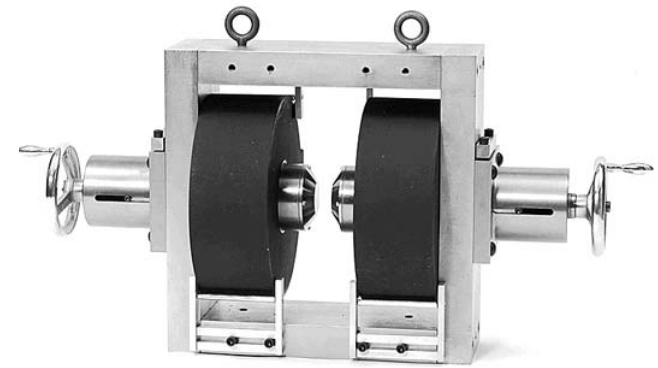


image source: appliedmagnetics.com

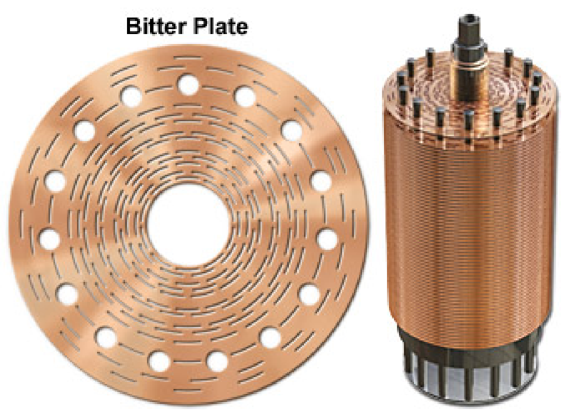
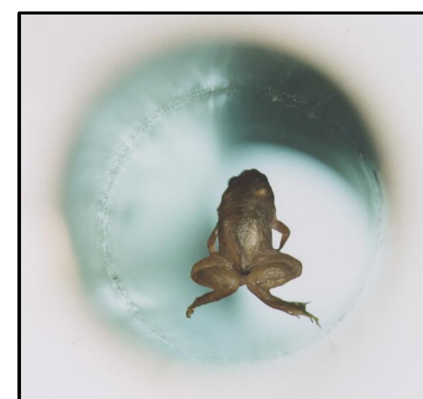


image source: National High Magnetic Field Laboratory
The Florida State University, USA, www.magnet.fsu.edu

A live frog levitates inside the Ø32mm vertical bore of a Bitter solenoid in a magnetic field of about 16 T at the Nijmegen High Field Magnet Laboratory



source Wikimedia Commons; author Lijnis Nelemans

*C.M. Fowler, L.L. Altgilbers, Электромагнитные Явления **3**, 306 (2003) http://emph.com.ua/11/pdf/fowler.pdf

Special sources of magnetic fields- examples

Field of electron beam:
- Biot-Savart field
- ~1 μm^2, ~10^{15} Am^{-2}
- beam parameters (σ_x=3.5μm, σ_y=0.2μm, σ_z=9mm)
- 2-10 ps

