

Giant magnetoresistance

GMR

Giant magnetoresistance

- Introduction
- Magnetoresistance
- Giant magnetoresistance

Introduction

Normal metal resistance
intrinsic resistivity – due solely to
phonons in a perfect lattice [5]

Bloch-Grüneisen formula [5]:

$$\rho_i(T) = \frac{C}{M\theta} \left(\frac{T}{\theta}\right)^5 \int_0^{\theta/T} \frac{z^5 e^z}{(e^z - 1)^2} dz$$

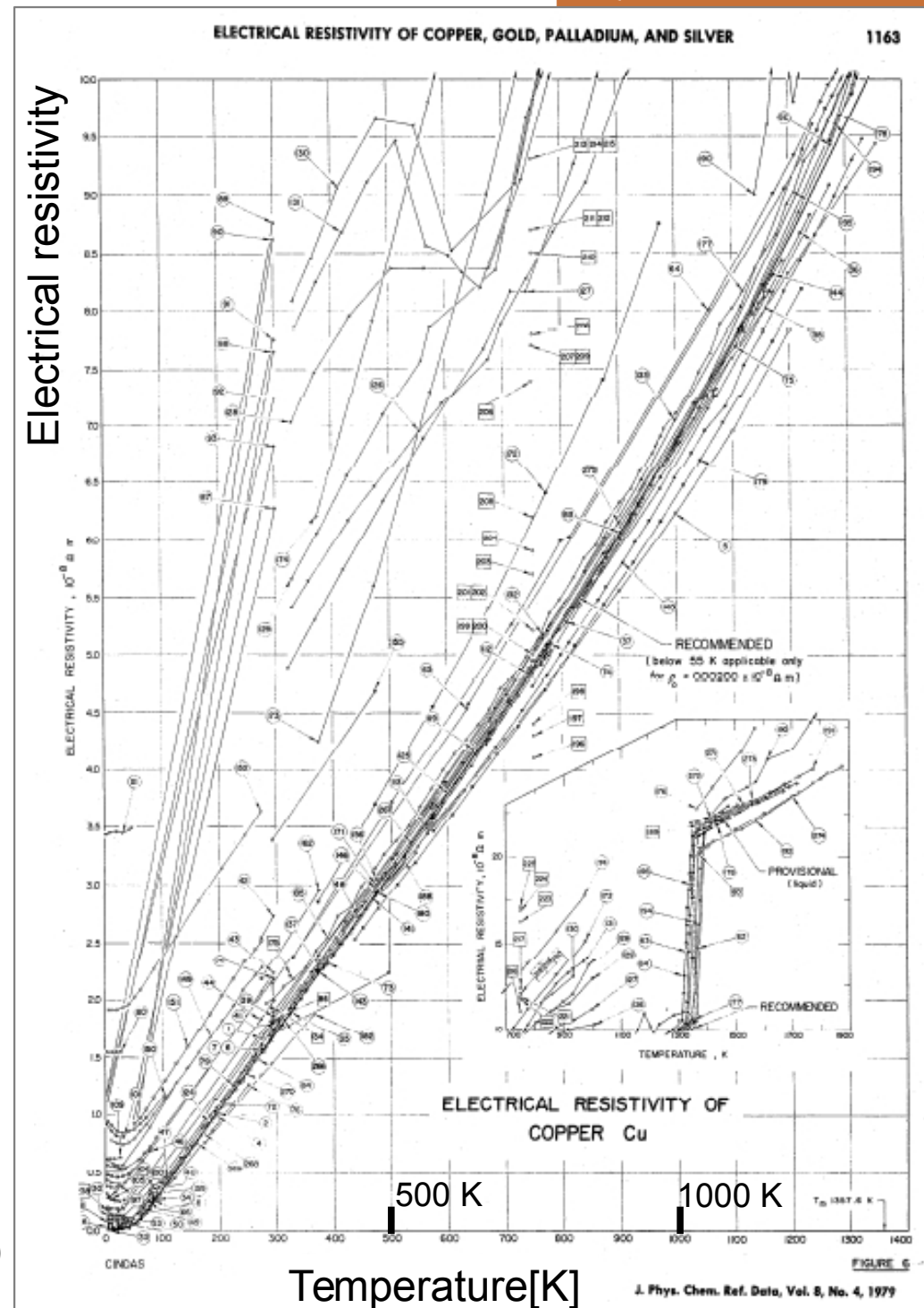
θ – Debye temperature, M – atomic
mass

$$\theta = \frac{\hbar v_D}{k_B}$$

maximal frequency of acoustic
phonons in isotropic Debye model of
specific heat (determined by lattice
constant and sound velocity) [9]

- the formula was derived for
monovalent metal with spherical
Fermi surface and phonon spectrum
from Debye model
- despite this the formula is useful for
initial analysis of experimental results

image from: R.A. Matula, J.Phys.Chem.Ref.Data 8, 1147 (1979)



Introduction

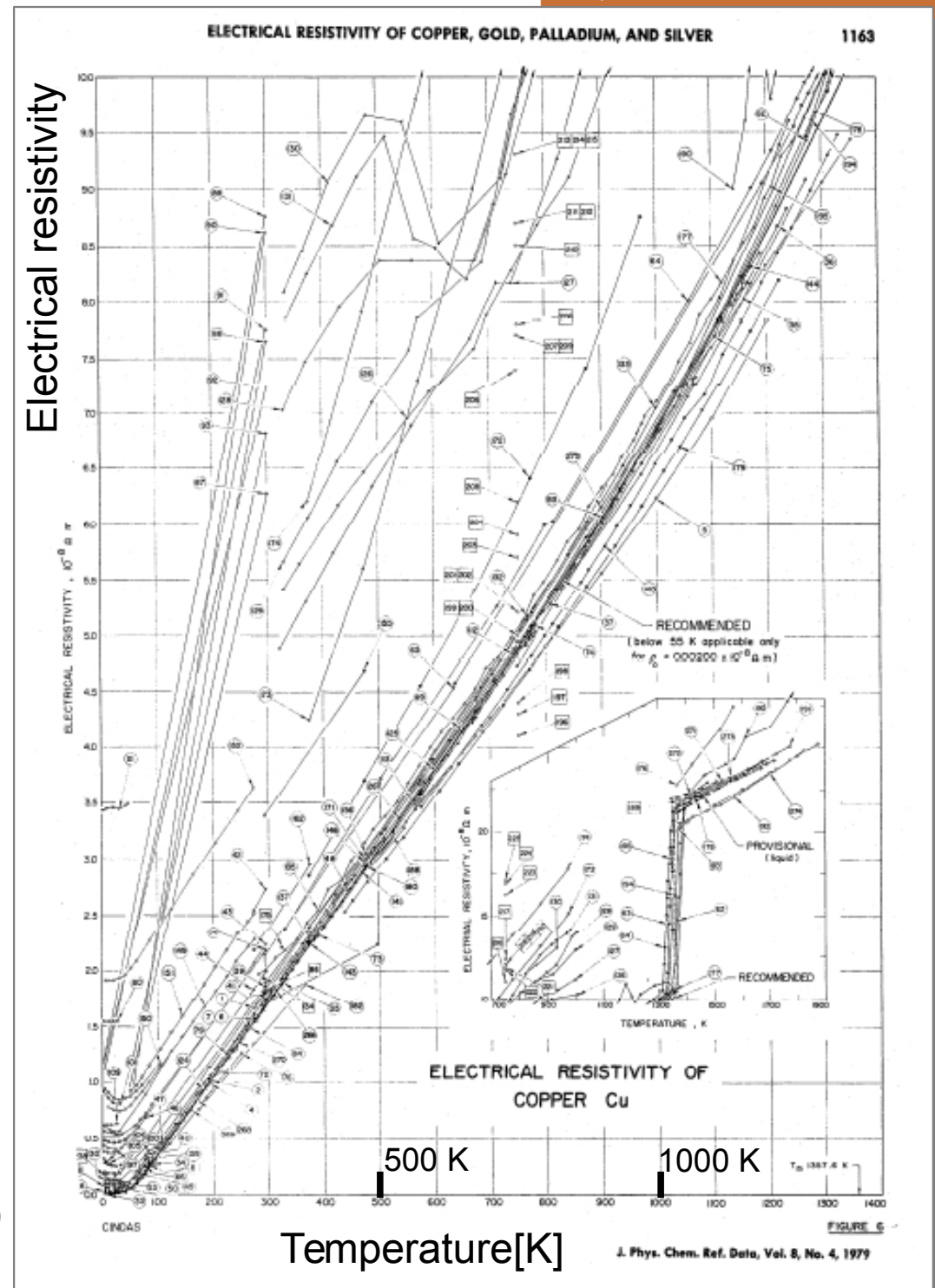
Bloch-Grüneisen equation for limiting temperatures [5]:

$$T \rightarrow 0: \rho_i(T) \rightarrow 124.431 \left(\frac{T}{\theta} \right)^5$$

$$T \rightarrow \infty: \rho_i(T) \rightarrow \frac{C}{4M\theta} \left(\frac{T}{\theta} \right)$$

	θ at 298 K [K] [5,11]
Au	178±8
Ag	221
Cu	320
Fe	467 [11]
Co	445 [11]
Ni	450 [11]

image from: R.A. Matula, J.Phys.Chem.Ref.Data 8, 1147 (1979)



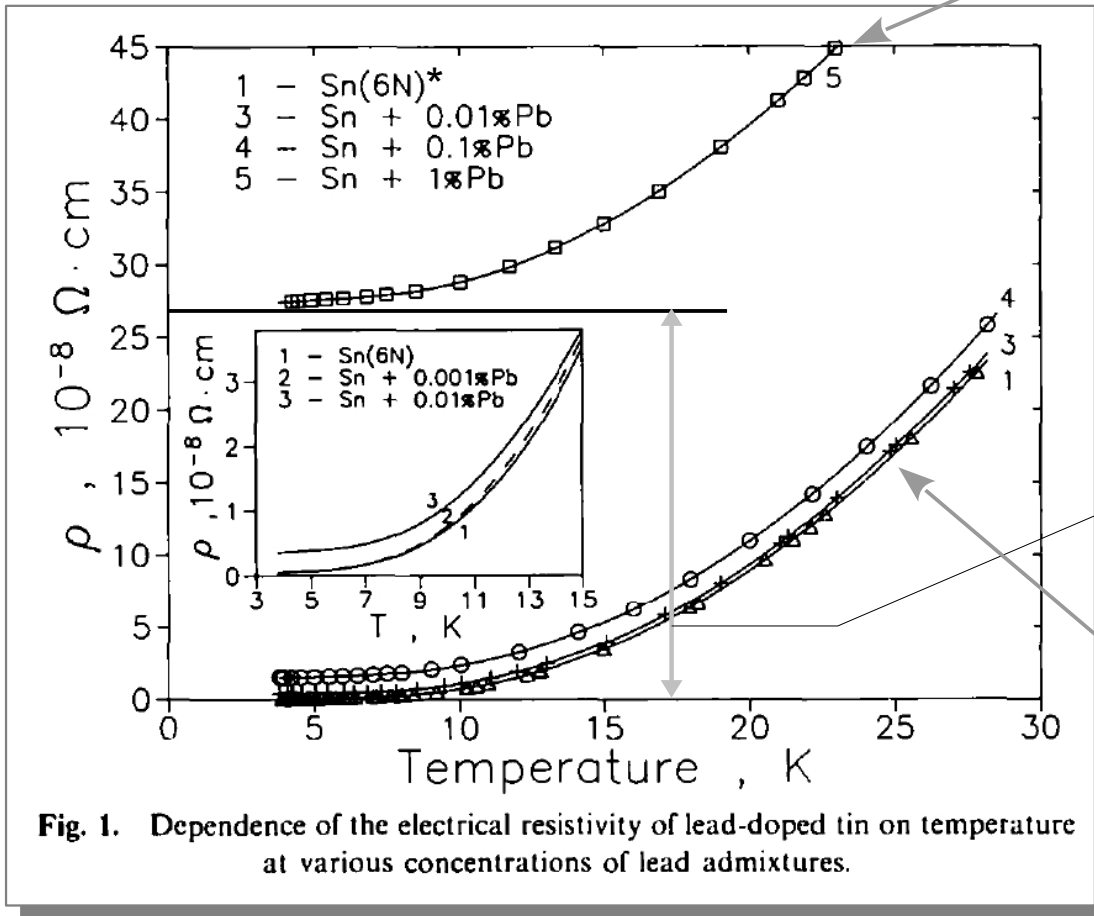
Introduction

Mathiessen rule (empirical) – the total resistivity of a specimen is a sum of resistivities due to phonons, impurities, defects, etc. (approx. valid if scattering events are independent)

$$\rho = \rho_{\text{phonons}} + \rho_{\text{impurities}} + \dots$$

high Pb concentration
“lots” of defects

- tin + lead, $T \leq 28$ K
- the resistivity of the Sn+Pb mixture increases with Pb concentration
- the dependence of a residual resistivity on Pb concentration is not linear



low Pb concentration

Fig. 1. Dependence of the electrical resistivity of lead-doped tin on temperature at various concentrations of lead admixtures.

image from: R. Wawryk, J. Rafalowicz, Cz. Marucha, K. Balcerek, International Journal of Thermophysics **15**, 379 (1994)

*99.9999% purity; see “Why do we need high purity metals?” at <https://www.ameslab.gov/mpc/purityFAQ> for absolute and metals basis purity

(retrieved on 2014.03.11)

Introduction

Size effect in resistivity

In thin films* the resistivity depends additionally on the thickness t of the layer

- in bulk samples only small fraction of electrons experiences the scattering at the outer boundaries (**red** fragments on the trajectories in the image)
- in thin films the contribution from surface scattering becomes important and the resistivity increases

The theoretical determination of the $\rho(t)$ dependence is very difficult. The approximate Fuchs- Sondheimer theory predicts the following $\rho(t)$ dependence [6,34]:

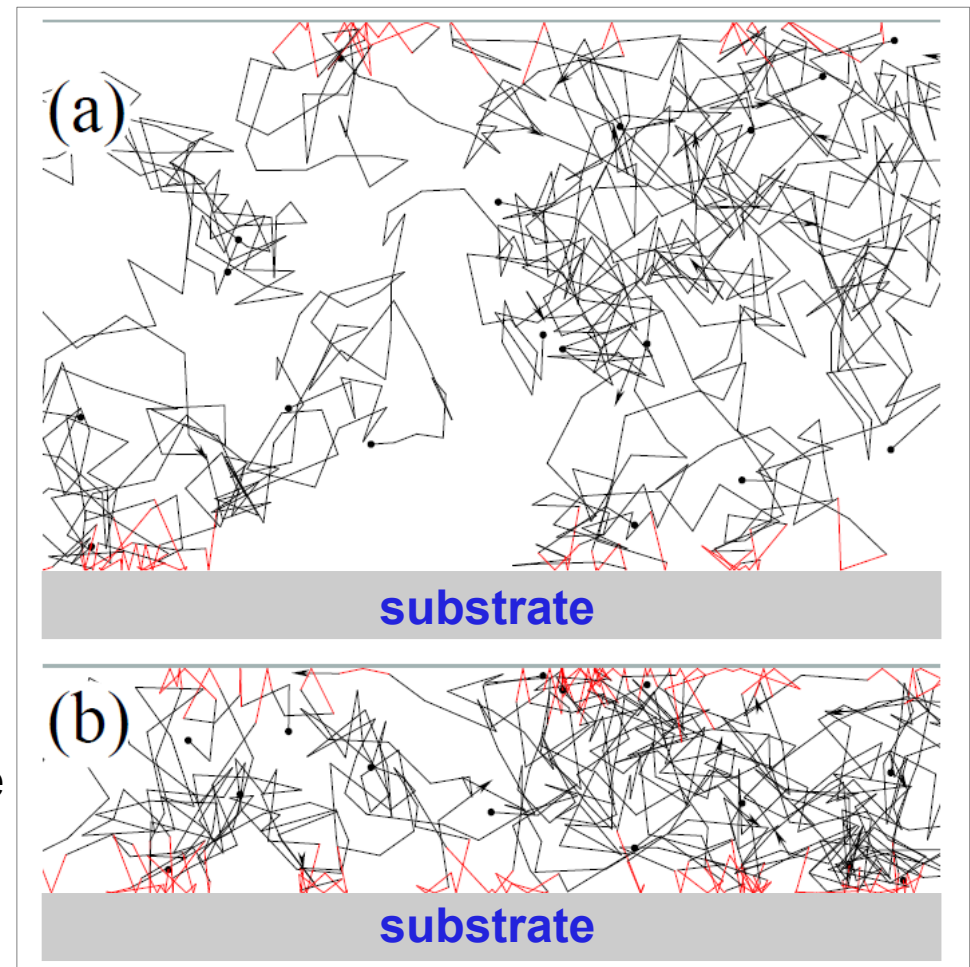
$$\frac{\sigma_0}{\sigma} = 1 + \frac{3}{8} \frac{\lambda}{t} (1 - p) \quad \left(\frac{t}{\lambda} \gg 1 \right)$$

$$\frac{\sigma_0}{\sigma} = \frac{4\lambda}{3t(1+2p)\ln(\lambda/t)} \quad \left(\frac{t}{\lambda} \ll 1 \right)$$