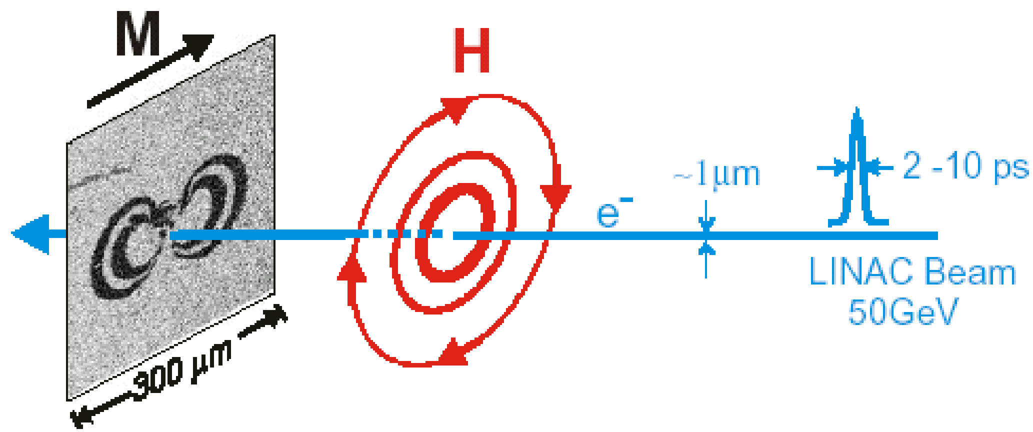


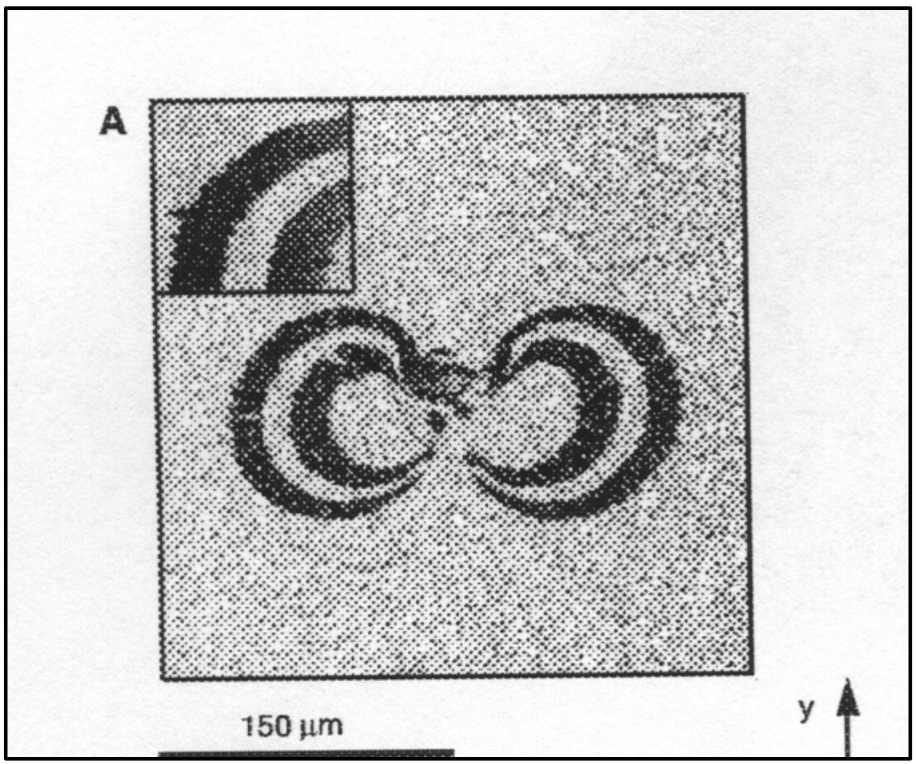
Fig. 1: Principle of the experiment with the SLAC FFTB*. The highly relativistic electron bunch generates magnetic field lines in the laboratory frame that are **equivalent to the ones from a straight current carrying wire**.

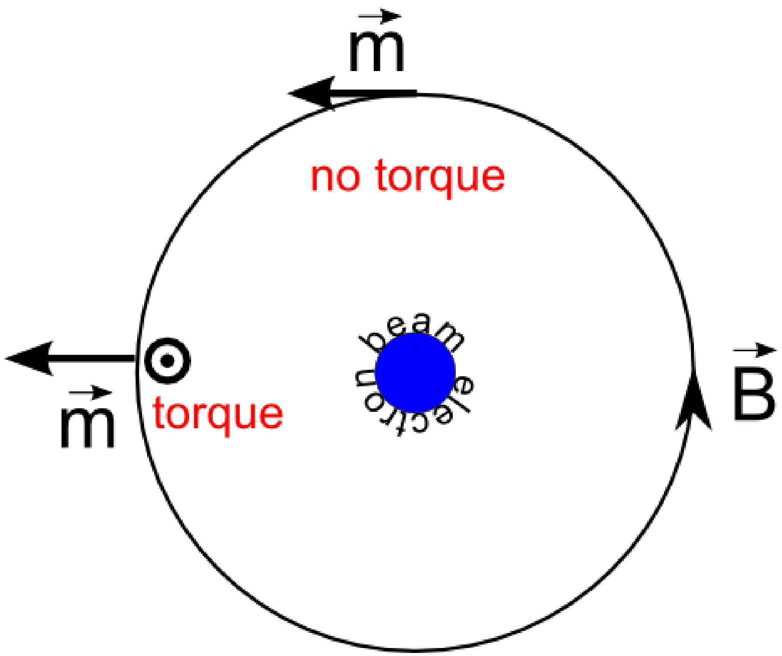
*Stanford Linear Accelerator Center (SLAC) Final Focus Test Beam (FFTB)

C.H. Back, R.Allenspach, W.Weber, S.S.P. Parkin, D. Weller, E.L. Garwin, H.C. Siegmann, Science 1999, **285**

Special sources of magnetic fields- examples

Field of electron beam:
- initially magnetization points in *x*-direction
- on the line with zero magnetic torque (y=0) no switching (see $\vec{N}=\vec{m}\times\vec{B}(\vec{r})$ later in this talk)





C.H. Back, R.Allenspach, W.Weber, S.S.P. Parkin, D. Weller, E.L. Garwin, H.C. Siegmann, Science 1999, **285**

Special sources of magnetic fields- examples

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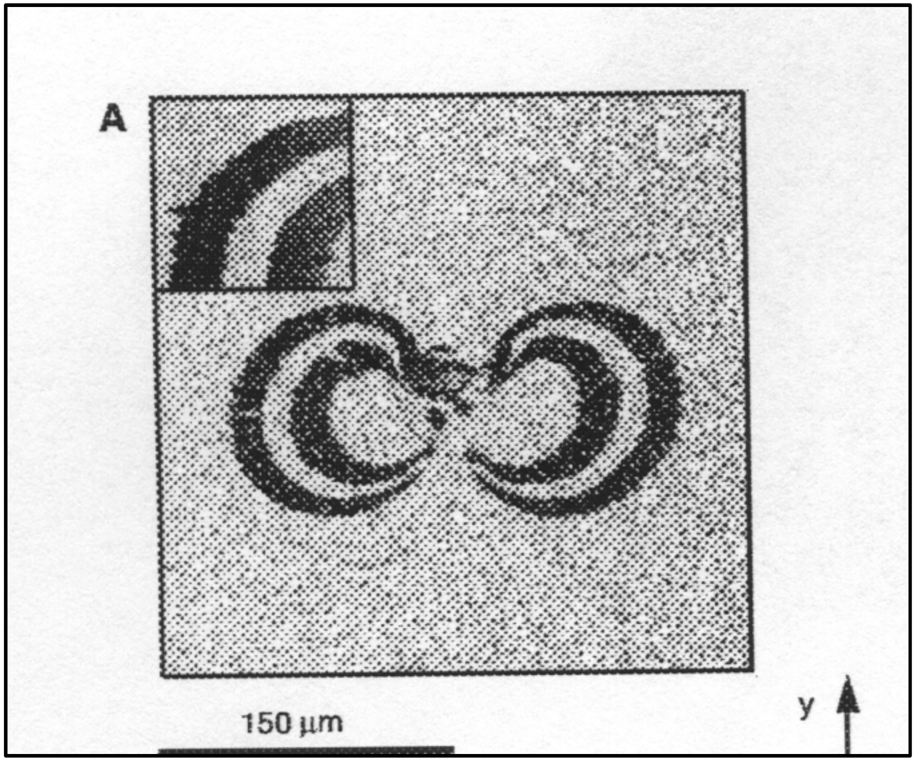
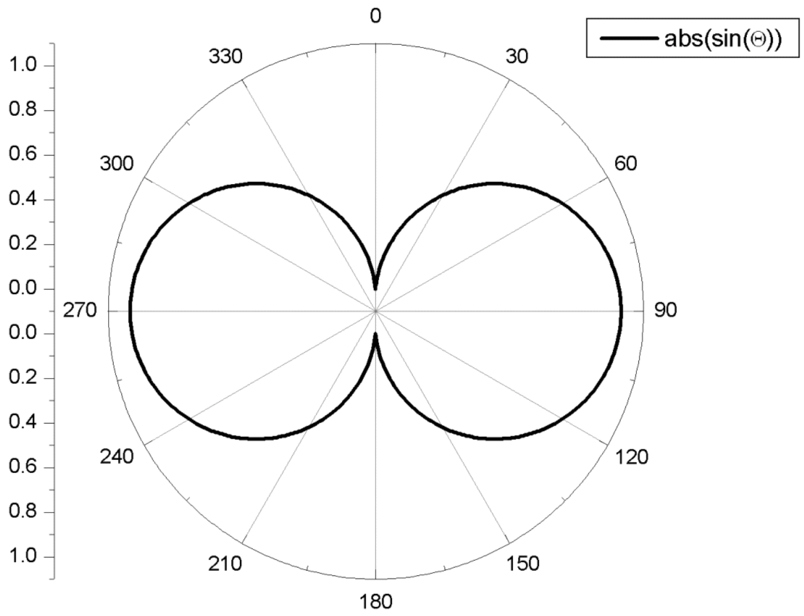