λ – mean free path (mfp), p – fraction of electrons that are specularly** reflected at the outer boundaries, σ_0 – bulk conductivity

**speculum – Latin. mirror

*a film is said to be **thin** if the mean free path of current carriers is comparable with its thickness (compare the definition of magnetic thin film)



Introduction

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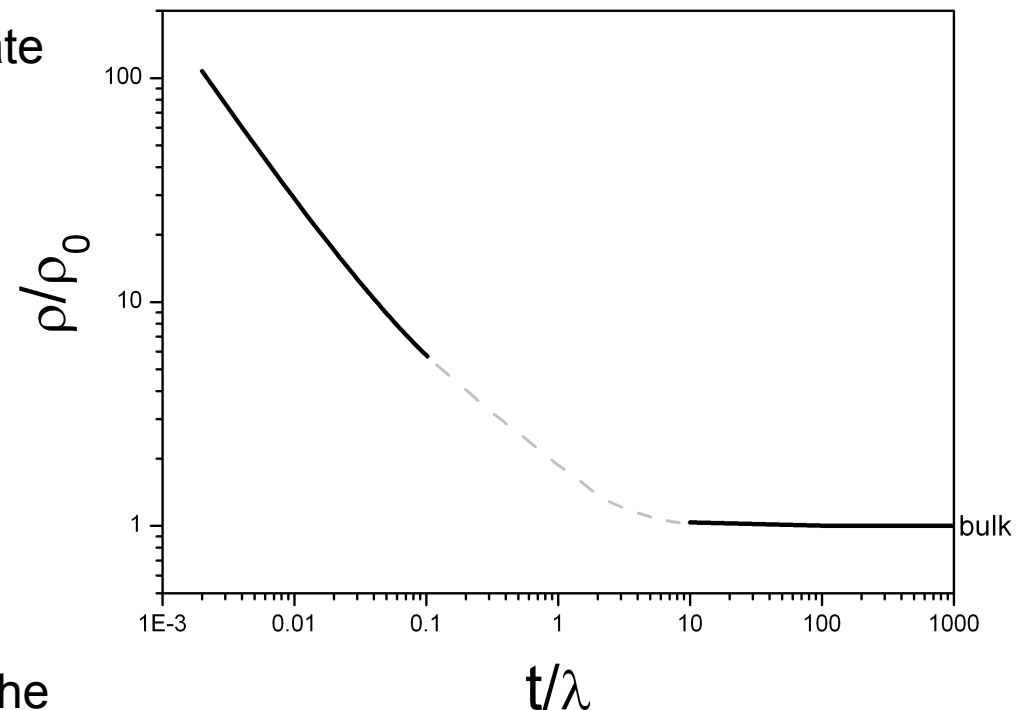
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thermal evaporation onto 500-nm thick SiO_2 on Si(100) substrates in an ultra high vacuum UHV

- Note that the crystalline structure of the films changes with thickness too.

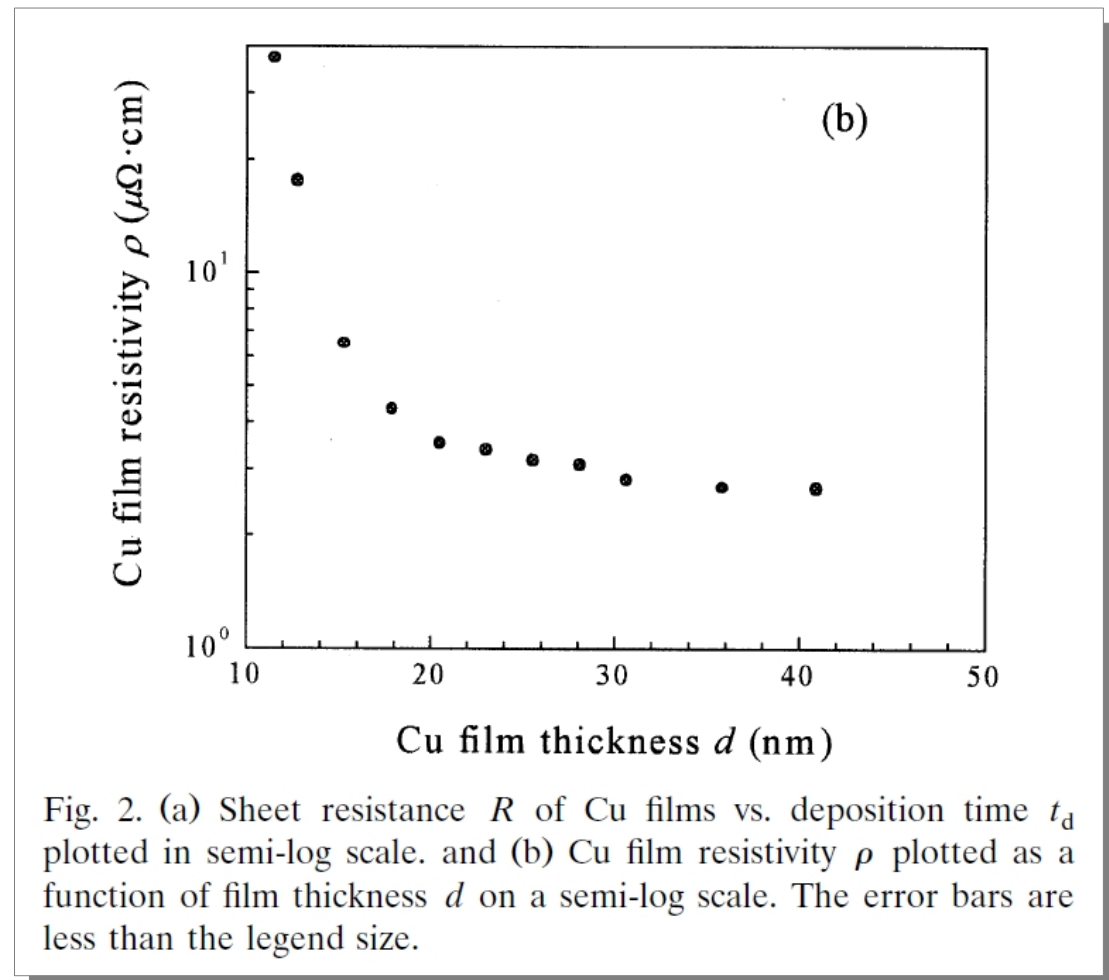
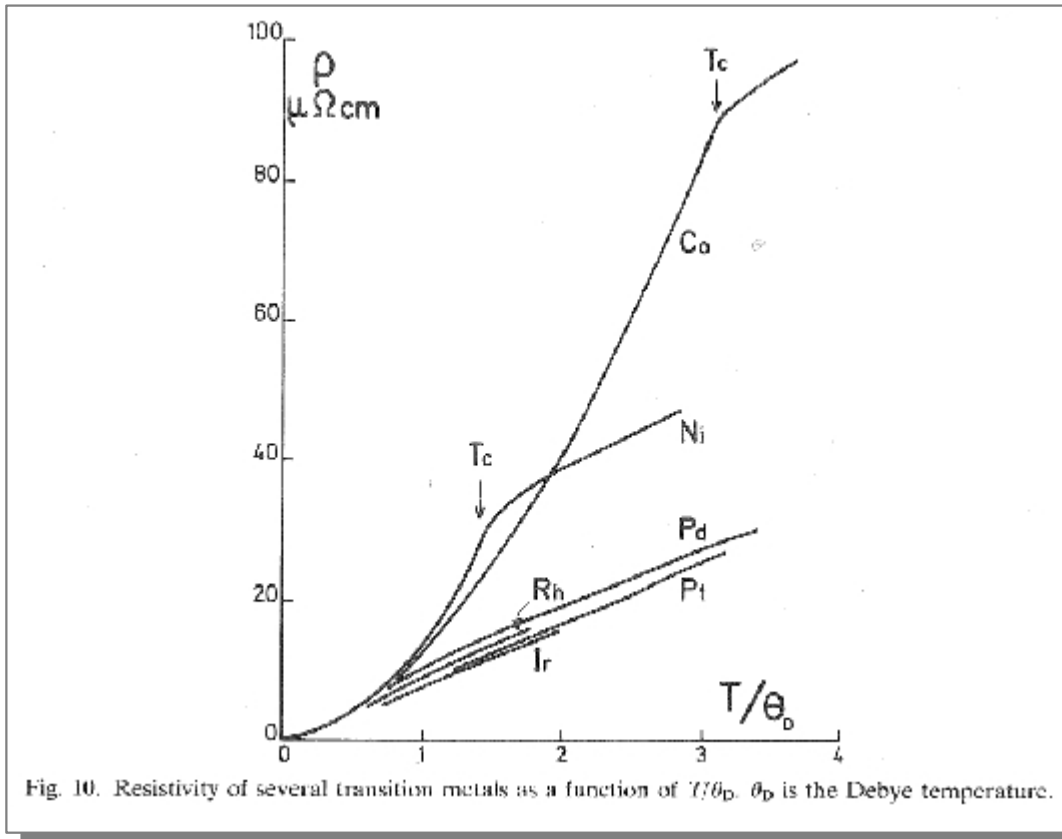


image (fragment) from:
H.-D. Liu, Y.-P. Zhao, G. Ramanath, S.P. Murarka,
G.-C. Wang, Thin Solid Films **384**, 151 (2001)

Dependence of resistivity on temperature in magnetic metals:



- below Curie temperature T_C resistivity of a magnetic metals increases with temperature faster than above it
- below T_C temperature increase leads to increased magnetic disorder
- resistivity and magnetic order correlate

T_{Curie} :

Fe 1044 K

Co 1388 K

Ni 627 K

image from: I.A. Campbell, A. Fert, in "Ferromagnetic Materials" 1982

Dependence of resistivity on temperature in magnetic metals:

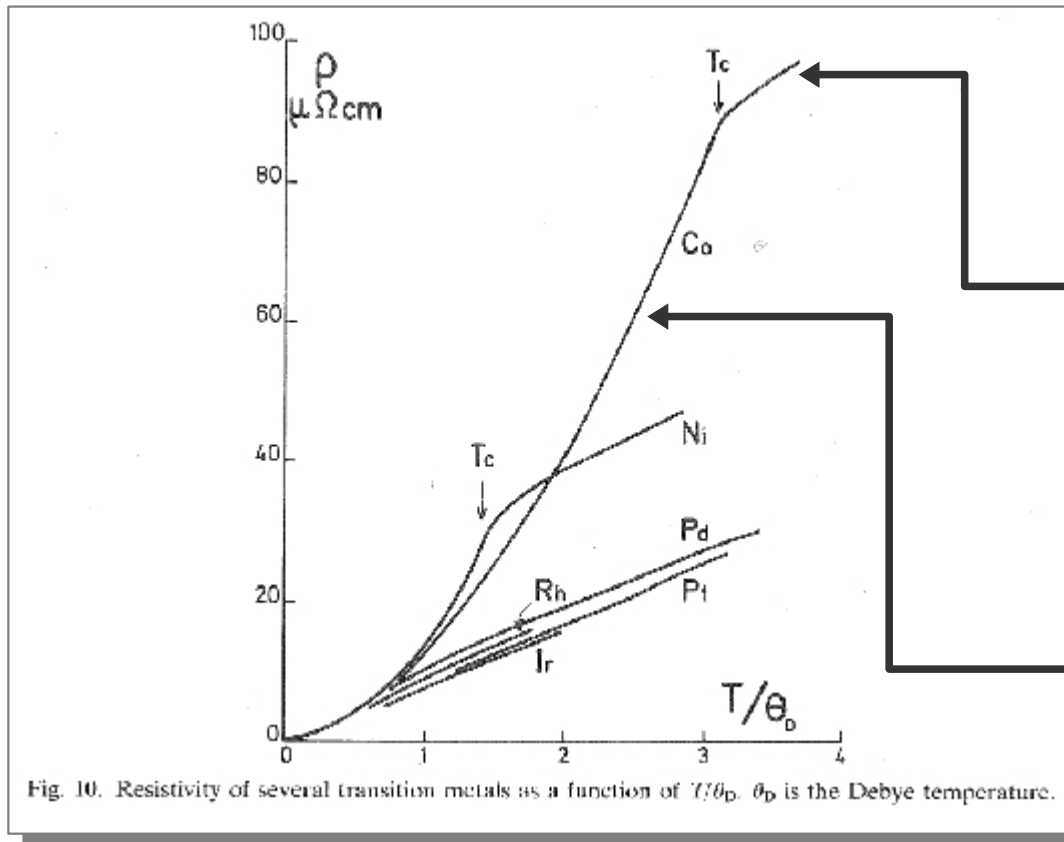
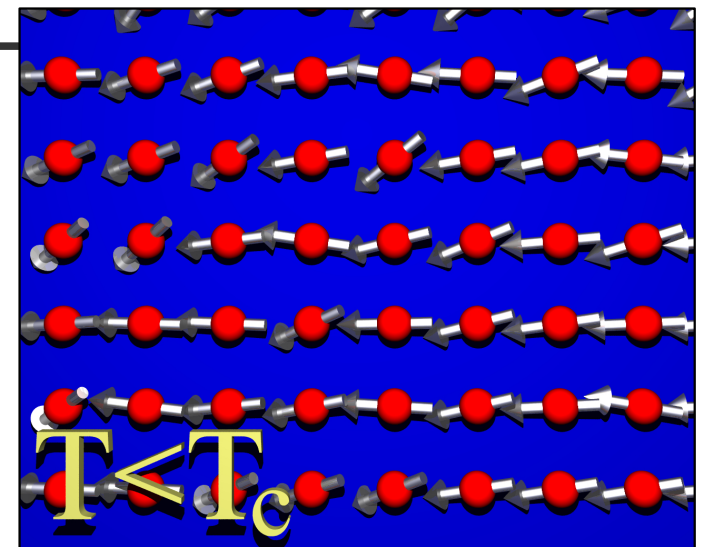
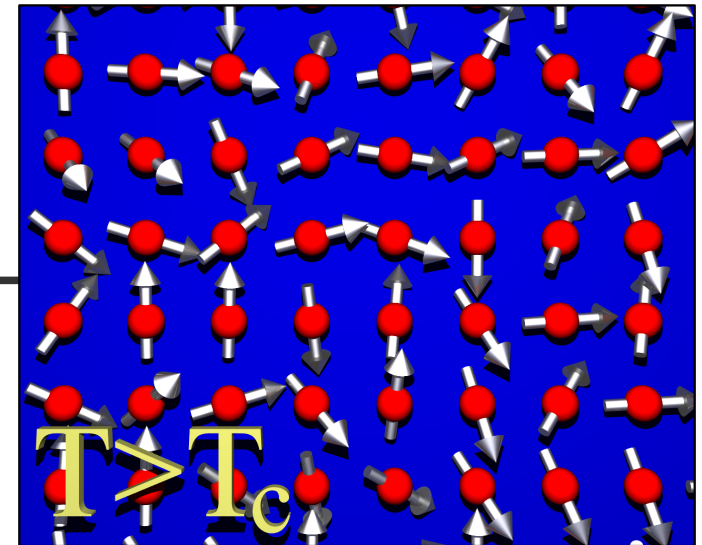


image from: I.A. Campbell, A. Fert, in "Ferromagnetic Materials" 1982



below Curie temperature magnetic moments are ordered

Introduction Pauli principle – fermions, bosons

In the system composed of indistinguishable particles the exchange of the particles does not change the wave function

$$H \psi(r_1, r_2) = E \psi(r_1, r_2) \quad H \psi(r_2, r_1) = E \psi(r_2, r_1)$$