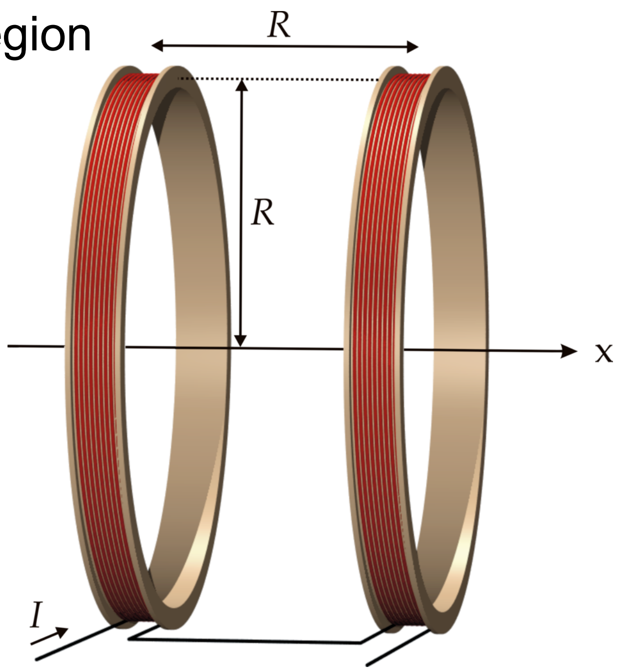
$$\vec{N} = \vec{m} \times \vec{B}(\vec{r})$$

C.H. Back, R.Allenspach, W.Weber, S.S.P. Parkin, D. Weller, E.L. Garwin, H.C. Siegmann, Science 1999, **285**

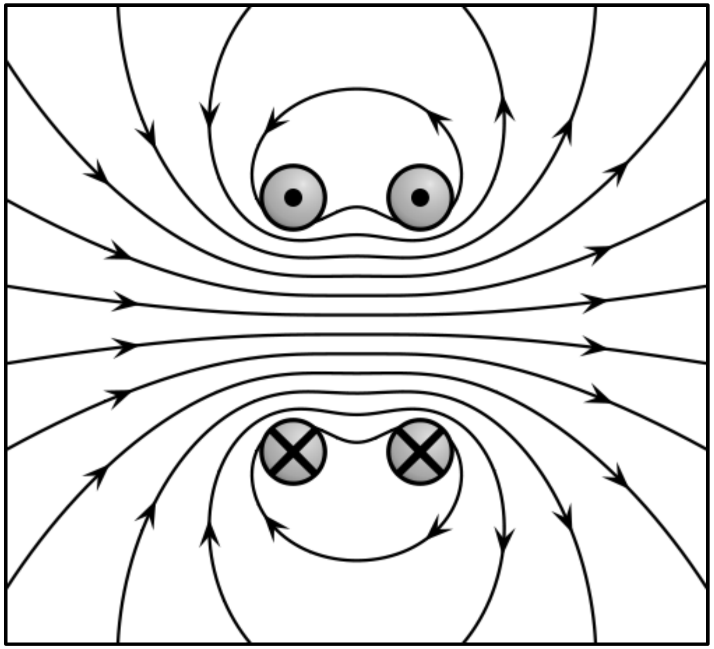
Special sources of magnetic fields- examples

Helmholz coils:

- to obtain nearly uniform field (usually weak) within a large region
- coils placed apart a distance equal to their radii
- each coil carries equal current



source: Wikimedia Commons; author Ahellwig



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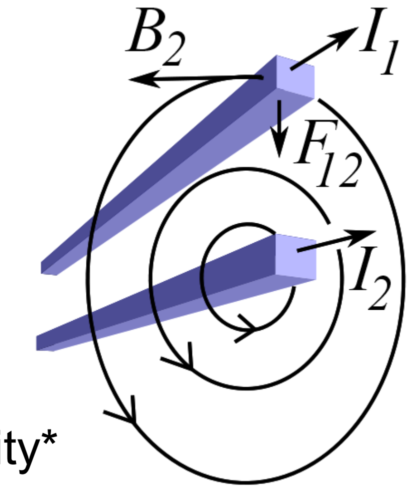
Force between two current carrying wires

There is a force acting on a moving electric charge placed in magnetic field:

$$\vec{F}_{Lorentz} = q\vec{E} + q\vec{v} \times \vec{B}$$

Lorentz force



source: Wikimedia Commons

The magnetic force acting on the volume element carrying current is [4]:

$$d\vec{F} = \varrho_V d^3 r \vec{v} \times \vec{B} = \vec{j}_V \times \vec{B} d^3 r$$

ϱ_V - volume charge density*

The overall force is obtained by the integration:

$$\vec{F} = \int_V \vec{j}_V \times \vec{B} d^3 r$$

Integrating that expression for two infinite, parallel, straight wires gives the expression for the attraction force (if currents in both of them flow in the same direction) per unit length:

$$\vec{F} = \mu_0 \frac{I_1 I_2}{2\pi d}$$

This equation is the basis of *Ampere* definition.

Or from field of straight wire (L.1):

$$B = \frac{\mu_0 I}{2\pi r}, \quad q \cdot v = (S\rho)\frac{I_2}{S\rho} = I_2 \Rightarrow F = \mu_0 \frac{I_1 I_2}{2\pi r}$$

cross section of wire

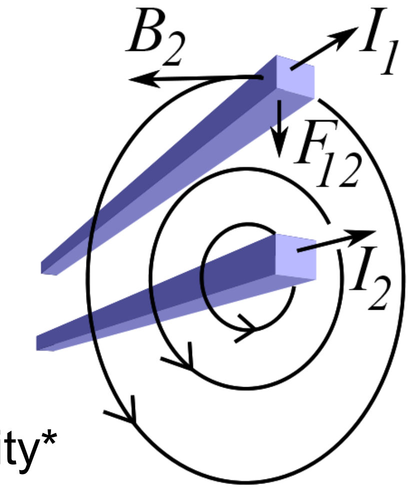
electron charge density

*local density of electric charge is usually zero so that electrostatic interaction is negligible