Since the observables do not change when particles are exchanged we must have*

$$\langle \psi(r_1, r_2) | \text{some operator} | \psi(r_1, r_2) \rangle = a \langle \psi(r_1, r_2) | \text{some operator} | \psi(r_1, r_2) \rangle \quad \text{or}$$

$$\psi(r_1, r_2) = a \psi(r_1, r_2)$$

But exchanging the particles twice brings us back to the initial state

$$\psi(r_1, r_2) \rightarrow a \psi(r_1, r_2) \rightarrow a^2 \psi(r_1, r_2)$$

It follows **a=±1**

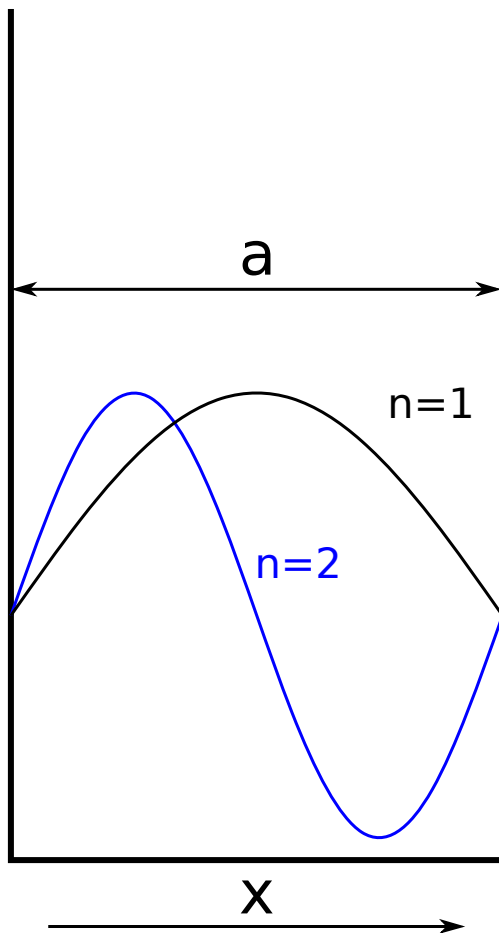
- **a=1** – symmetric wave functions – bosons
- **a=-1** – antisymmetric wave functions – fermions (electrons, protons, neutrons)

*argument, leading to identical conclusions, is different if energy levels are degenerate

Introduction Pauli principle – fermions, bosons

Assume that two identical particles are confined to a potential well of the **infinite** depth and width **a**. The normalized 1-D solutions to Schrödinger equation are of the form [9]:

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(n \frac{\pi}{a} x\right), \quad n = \pm 1, \pm 2, \dots$$



Depending on whether the particles are distinguishable (classical case) or not and on whether they are bosons or fermions the wave function of two non-interacting particles can be written in one of three ways*:

- distinguishable particles

$$\psi(r_1, r_2) = \psi_{n_1}(r_1) \psi_{n_2}(r_2)$$

which means that particle no.1 is in state n_1 at position r_1 and particle no.2 is in state n_2 at position r_2

- bosons (symmetric wave function)

$$\psi(r_1, r_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(r_1) \psi_{n_2}(r_2) + \psi_{n_1}(r_2) \psi_{n_2}(r_1)]$$

- fermions (antisymmetric function)

$$\psi(r_1, r_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(r_1) \psi_{n_2}(r_2) - \psi_{n_1}(r_2) \psi_{n_2}(r_1)]$$

*see the video lectures 7.1-7.5 of Ron Reifenberger () [26]

Assume that two identical particles are confined to a potential well of the **infinite** depth and width **a**. The normalized 1-D solutions to Schrödinger equation are of the form [9]:

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(n \frac{\pi}{a} x\right), \quad n = \pm 1, \pm 2, \dots$$

Depending on whether the particles are distinguishable (classical case) or not and on whether they are bosons or fermions the wave function of two non-interacting particles can be written in one of three ways*:

Both particles in the same quantum state [26]
probability density

- distinguishable particles

$$\psi(r_1, r_2) = \psi_{n_1}(r_1) \psi_{n_2}(r_2)$$

$$P(r_1, r_2) = \psi_n(r_1) \psi_n(r_2) \psi_n(r_1)^* \psi_n(r_2)^*$$

- bosons (symmetric wave function)

$$\psi(r_1, r_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(r_1) \psi_{n_2}(r_2) + \psi_{n_1}(r_2) \psi_{n_2}(r_1)]$$

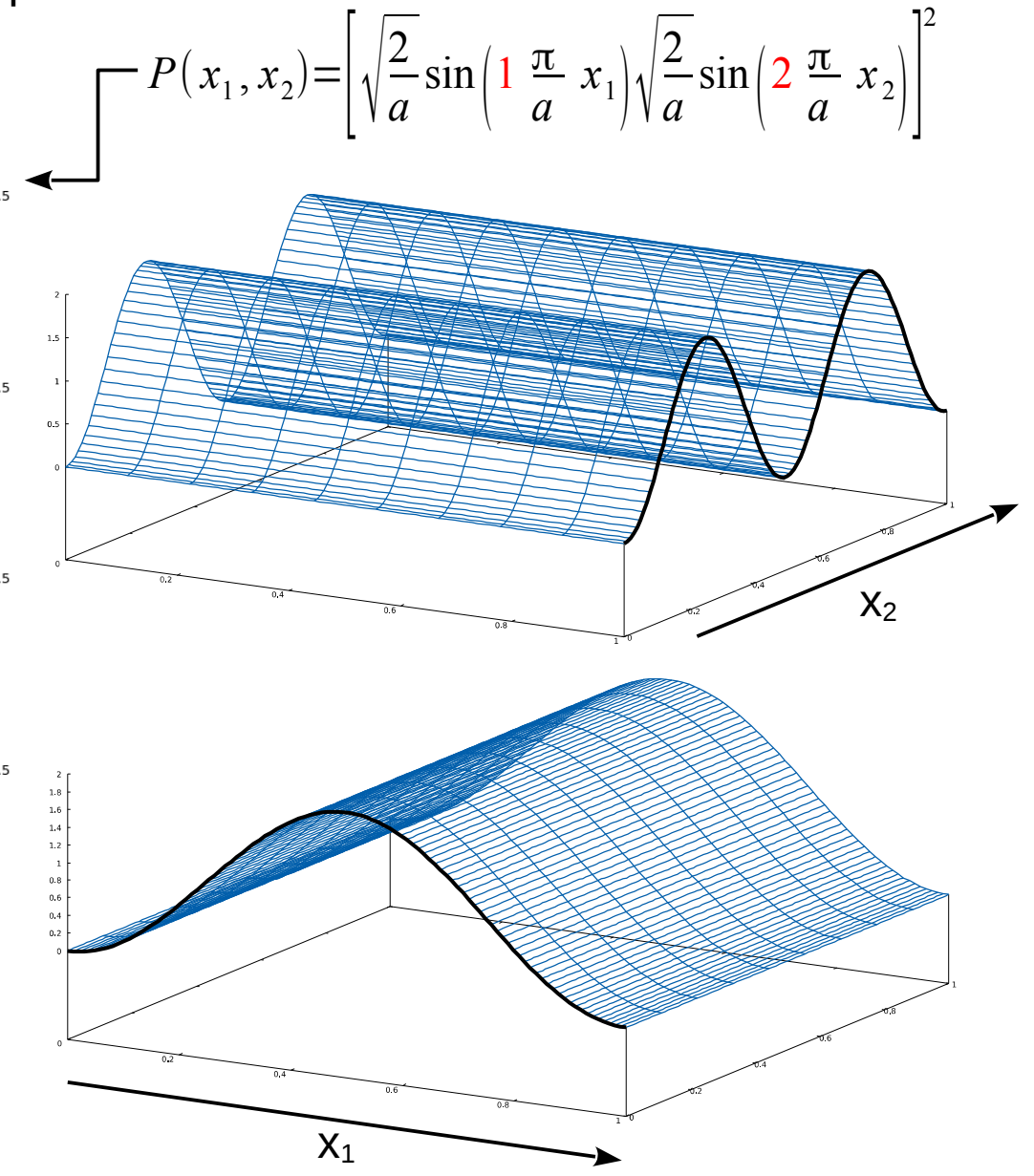
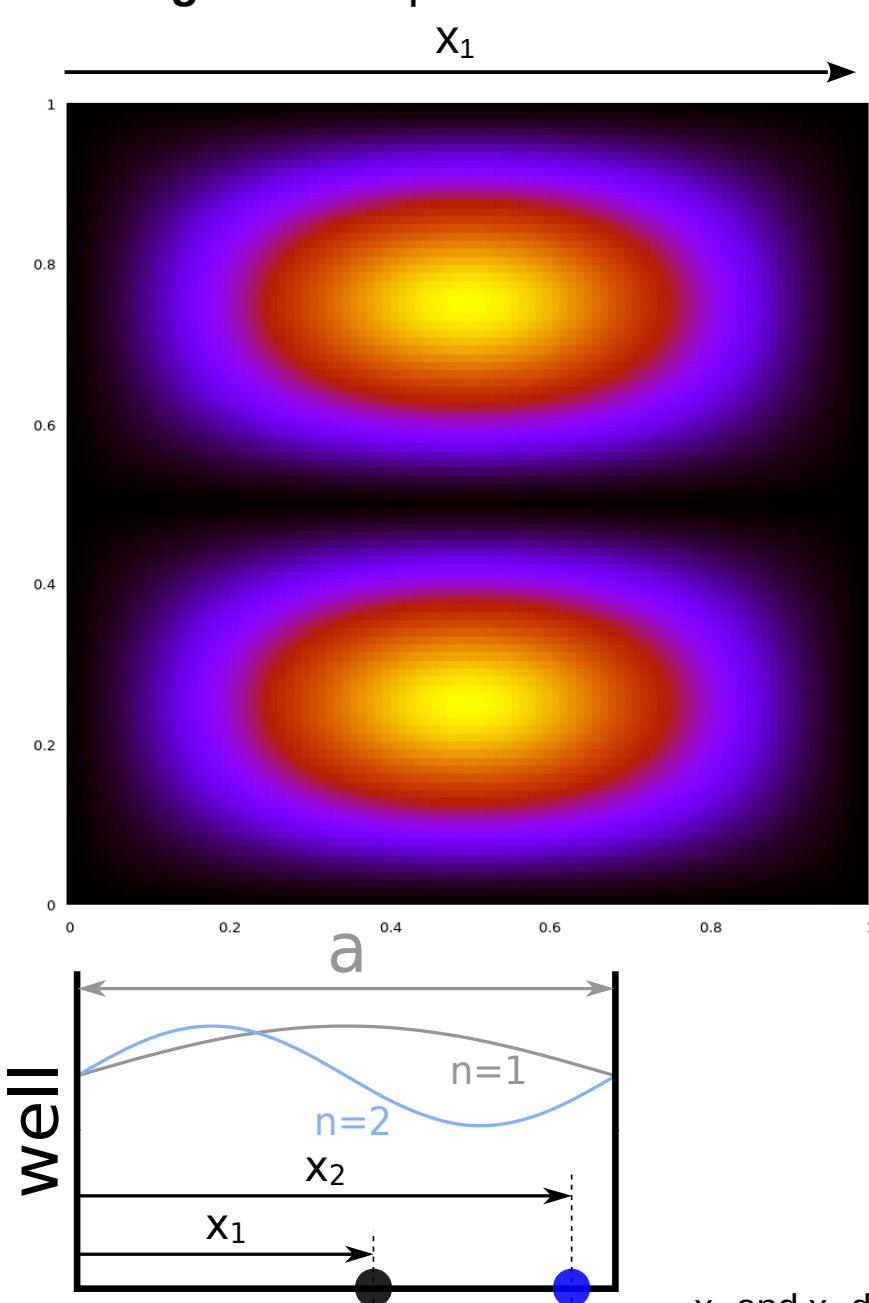
$$P(r_1, r_2) = \left(\frac{1}{\sqrt{2}} [2 \psi_n(r_1) \psi_n(r_2)] \right) \left(\frac{1}{\sqrt{2}} [2 \psi_n(r_1) \psi_n(r_2)] \right)^* \\ = 2 \psi_n(r_1) \psi_n(r_2) \psi_n(r_1)^* \psi_n(r_2)^*$$

Bosons have enhanced probability of being in the same quantum state

Introduction Pauli principle – fermions, bosons

We consider first two levels in a well: one particle in state with $n=1$ and the other with $n=2$

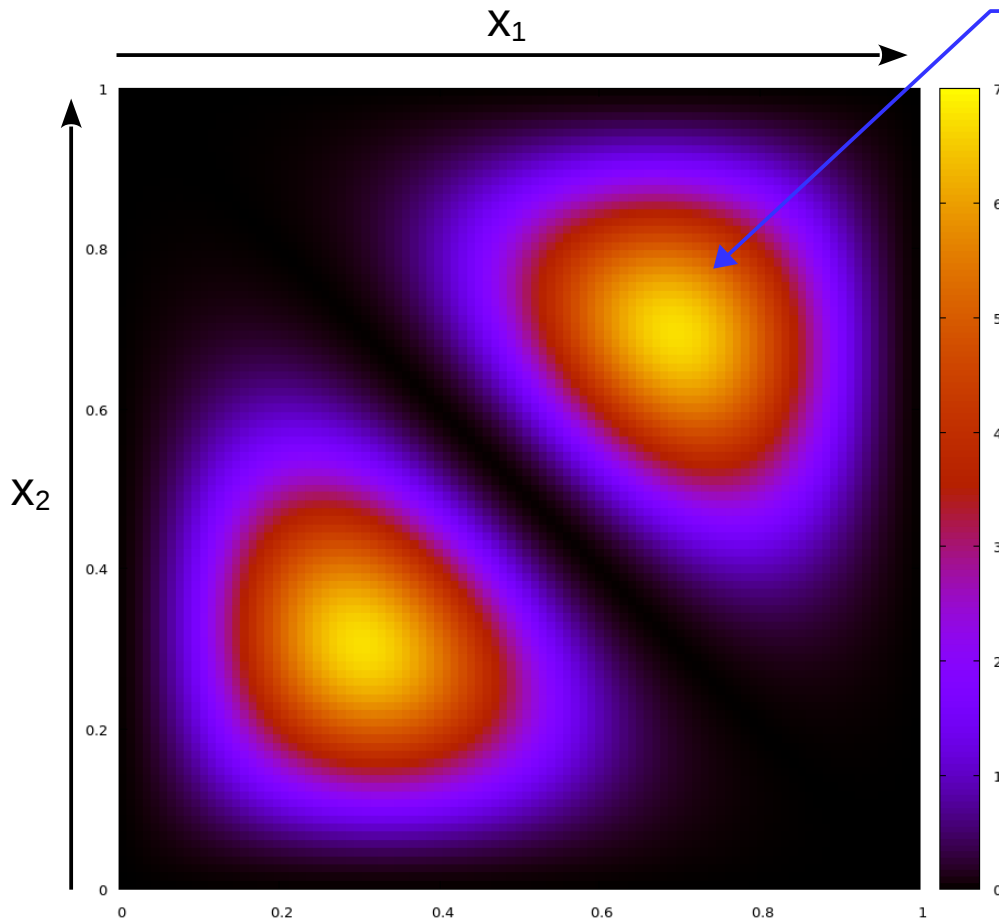
Distinguishable particles – classical description



$$P(x_1, x_2) = \left[\sqrt{\frac{2}{a}} \sin\left(1 \frac{\pi}{a} x_1\right) \sqrt{\frac{2}{a}} \sin\left(2 \frac{\pi}{a} x_2\right) \right]^2$$

x_1 and x_2 denote the positions of particles along x-axis

We consider first two levels in a well: one particle in state with $n=1$ and the other with $n=2$
Indistinguishable boson particles

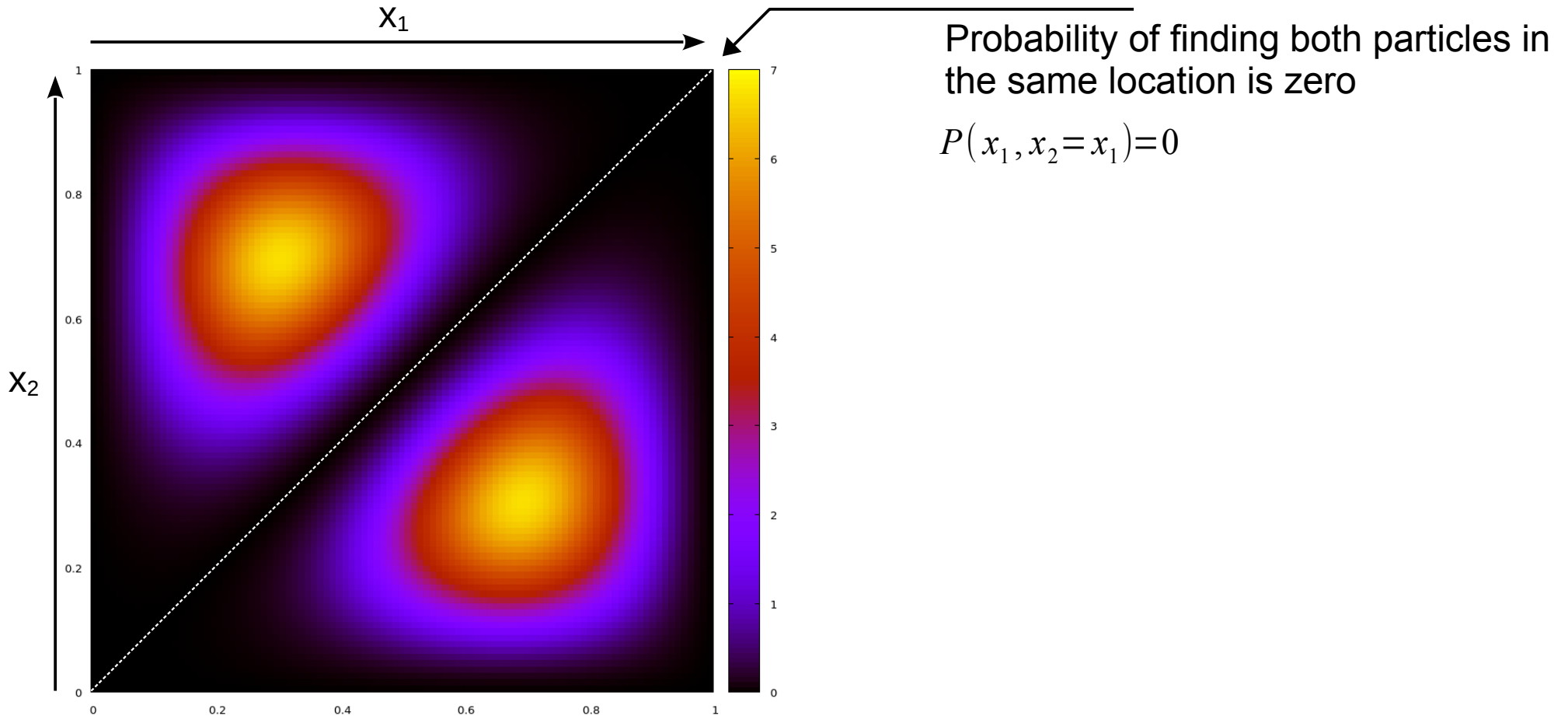


Probability maxima correspond to both particles being at the same location

$$x_2 = x_1$$

$$P(x_1, x_2) = \left[\sqrt{\frac{2}{a}} \sin\left(1 \frac{\pi}{a} x_1\right) \sqrt{\frac{2}{a}} \sin\left(2 \frac{\pi}{a} x_2\right) + \sqrt{\frac{2}{a}} \sin\left(1 \frac{\pi}{a} x_2\right) \sqrt{\frac{2}{a}} \sin\left(2 \frac{\pi}{a} x_1\right) \right]^2$$

We consider first two levels in a well: one particle in state with $n=1$ and the other with $n=2$
Indistinguishable fermions (with the same spin- see next slides)



$$P(x_1, x_2) = \left[\sqrt{\frac{2}{a}} \sin\left(1 \frac{\pi}{a} x_1\right) \sqrt{\frac{2}{a}} \sin\left(2 \frac{\pi}{a} x_2\right) - \sqrt{\frac{2}{a}} \sin\left(1 \frac{\pi}{a} x_2\right) \sqrt{\frac{2}{a}} \sin\left(2 \frac{\pi}{a} x_1\right) \right]^2$$

Introduction Exchange coupling

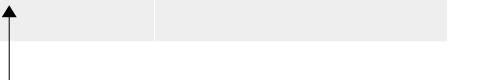
Consider a system composed of two particles with spin $\frac{1}{2}$. For one spin we have a set of matrices

$$S_x = \frac{1}{2}\hbar \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad S_y = \frac{1}{2}\hbar \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad S_z = \frac{1}{2}\hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \leftarrow \begin{matrix} \text{Pauli matrices} \\ \sigma_1, \sigma_2, \sigma_3 \end{matrix}$$

with eigenvalues $\pm \frac{1}{2}\hbar$ and corresponding eigenvectors $\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for $S_z = \frac{1}{2}\hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

With 2 spins we should work in 4-dimensional representation*. Each spin has two eigenvectors so there are 4 possibilities:

spin1/spin2	$\uparrow \uparrow$	$\uparrow \downarrow$	$\downarrow \uparrow$	$\downarrow \downarrow$
traditional notation	$\alpha(1)\alpha(2)$	$\alpha(1)\beta(2)$	$\beta(1)\alpha(2)$	$\beta(1)\beta(2)$



meaning first spin down, second spin up

α and β traditionally mean up and down, respectively

Introduction Exchange coupling

From single spin vectors we can construct symmetric and antisymmetric functions (with respect to spin exchange) [B. Średniawa, 39_{p.233}]:

$$\alpha(1)\alpha(2), \quad \beta(1)\beta(2), \quad \frac{1}{\sqrt{2}}[\alpha(1)\beta(2)+\beta(1)\alpha(2)],$$

$$\frac{1}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)] = -\left[\frac{1}{\sqrt{2}}[\alpha(2)\beta(1)-\beta(2)\alpha(1)]\right]$$

note the minus sign

antisymmetric combination

Combining Pauli matrices into vector we get:

$$\vec{\sigma} = \hat{x} \sigma_x + \hat{y} \sigma_y + \hat{z} \sigma_z = \hat{x} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \hat{y} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \hat{z} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \hat{z} & \hat{x} - i \hat{y} \\ \hat{x} + i \hat{y} & -\hat{z} \end{pmatrix}$$

and for a resultant spin momentum

$$\vec{J} = \frac{\hbar}{2}(\vec{\sigma}(1) + \vec{\sigma}(2))$$

each operator acts on its "own" spin

For a square of the momentum we have

$$J^2 = \frac{\hbar^2}{4} [(\sigma_x(1) + \sigma_x(2))^2 + (\sigma_y(1) + \sigma_y(2))^2 + (\sigma_z(1) + \sigma_z(2))^2] = \frac{\hbar^2}{2} [3 + \vec{\sigma}(1) \cdot \vec{\sigma}(2)]$$

We act now with the operator $\vec{\sigma}(1) \cdot \vec{\sigma}(2)$ on constructed spin functions (using explicit forms of Pauli matrices):

$$\begin{aligned} [\vec{\sigma}(1) \cdot \vec{\sigma}(2)]\alpha(1)\alpha(2) &= [\sigma_x(1)\sigma_x(2) + \sigma_y(1)\sigma_y(2) + \sigma_z(1)\sigma_z(2)]\alpha(1)\alpha(2) \\ &= \beta(1)\beta(2) + i\beta(1)i\beta(2) + \alpha(1)\alpha(2) = 1 \cdot \alpha(1)\alpha(2) \end{aligned}$$

Introduction Exchange coupling

Which means that eigenvalue of $\vec{\sigma}(1) \cdot \vec{\sigma}(2)$ for $\alpha(1)\alpha(2)$ function is **1**:

$$[\vec{\sigma}(1) \cdot \vec{\sigma}(2)] \alpha(1)\alpha(2) = 1 \cdot \alpha(1)\alpha(2)$$

Inserting this “1” into the expression for the square of the momentum yields:

$$J^2 = \frac{\hbar^2}{2} [3 + \vec{\sigma}(1) \cdot \vec{\sigma}(2)] = \frac{\hbar^2}{2} 4 \quad \rightarrow \quad J = \hbar \sqrt{2}$$

From the expression of the momentum corresponding to a spin ($L_S = \sqrt{S(S+1)}\hbar$) we see that:

- this value of momentum ($\hbar\sqrt{2}$) corresponds to resultant **spin 1** \leftarrow $S=1$:
 $L_S = \sqrt{1(1+1)}\hbar = \hbar\sqrt{2}$
- and consequently the function $\alpha(1)\alpha(2)$ corresponds to **spin 1**

Analogous calculations show that all three symmetric two spin functions correspond to **spin 1** (each of them corresponds to different component of momentum along z-axis)

$$\alpha(1)\alpha(2), \quad \frac{1}{\sqrt{2}} [\alpha(1)\beta(2) + \beta(1)\alpha(2)], \quad \beta(1)\beta(2)$$

$$S_z: \quad +\hbar \qquad \qquad 0 \qquad \qquad -\hbar$$

resultant spin $S=1$

triplet

$$\frac{1}{\sqrt{2}} [\alpha(1)\beta(2) - \beta(1)\alpha(2)]$$

resultant spin $S=0$

singlet

When we are dealing **with fermions the total wave function must be asymmetric.**

If Hamiltonian has no terms dependent on spin we can write the total wave function as a product of spatial and spin wave functions. We can have thus two cases [Enge..]:

- spatial function is asymmetric, spin function is symmetric (triplet)

$$\frac{1}{\sqrt{2}} [\psi_{n_1}(r_1)\psi_{n_2}(r_2) - \psi_{n_1}(r_2)\psi_{n_2}(r_1)] \times \alpha(1)\alpha(2)$$

$$\frac{1}{\sqrt{2}} [\psi_{n_1}(r_1)\psi_{n_2}(r_2) - \psi_{n_1}(r_2)\psi_{n_2}(r_1)] \times \frac{1}{\sqrt{2}} [\alpha(1)\beta(2) + \beta(1)\alpha(2)]$$

$$\frac{1}{\sqrt{2}} [\psi_{n_1}(r_1)\psi_{n_2}(r_2) - \psi_{n_1}(r_2)\psi_{n_2}(r_1)] \times \beta(1)\beta(2)$$

spin functions

- spatial function is symmetric, spin function is asymmetric (singlet)

$$\frac{1}{\sqrt{2}} [\psi_{n_1}(r_1)\psi_{n_2}(r_2) + \psi_{n_1}(r_2)\psi_{n_2}(r_1)] \times \frac{1}{\sqrt{2}} [\alpha(1)\beta(2) - \beta(1)\alpha(2)]$$

Assume now that the two particles (electrons with a spin) interact via Coulomb electrostatic interactions.

If the interaction is weak we can use a non-degenerate perturbation method for which we have

$$E_m^{(1)} = V_{mm} = \int \psi_m^{(0)*} \hat{V} \psi_m^{(0)} dV$$

$$E_m = E_m^{(0)} + \lambda V_{mm} \quad E_m = E_0 + \langle 0 m | H^{(1)} | 0 m \rangle$$

The correction to the eigenvalues in the first order approximation is equal to the average energy of the perturbation in the unperturbed state

Depending on the spin state (triplet, singlet) the spatial wave function is either symmetric or antisymmetric, and because the spin function is not acted upon by the perturbation we get [39]

$$E^{(1)} = \int \frac{1}{\sqrt{2}} [\psi_{n_1}(r_1) \psi_{n_2}(r_2) \pm \psi_{n_1}(r_2) \psi_{n_2}(r_1)]^* \frac{e^2}{r_{1,2}} \frac{1}{\sqrt{2}} [\psi_{n_1}(r_1) \psi_{n_2}(r_2) \pm \psi_{n_1}(r_2) \psi_{n_2}(r_1)] dV$$

which yields [39]

$$\hat{V} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_{1,2}}$$

$$E^{(1)} = \int |\psi_{n_1}(r_1)|^2 \frac{e^2}{r_{1,2}} |\psi_{n_2}(r_2)|^2 dV \pm \int \psi_{n_1}(r_1)^* \psi_{n_2}(r_2)^* \frac{e^2}{r_{1,2}} \psi_{n_1}(r_2) \psi_{n_2}(r_1) dV$$

Coulomb integral

exchange integral

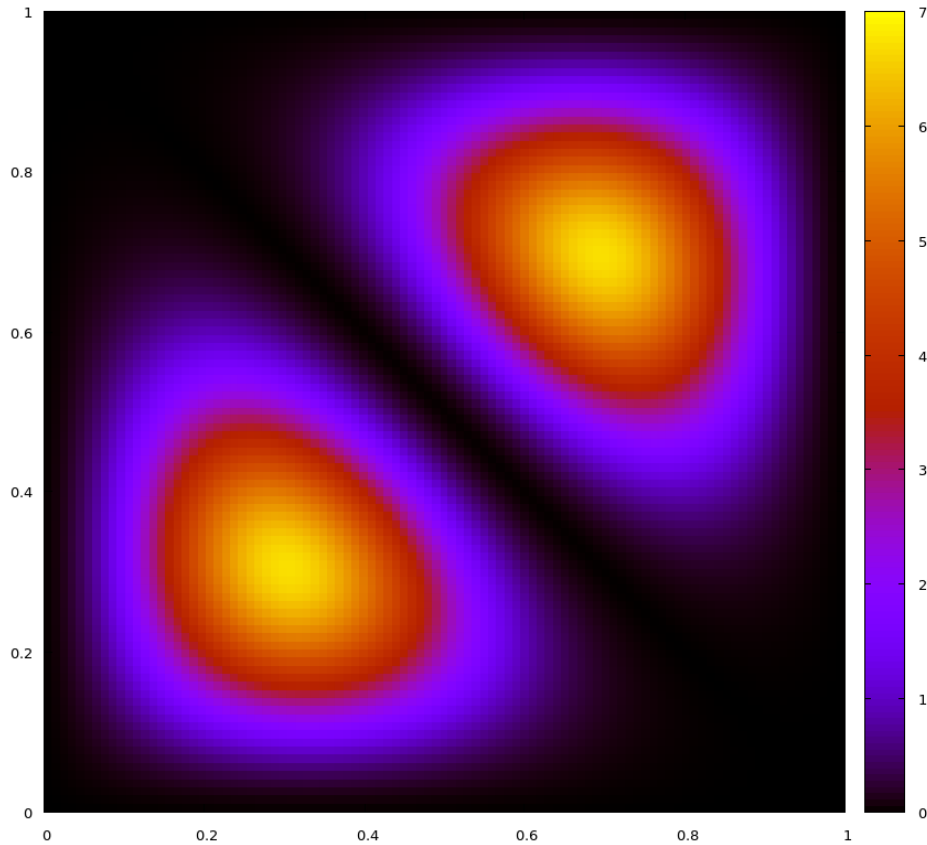
Introduction Exchange coupling

$$E^{(1)} = \int |\psi_{n_1}(r_1)|^2 \frac{e^2}{r_{1,2}} |\psi_{n_2}(r_2)|^2 dV \pm \int \psi_{n_1}(r_1)^* \psi_{n_2}(r_2)^* \frac{e^2}{r_{1,2}} \psi_{n_1}(r_2) \psi_{n_2}(r_1) dV$$

Coulomb integral

exchange integral

Two particles in an infinite potential well – symmetric function



If spatial function is **symmetric** the particles tend to be closer to each other than in a classical case (due to statistical forces*) and **the electrostatic interactions increase energy of the system**

If spatial function is antisymmetric (**triplet**) the particles are repelled by statistical forces and the electrostatic interaction energy is lower than in the classical case [39, p. 301]

The exchange interactions favor parallel orientations of spins

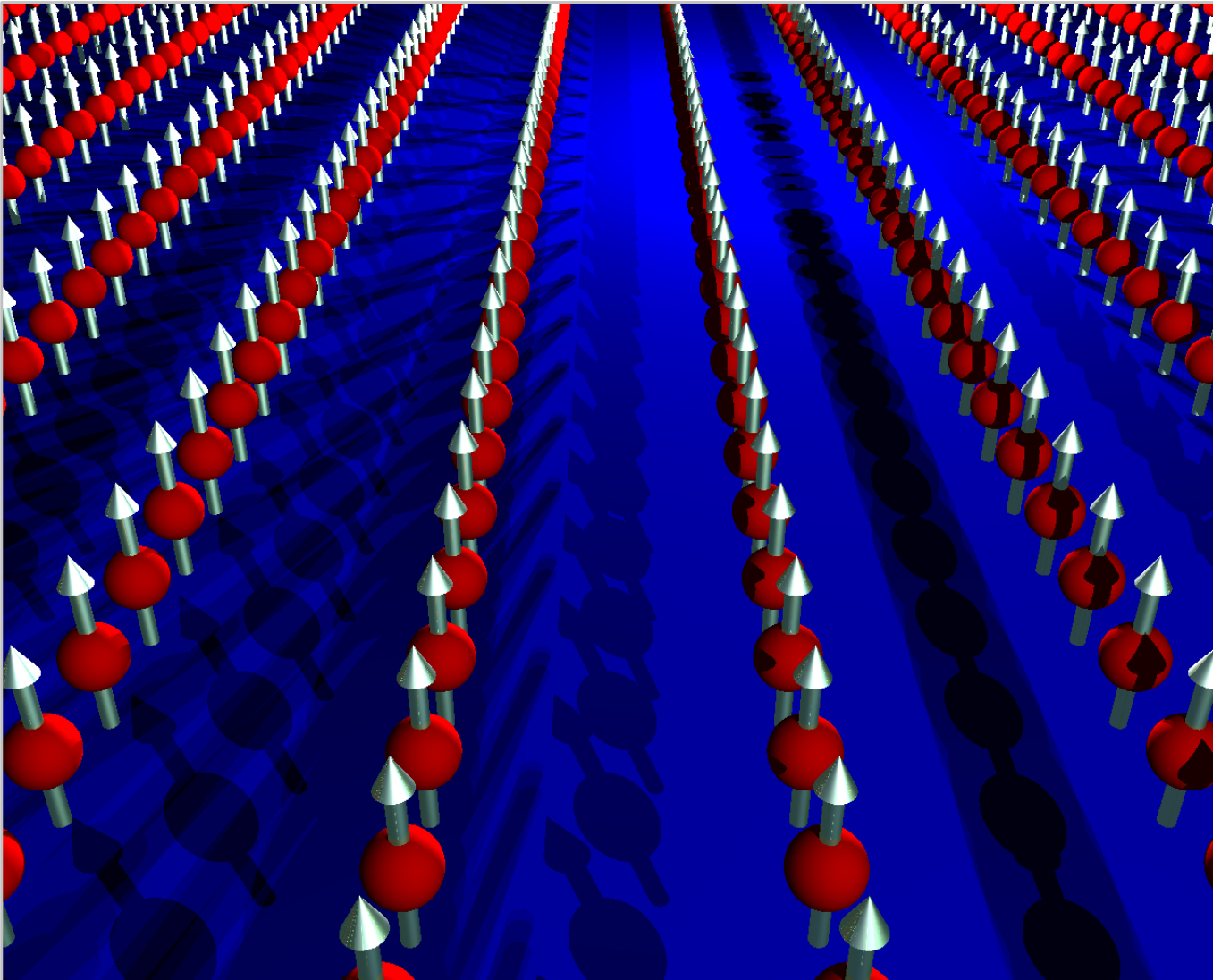
For the description of the image see 6 slides back