Force between two current carrying wires

There is a force acting on a moving electric charge placed in magnetic field:

$$\boxed{\vec{F}_{Lorentz}=q\vec{E}+q\vec{v}\times\vec{B}}$$ Lorentz force



source: Wikimedia Commons

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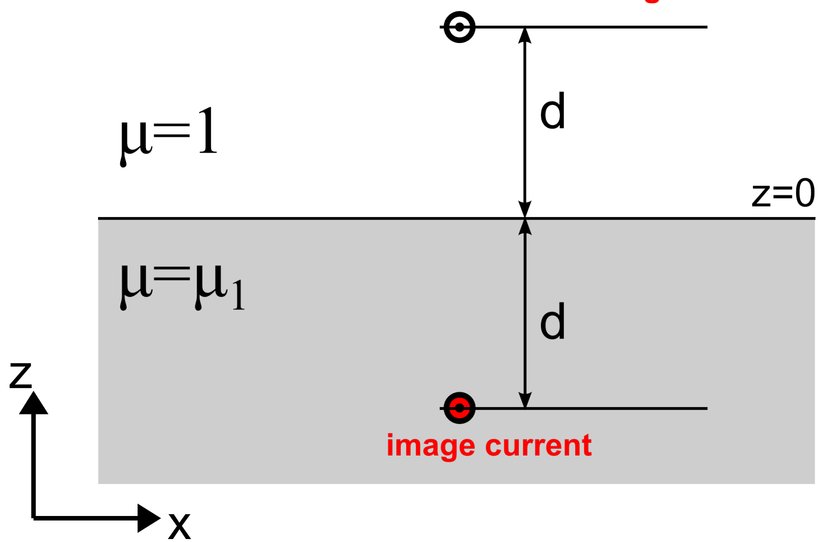
The force between current and magnetic body:
-the force between current and its image current

$$\frac{\mu-1}{\mu+1}J_x(x,y,-z),\qquad \frac{\mu-1}{\mu+1}J_y(x,y,-z),\qquad -\frac{\mu-1}{\mu+1}J_z(x,y,-z)$$

$$\boxed{\vec{F}=\mu_0\frac{I_1 I_2}{2\pi d}}$$



Force on a magnetic dipole

Applying the Lorentz force to general current distribution we have for a force and torque acting on the current distribution*:

$$\vec{F}=\int_V \vec{j}_V(\vec{r})\times\vec{B}(\vec{r})\,d^3r \qquad \vec{N}=\int_V \vec{r}\times\vec{j}_V(\vec{r})\times\vec{B}(\vec{r})\,d^3r$$

We assume that the volume occupied by the current distribution is much smaller than the length scale over which induction ***B*** varies. We can then Taylor expand ***B*** relative to some point in the vicinity of the current [1] (*k*-cartesian component):

$$B_k(\vec{r})=B_k(\vec{r}=0)+\vec{r}\cdot\nabla B_k|_{\vec{r}=0}\,d^3r'+\ldots$$

Inserting the expansion into the expression for ***F*** we obtain:

$$F_i=\sum_{jk}\varepsilon_{ijk}\Big[B_k(0)\underbrace{\int j_i(\vec{r}\,')\,d^3r'}_{0}+\int j_j(\vec{r}\,')\,\vec{r}\,'\cdot\nabla B_k|_{\vec{r}=0}\,d^3r'+\ldots\Big]$$

Following rather *lengthy* calculations [1,6] we obtain:

$$\vec{F}=(\vec{m}\times\nabla)\times\vec{B}=\nabla(\vec{m}\cdot\vec{B})-\vec{m}(\nabla\cdot\vec{B})$$