*see the lecture of prof. T. Dietl: Physics of Exchange Interactions in Solids, Osaka/Japan, 2010.05.30 (youtube)

Introduction

Magnons

- at very low temperatures all spins point in almost exactly one direction* determined by the effective magnetic anisotropy

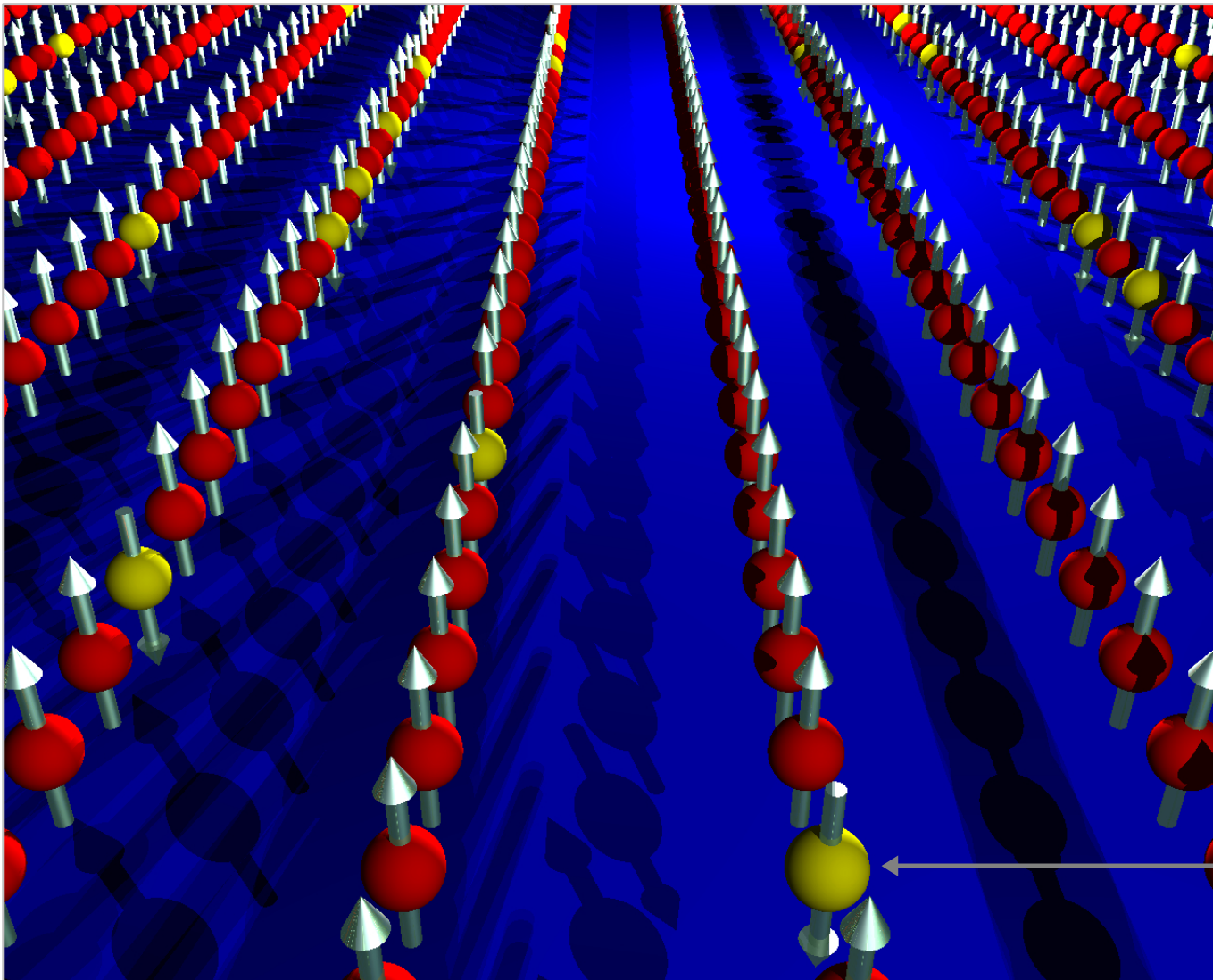


*we assume the bulk sample (negligible fraction of surface spins) and lack of domain walls

Introduction

Magnons

- at higher temperatures the thermal energy coming from spin-phonon coupling must be accommodated by rearrangement of spins*



It is energetically costly to reverse single spins S [9]:

$$\Delta E = N_{nn} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

N_{nn} - number of nearest neighbors

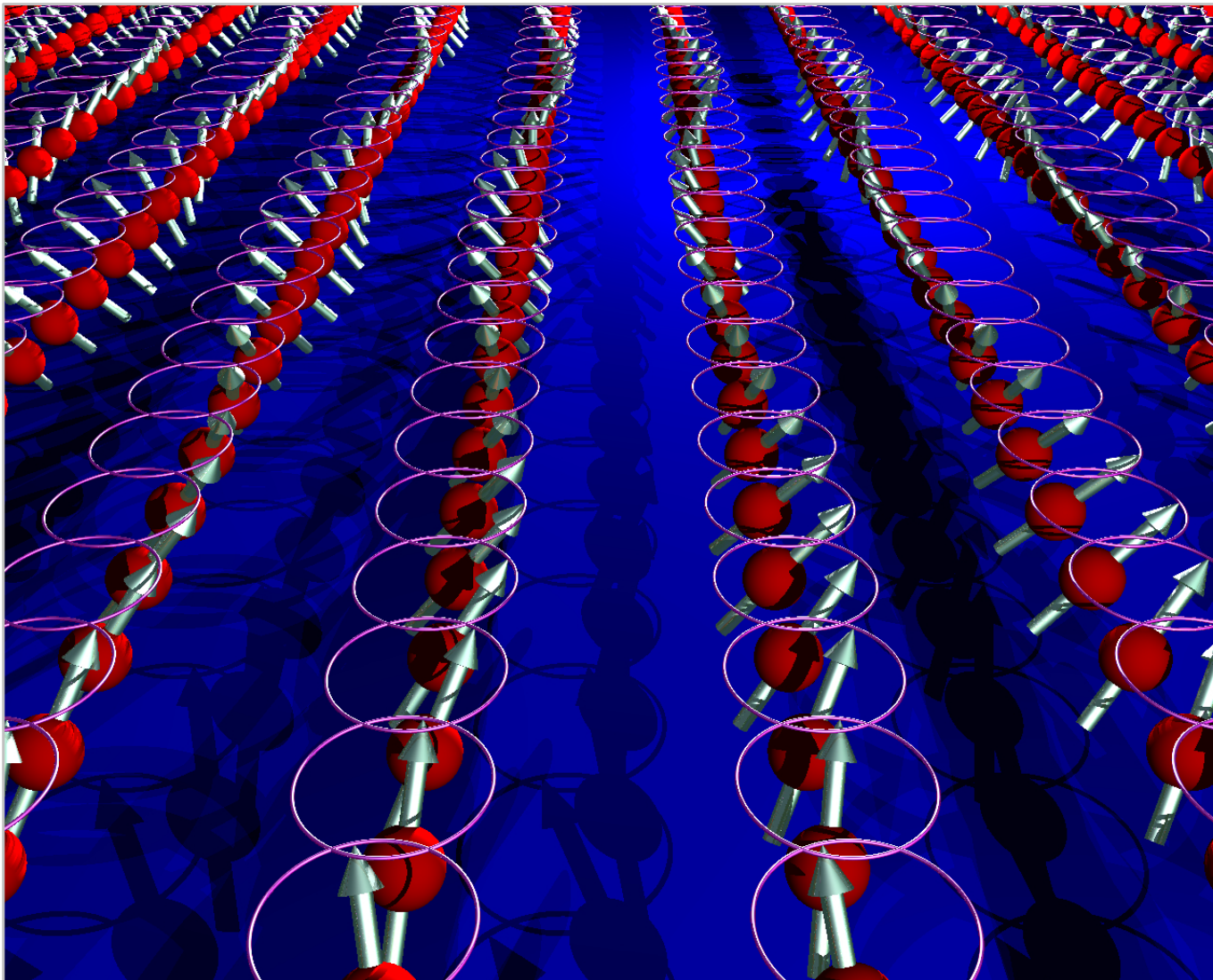
J_{ij} - exchange integral

...and a state with single flipped spin is not an eigenstate of Hamiltonian

*the spins may be excited directly by magnetic field too.

Magnons

- at higher temperatures the thermal energy coming from spin-phonon coupling must be accommodated by rearrangement of spins*



It is energetically costly to reverse single spins S [9]:

$$\Delta E = N_{nn} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

N_{nn} - number of nearest neighbors

J_{ij} - exchange integral

Correlated movement/ precession of spins (pictured here as classical moments) reduces the energy of a spin system by [9]:

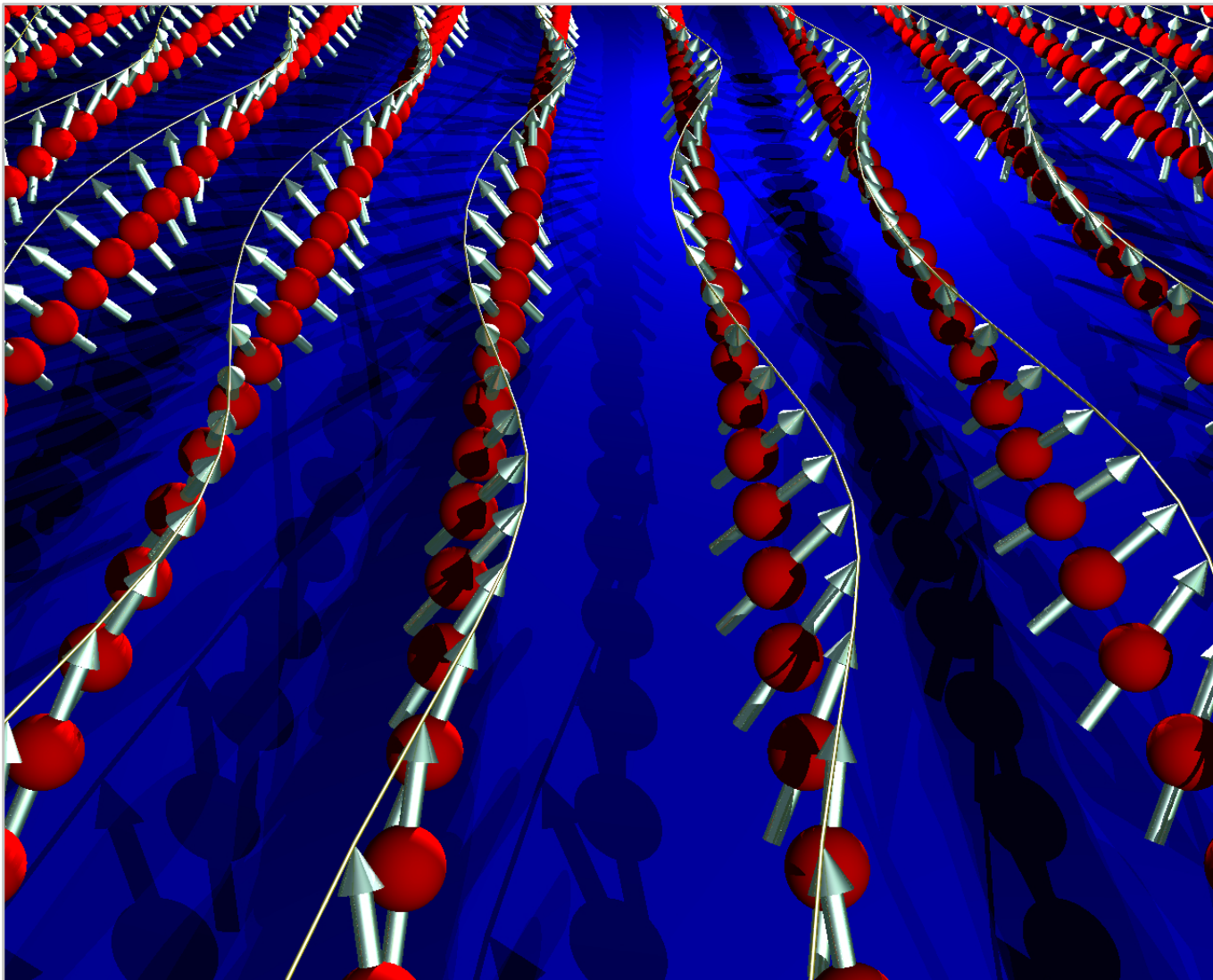
$$\Delta E = \left(\frac{\hbar}{2\pi} \right) \omega_q \approx 2SJq^2 a^2$$

*the spins may be excited directly by magnetic field too.

Introduction

Magnons

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energy of a spin system by
[9]:

$$\Delta E = \left(\frac{\hbar}{2\pi} \right) \omega_q \approx 2SJq^2 a^2$$

Each elementary excitation
reduces the total spin NS
of the system by one unit.

*the spins may be excited directly by magnetic field too.