Because ***B*** is divergenceless we finally have (up to the second term of the expansion of ***B***):

$$\boxed{\vec{F}=\nabla(\vec{m}\cdot\vec{B})}$$

this expression holds for time varying fields too

*current distribution is assumed to be independent of ***B*** - see Faraday induction

Force on a magnetic dipole

In stationary field B the force expression can be rewritten to the form often used in biosciences (magnetophoresis etc.):

$$E=-\vec{m}\cdot\vec{B}$$

$$\vec{F}=\nabla(\vec{m}\cdot\vec{B})=-\vec{i}\,(m_x\frac{\partial B_x}{\partial x}+m_y\frac{\partial B_y}{\partial x}+m_z\frac{\partial B_z}{\partial x})-\vec{j}\,(\ldots)-\ldots$$

$$\nabla\times\vec{B}=0\quad : \qquad (\vec{J}+\frac{\partial\vec{D}}{\partial t}=0 \leftarrow \mathit{current\ free\ space,\ Maxwell})$$

$$\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}=0 \qquad \frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}=0 \qquad \frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=0$$

$$\vec{F}=-\vec{i}\,(m_x\frac{\partial B_x}{\partial x}+m_y\frac{\partial B_x}{\partial y}+m_z\frac{\partial B_x}{\partial z})-\vec{j}\,(\ldots)-\ldots=(m_x\frac{\partial}{\partial x}+\ldots)(\vec{i}\,B_x+\ldots)$$

$$\boxed{\vec{F}=(\vec{m}\cdot\nabla)\vec{B}}$$

Force on a paramagnetic particle (small digression)

In case where the moment is proportional to induction B – small fields*:

$$\vec{F}=(\vec{m}\cdot\nabla)\vec{B}=(\chi\frac{\vec{B}}{\mu_0}V\cdot\nabla)\vec{B}=\chi\frac{V}{\mu_0}\boxed{(\vec{B}\cdot\nabla)\vec{B}}$$

$$2\vec{B}\times\underbrace{(\nabla\times\vec{B})}_{0}+2\boxed{(\vec{B}\cdot\nabla)\vec{B}}=\nabla(\vec{B}\cdot\vec{B})$$