Introduction

Magnon contribution to resistivity

- measurement in pulsed magnetic fields
- almost linear and non-saturating decrease of resistivity in fields above technical saturation (paraprocess)
- spin-flip and non-spin-flip (on phonons, other electrons [8]) scattering events are responsible for resistance
- spin-flip scattering on magnons (s-d interband transitions) is responsible for magnetoresistance
- “effect results from a reduction of electron-magnon scattering processes due to a damping of the spin waves at high fields” [8]
- magnon magnetoresistance is estimated to saturate at 80T at 50K and **2000 T** [sic] at 450 K

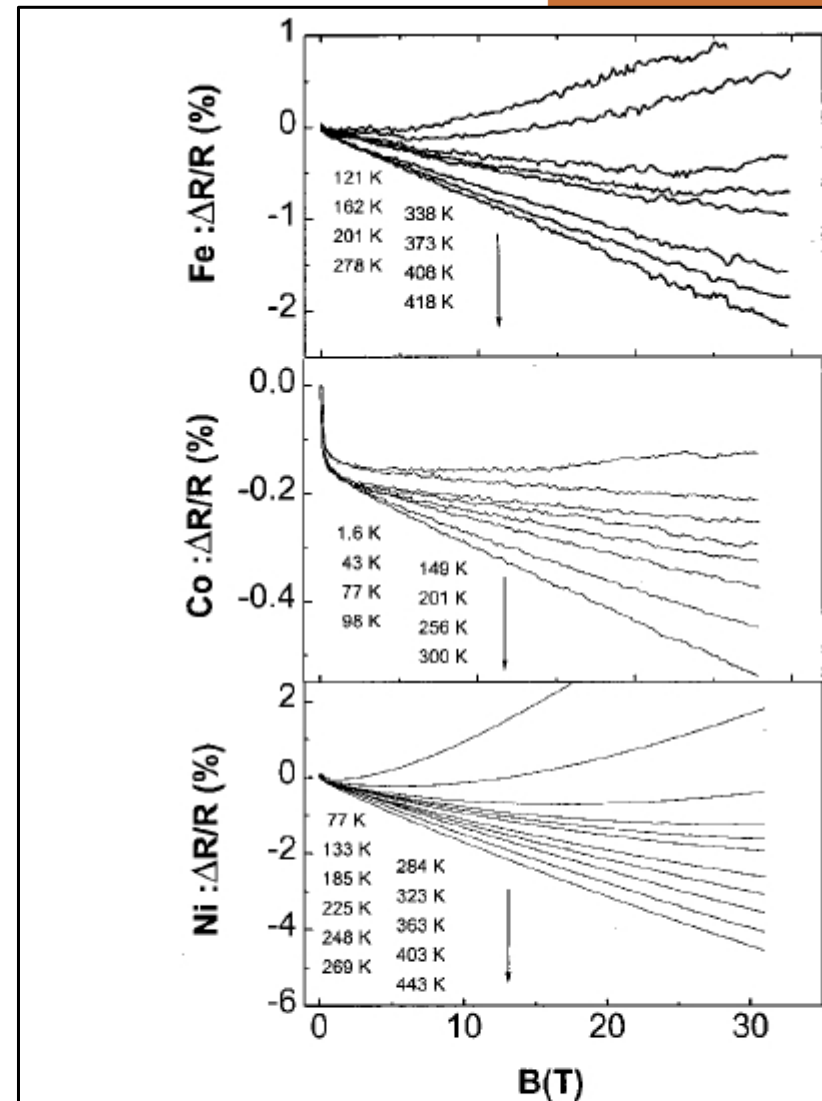
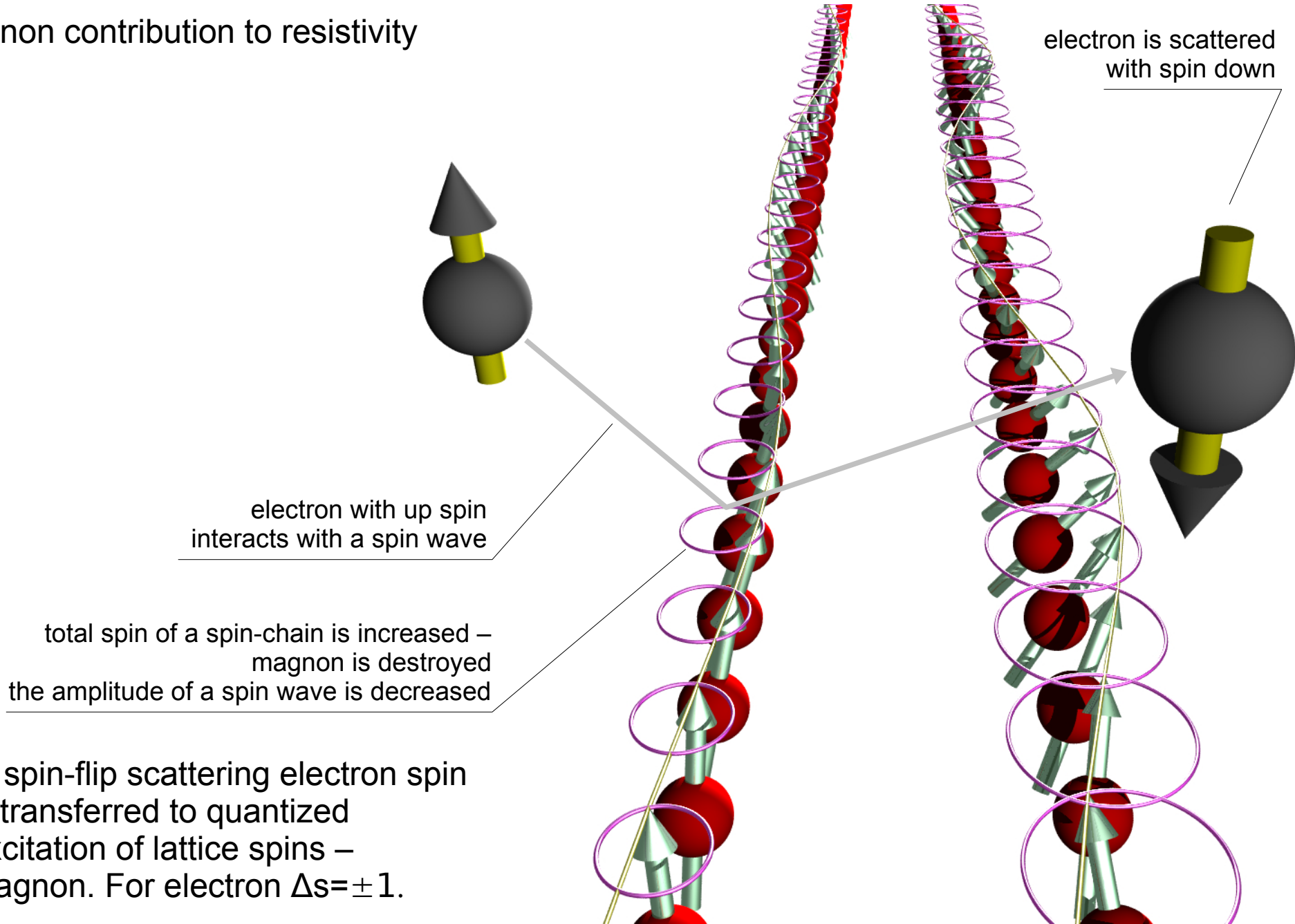


FIG. 1. Longitudinal high-field magnetoresistance ($B \parallel i$) at various temperatures for Fe, Co, and Ni epitaxial films – $\text{Fe}_{80\text{nm}}/\text{MgO}$, $\text{Co}_{7\text{nm}}/\text{Al}_2\text{O}_3$, and $\text{Ni}_{20\text{nm}}/\text{MgO}$. Note the extrinsic negative MR below the technical saturation of the magnetization for Co due to grain boundary effects.

Introduction

Magnon contribution to resistivity

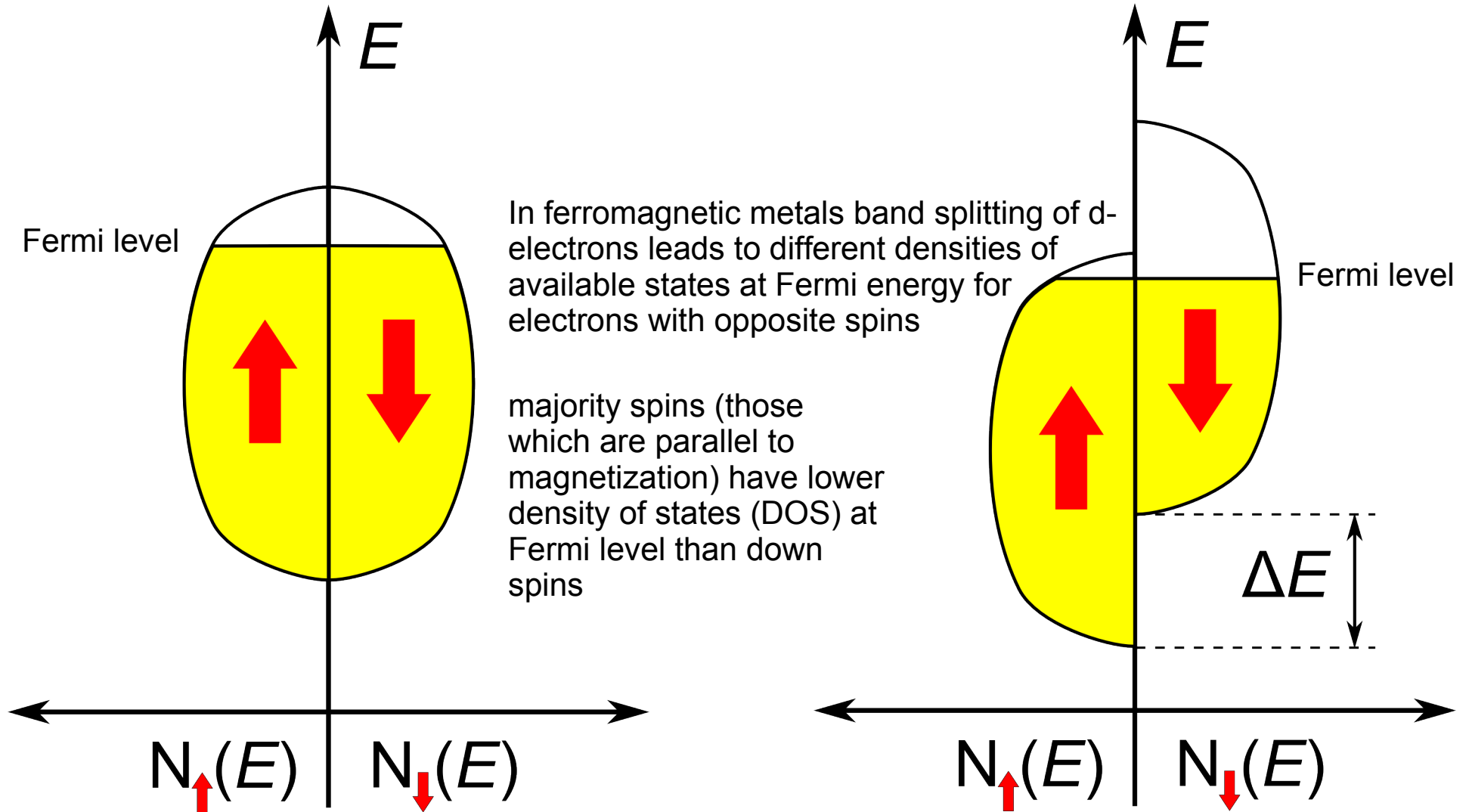


- In spin-flip scattering electron spin is transferred to quantized excitation of lattice spins – magnon. For electron $\Delta s = \pm 1$.

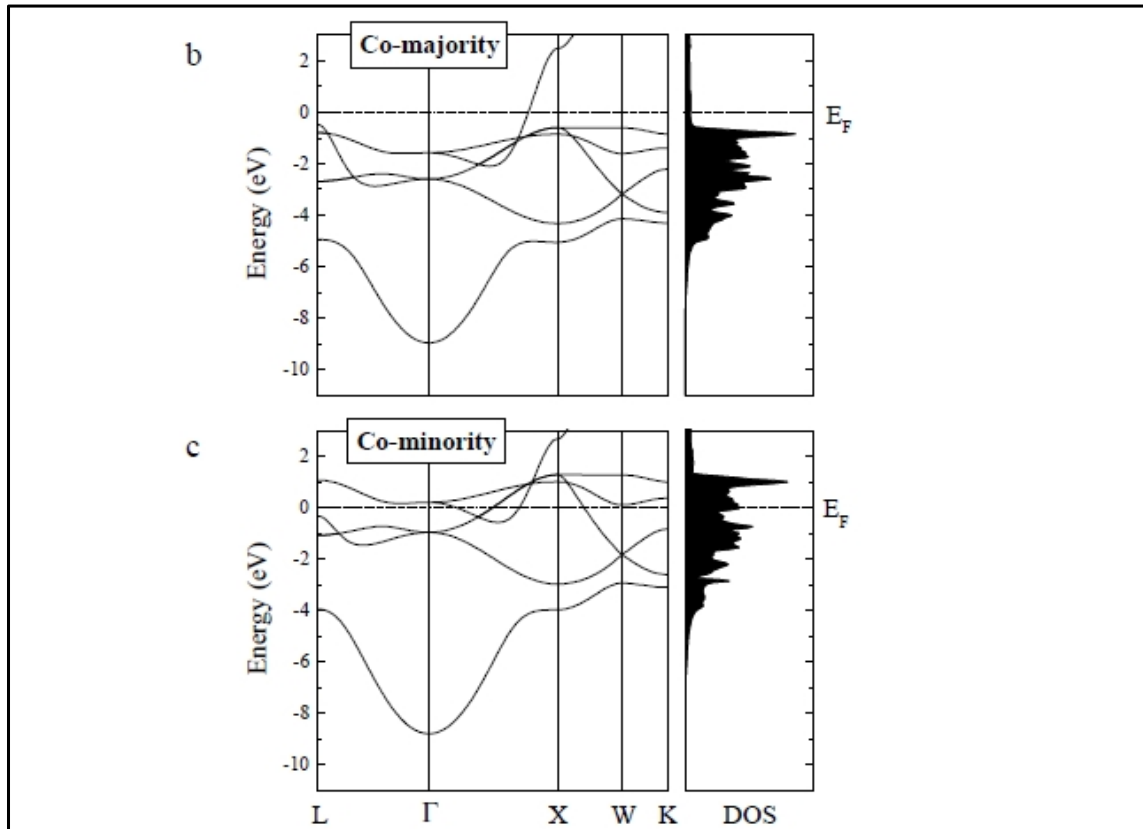
Band splitting in ferromagnetic materials

normal metal (Cu, Au, etc.)

ferromagnetic metal (Fe, Co, etc.)



Band splitting in ferromagnetic materials



Electronic band structures (left panels) and the density of states (right panels) of Cu (a) and fcc Co for the majority-spin (b) and minority-spin (c) electrons. The band structure of non-magnetic Cu is same for the up-spin and down-spin electrons. It is characterized by the fully occupied *d* bands and the presence of a dispersive *sp* band at the Fermi energy, which result in high conductivity of Cu. The electronic structure of ferromagnetic Co is different for the two spin orientations and is characterized by the exchange-split *d* bands. The Fermi level lies within the *sp* band for the majority-spin electrons, which leads to high conductivity of majority-spin channel. The Fermi level lies, however, within the *d* band for the minority-spin electrons resulting in low conductivity of the minority-spin channel. In the latter case the *sp* electrons are strongly hybridized with the *d* electrons, which diminishes their contribution to conduction.

- in some ferromagnetic materials the density of states of up-spins may be close to zero
- in Co the *d*-band DOS for spin-up electrons is about 10 times lower than that of down-spin electrons

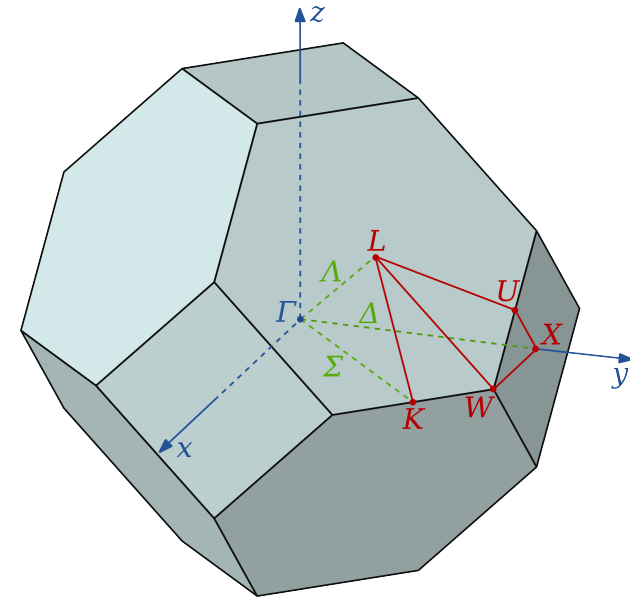


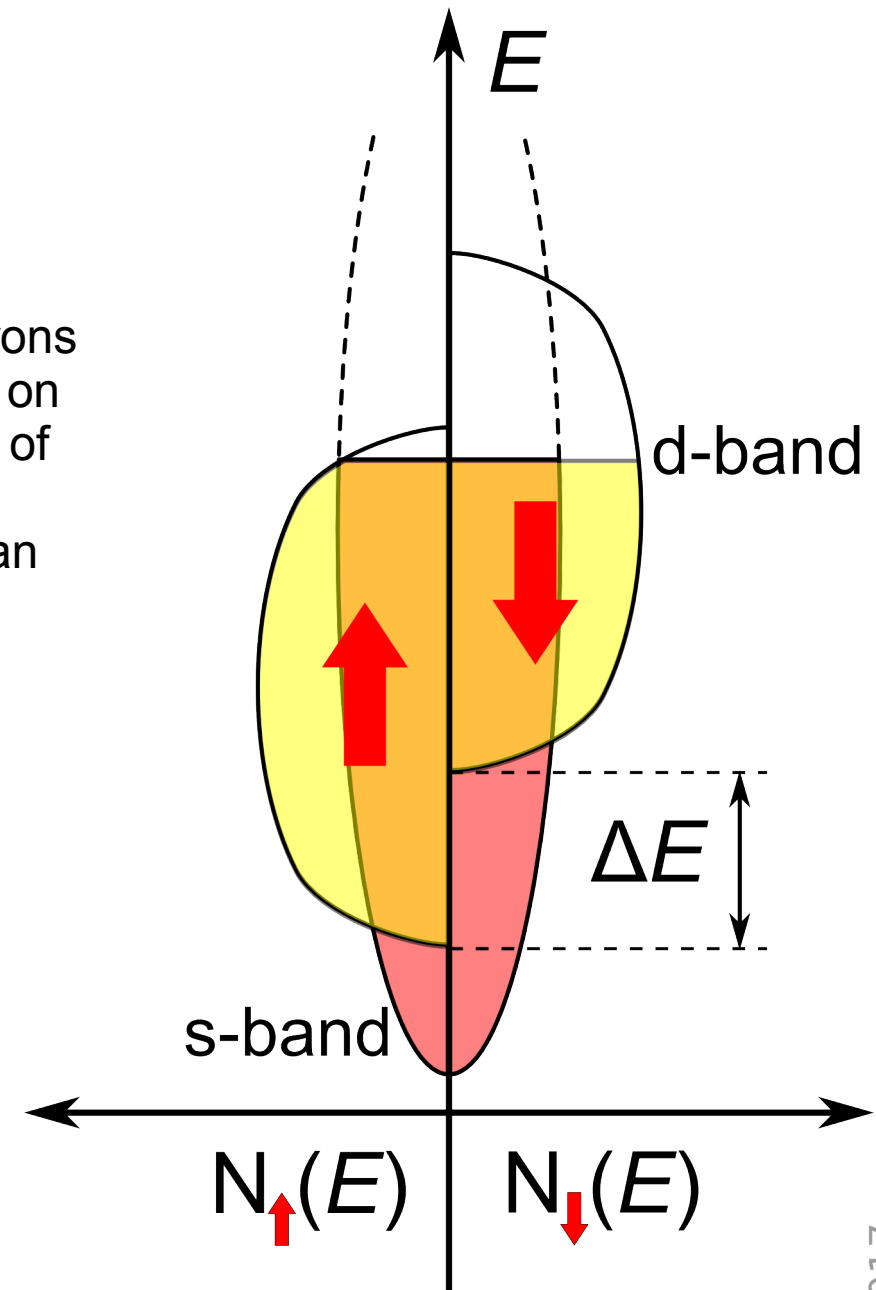
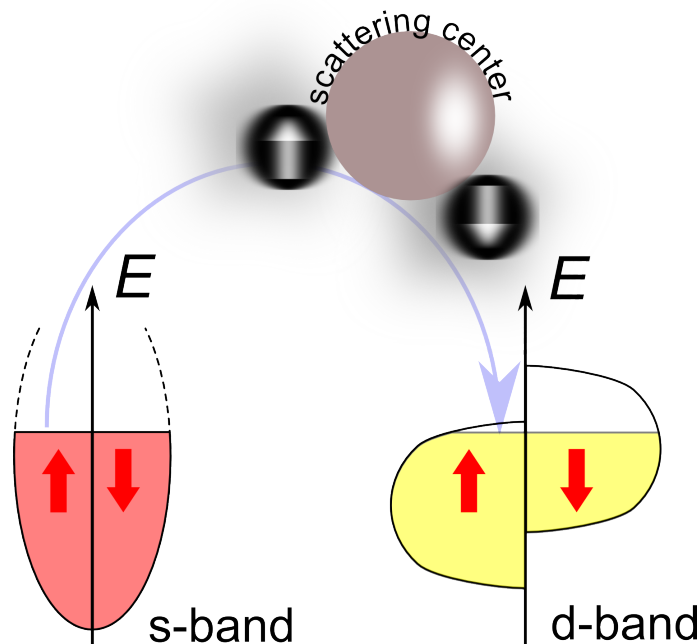
image from Wikimedia Commons;
author Inductiveload

graphics from: Perspectives of Giant Magnetoresistance, E. Y. Tsymlal, D. G. Pettifor, published in Solid State Physics, ed. by H. Ehrenreich and F. Spaepen, Vol. 56 (Academic Press, 2001) pp.113-237

Introduction

Mott – two current model, 1936.

- mobile sp-electrons are responsible for electronic conductivity [3]
- s- electrons can be scattered to free states near Fermi level
- density of states of d-electrons in ferromagnetic metal is different for spin-up and spin-down electrons
- the conductivity of given type of carriers depends on the number of free states available as final states of scattering events
- probability of spin-flip scattering is much lower than the probability of scattering without the change of spin



Introduction

Mott – two current model, 1936.

- the conductivity of a metal is a sum of independent conductivities of up and down-spins channels [3]

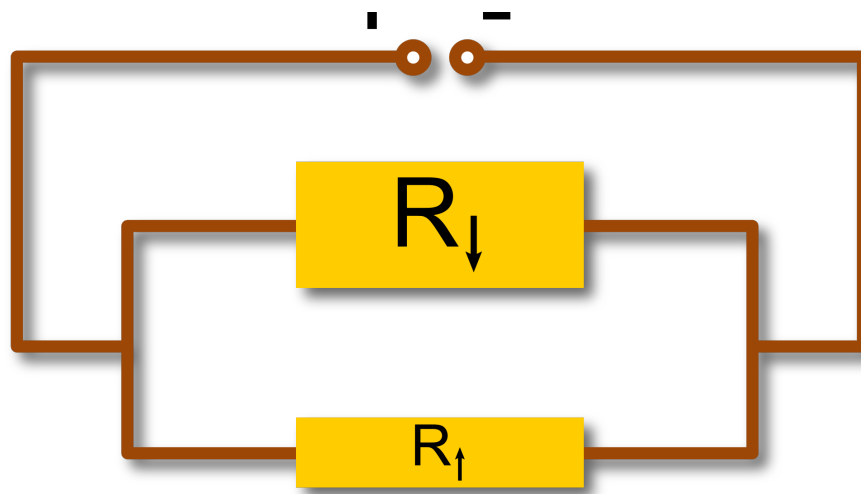
$$\sigma = \sigma_{\uparrow} + \sigma_{\downarrow}$$

- the relaxation time τ is given by Fermi golden rule (mean free path: $\lambda = \tau v_F$, v_F - Fermi velocity, of the order of $0.5 \times 10^6 \text{m/s}$) [3,9]:

$$\tau^{-1} \propto \langle V_{scatt}^2 \rangle D(E_F)$$

\uparrow
 average value of scattering potential

- the scattering potential needs not to be spin-dependent for resistivities of spin channels to be different – different DOS-es for opposite spins are enough

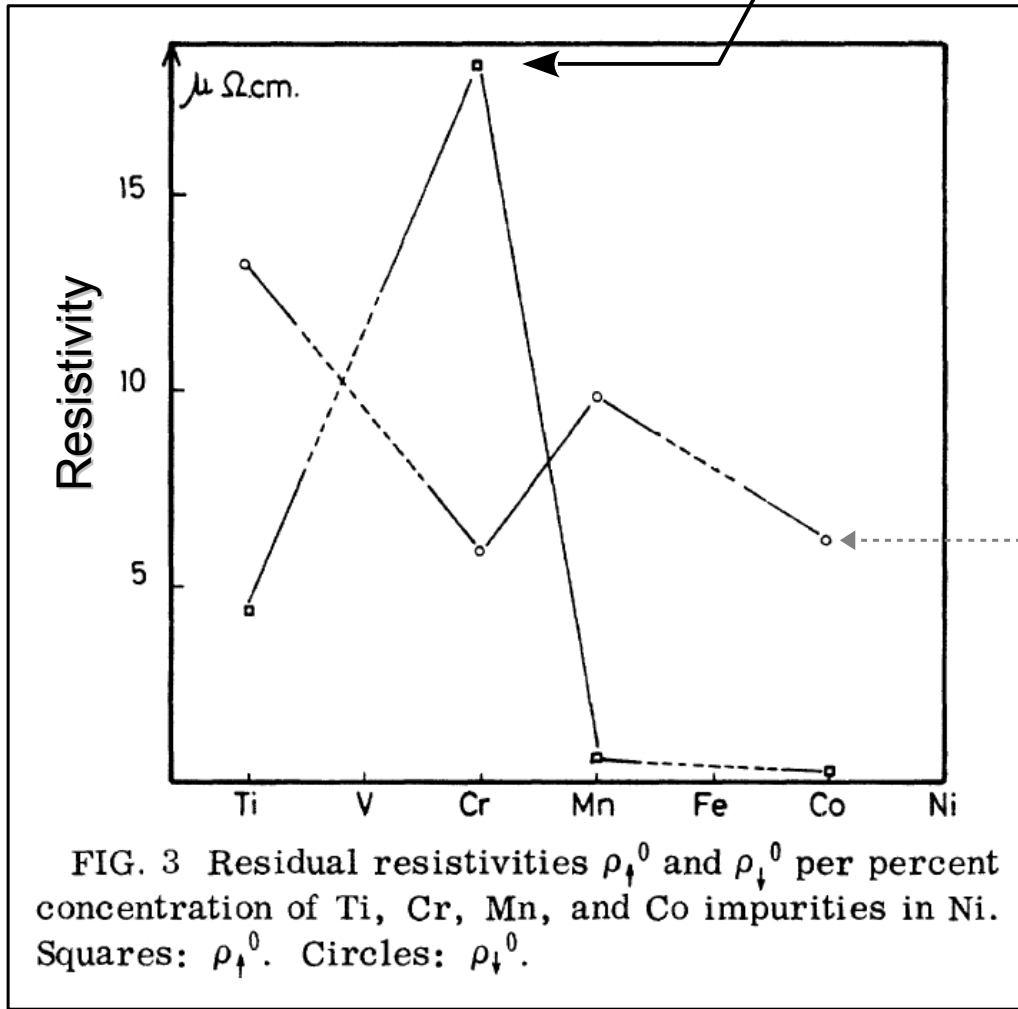


If spin-flip events are negligible current can be considered as carried in-parallel by two spin channels with spins parallel and antiparallel to quantization axis [12]

Introduction

not typical

Mott – two current model, 1936.



$$\alpha = \rho_{\downarrow}^0 / \rho_{\uparrow}^0$$

- Ni with impurities at low temperatures
- the spin-down channel resistivity can be many times lower than the resistivity of the up-spin channel

- $\alpha_{Co} = 30$ $\alpha_{Mn} = 16$
- the maximum for Cr impurity is associated with the bound state crossing the spin-up Fermi level [10]

spin-down channel

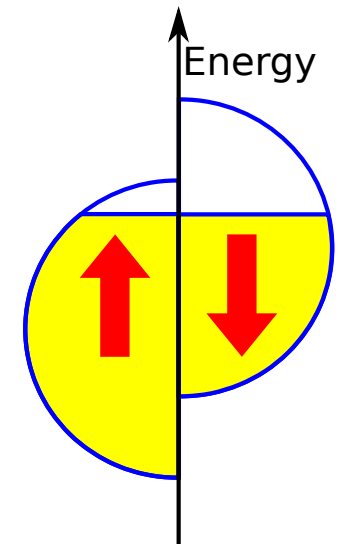
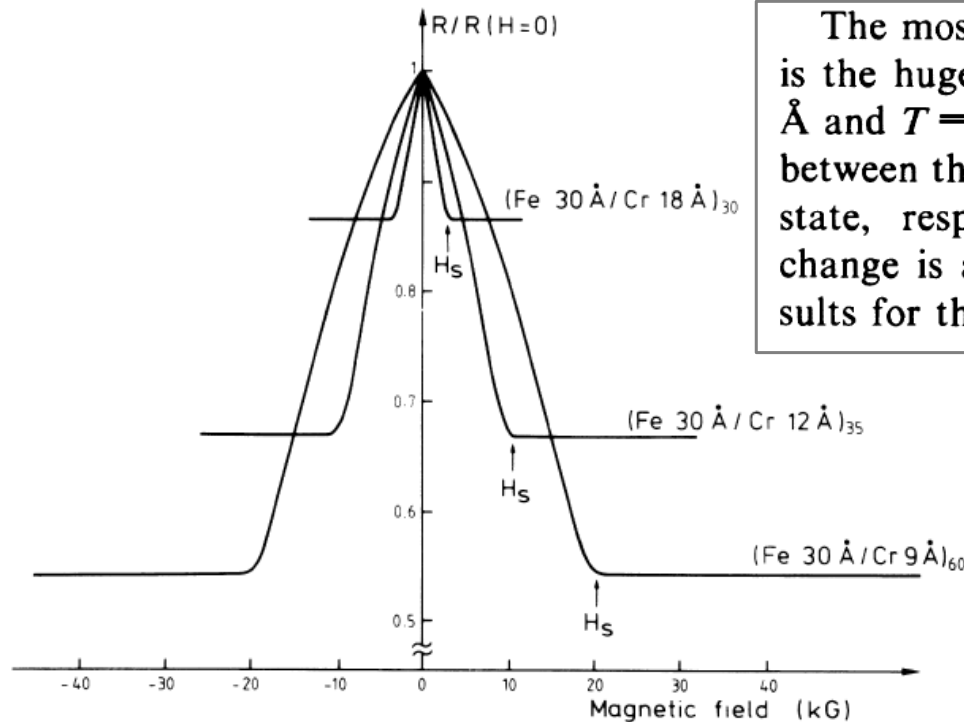


image from: A. Fert and I. A. Campbell, Phys. Rev. Lett. **21**, 1190 (1968)

Giant Magnetoresistance (GMR) – Nobel Prize 2007 (A. Fert, P. Grünberg)



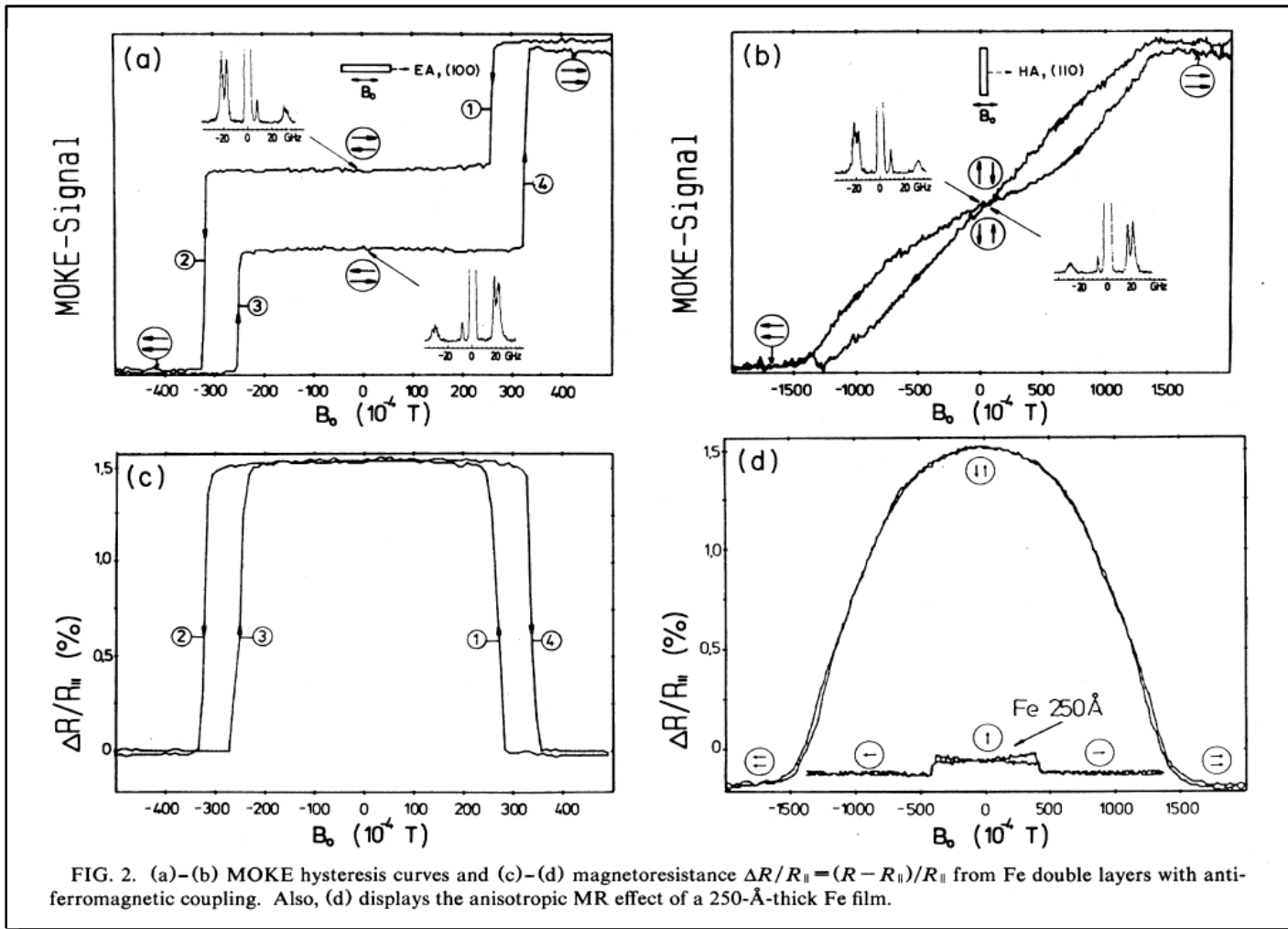
The most remarkable result exhibited in Figs. 2 and 3 is the huge value of the magnetoresistance. For $t_{\text{Cr}}=9 \text{ \AA}$ and $T=4.2 \text{ K}$, see Fig. 2, there is almost a factor of 2 between the resistivities at zero field and in the saturated state, respectively (in absolute value, the resistivity change is about $23 \mu\Omega \text{ cm}$). By comparison of the results for three different samples in Fig. 3, it can be seen

FIG. 3 Magnetoresistance of three Fe/Cr superlattices at 4.2 K. The current and the applied field are along the same [110] axis in the plane of the layers.