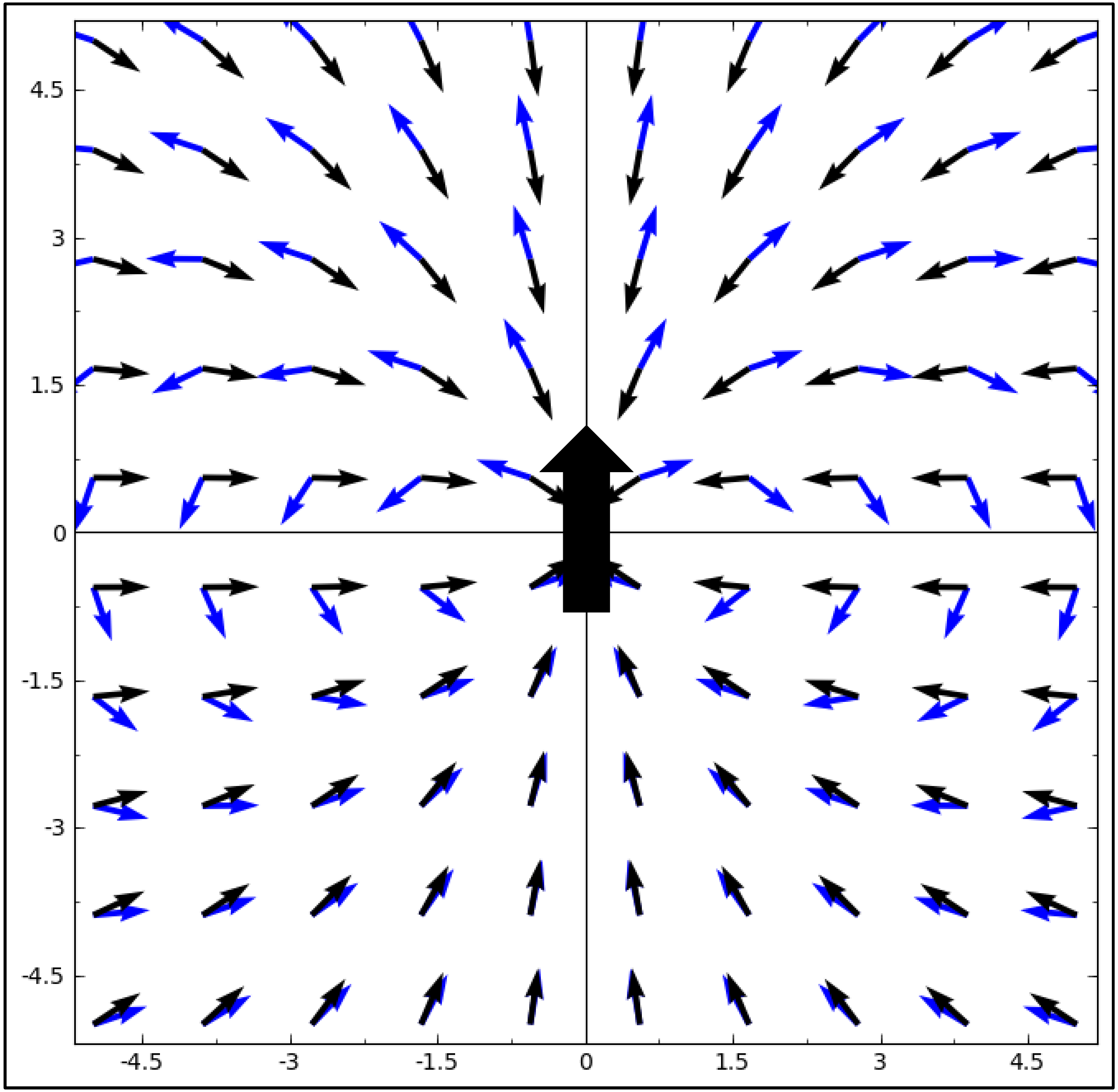
current free space,
no time-varying fields

$$\boxed{\vec{F}=\frac{1}{2\mu_0}\chi V\nabla(B^2)}$$

[Q. A. Pankhurst, J. Connolly, S. K. Jones, J. Dobson, J. Phys. D: Appl. Phys., **36**, R167 (2003)

*current distribution is assumed to be independent of ***B*** - see Faraday induction

Force on a paramagnetic particle (a small digression)



induction B

force

Force everywhere attractive !

Arrows show the directions of the fields
(not the magnitude!)

Torque on a magnetic dipole

The torque is calculated similarly from the general expression:

$$\vec{N} = \int_V \vec{r} \times \vec{j}_V(\vec{r}) \times \vec{B}(\vec{r})\, d^3 r$$

This time however already the first term of ***B*** expansion gives nonvanishing term:

$$\vec{N} = \vec{m} \times \vec{B}(\vec{r})$$

$$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$$

From each of these two equations it follows that the potential energy of a dipole in magnetic field can be expressed as:

$$E = -\vec{m} \cdot \vec{B}$$

- The above expression does not in general describe the total energy of a dipole; placing the moment in magnetic field requires the additional energy with which the current source maintains the magnitude of the moment under the influence of magnetic (Faraday) induction.
- In case of elementary particles with the spin (electron, neutron etc) their intrinsic magnetic moment is constant and the above expression gives the total energy.

Things to remember from today's talk:

- Magnetic charges although not physical are useful in solving magnetostatic problems
- Biot-Savart law and magnetic charges methods are equivalent
- Demagnetizing fields originate from magnetic charges of the magnetized body itself; they diminish magnetic field within ferromagnets
- The force on magnetic dipole is related to magnetic field gradient

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