In conclusion, we have found a giant magnetoresistance in (001)Fe/(001)/Cr superlattices when, for thin Cr layers (9, 12, and 18 Å), there is **an antiparallel coupling of the neighbor Fe layers** at zero field.

image from M.N. Baibich, J.M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Eitenne, G. Creuzet, A. Friederich, J. Chazelas, Phys. Rev. Lett. **61**, 2472 (1988)

Giant Magnetoresistance (GMR) – Nobel Prize 2007 (A. Fert, P. Grünberg)

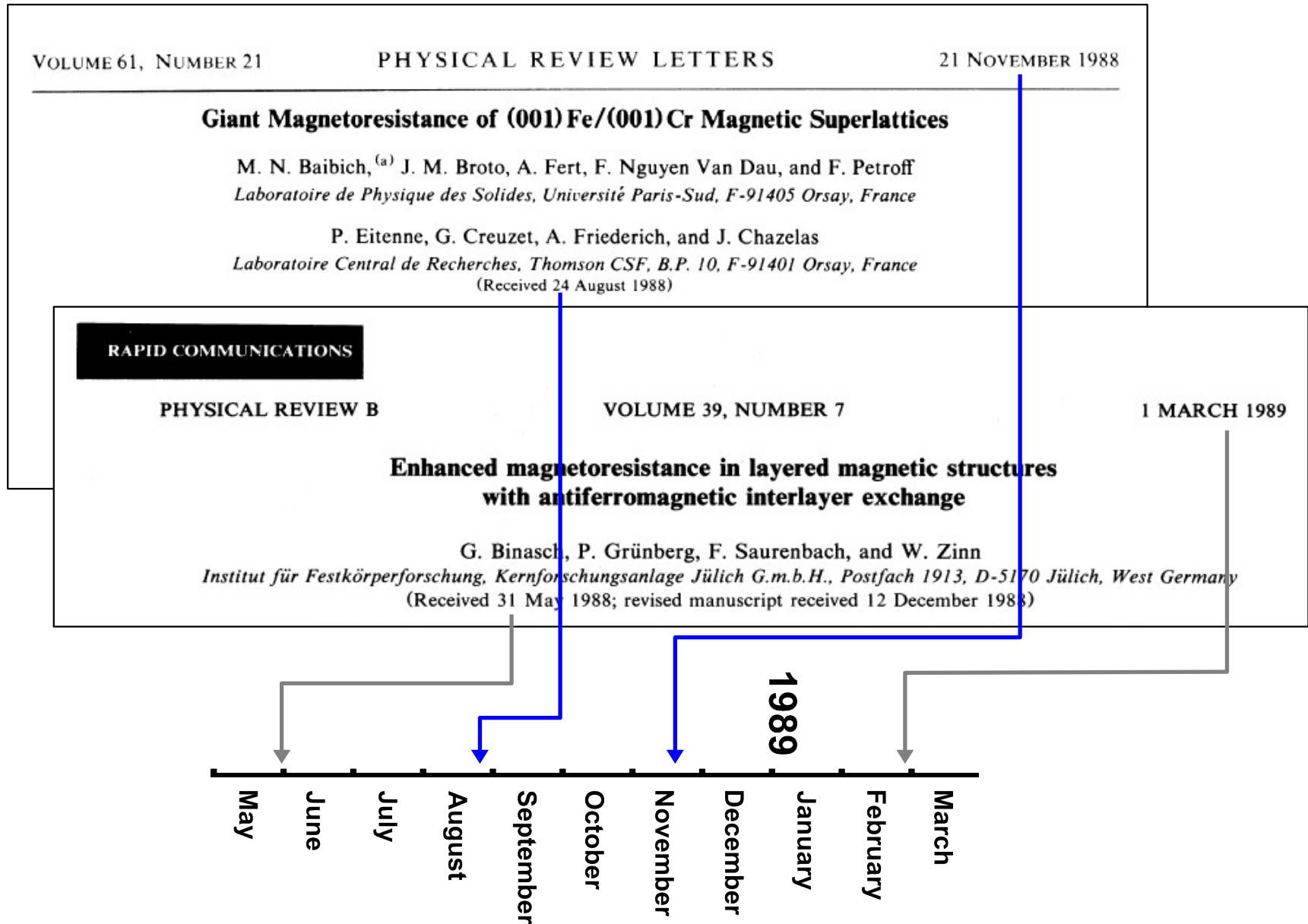


The experimental results reported here show that the **antiparallel alignment of the magnetizations in our double layers produces an appreciable increase of the electrical resistivity**. We propose that this is caused by spin-flip scattering. Electrons in one...

image from G. Binasch, P. Grünberg, F. Saurenbach, W. Zinn, Phys. Rev. B **39**, 4828 (1989)

the first interpretation was not correct – see later

Giant Magnetoresistance (GMR) – Nobel Prize 2007 (A. Fert, P. Grünberg)



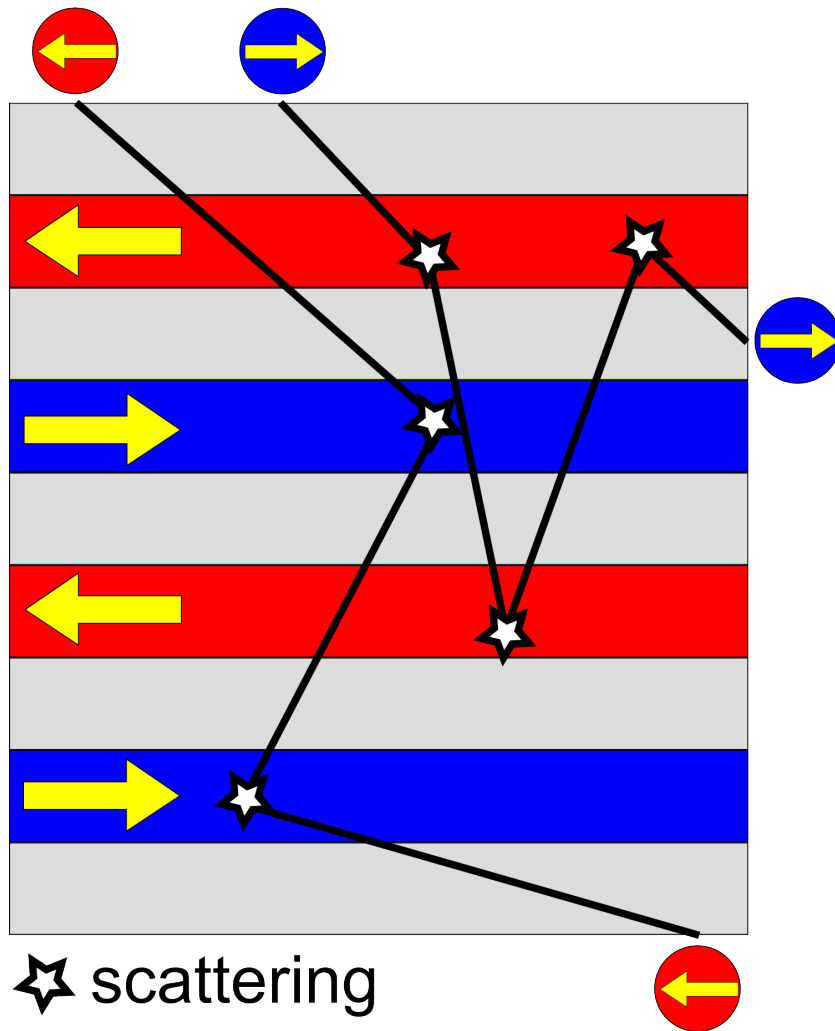
Magnetoresistance

Kinds of magnetoresistance (incomplete list)

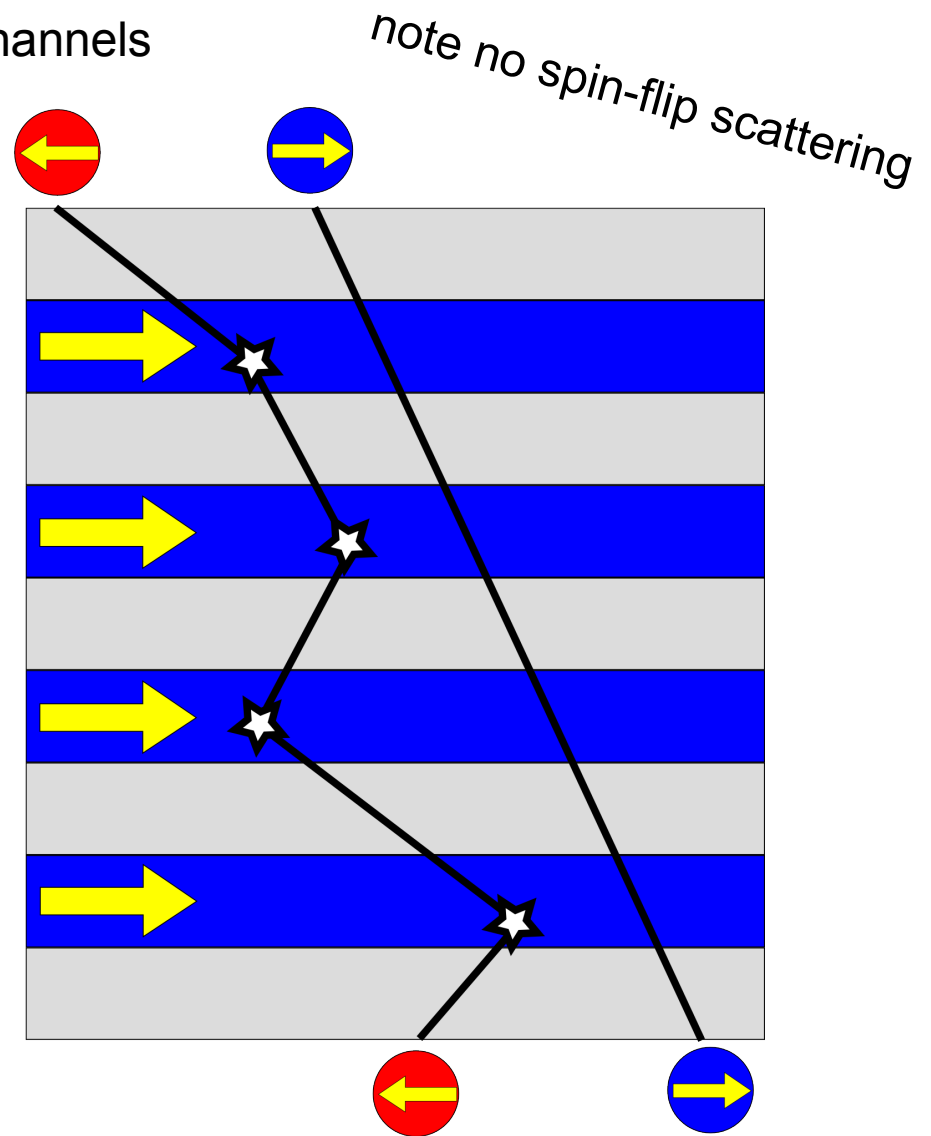
Type of magnetoresistance	Systems	Values	Note
ordinary (OMR)	all conducting materials	approx. 10^{-2} % in metals in RT in up to 2T in high fields (30T) 47% in Cu (at 78K) and 380% in Bi [13]	Lorentz force changes orbits of carriers De Haas-Shubnikov effect – high field oscillations of resistance
anisotropic (AMR)	ferromagnetic materials	several percent at RT (NiCo alloys [14]) in small fields	depends on orientation of current relative to magnetization; high field sensitivity (NiFe)
giant (GMR)	ferromagnetic materials	several dozens percent in small fields [3]	depends on relative orientation of magnetic moments
tunneling (TMR)	ferromagnetic materials	several hundred percent in small fields	depends on the orientation of magnetic moments of electrodes separated by insulating film
colossal (CMR)	transition metal oxides	several hundred percent in several Tesla	phase transition paramagnet-ferromagnet

Magnetoresistance

GMR - the simplistic explanation with two-spin channels



antiparallel – high resistance

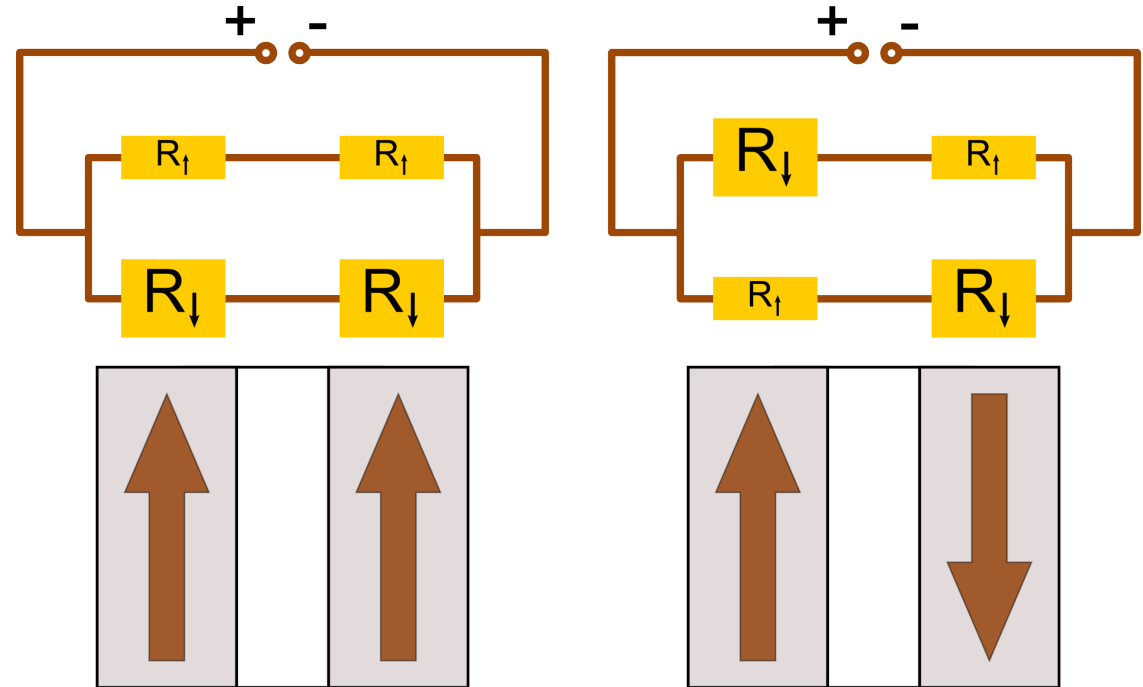


parallel – low resistance

Magnetoresistance

GMR - resistor network model [17]

- Mott two spin-channels – two conducting channels **do not mix**
- resistance of parallel configuration is lower



Resistance of parallel configuration

$$\frac{1}{R_P} = \frac{1}{R_{\uparrow} + R_{\uparrow}} + \frac{1}{R_{\downarrow} + R_{\downarrow}}$$

$$R_P = \frac{2R_{\uparrow}R_{\downarrow}}{R_{\uparrow} + R_{\downarrow}}$$

Resistance of antiparallel configuration

$$\frac{1}{R_{AP}} = \frac{1}{R_{\uparrow} + R_{\downarrow}} + \frac{1}{R_{\uparrow} + R_{\downarrow}}$$

$$R_{AP} = \frac{R_{\uparrow} + R_{\downarrow}}{2}$$

$$R_{\downarrow} > R_{\uparrow} : R_{\downarrow} = (1+a^2)R_{\uparrow} \rightarrow \frac{R_{AP}}{R_P} - 1 = \frac{a^4}{4+4a^2} \geq 0 \rightarrow R_{AP} \geq R_P$$

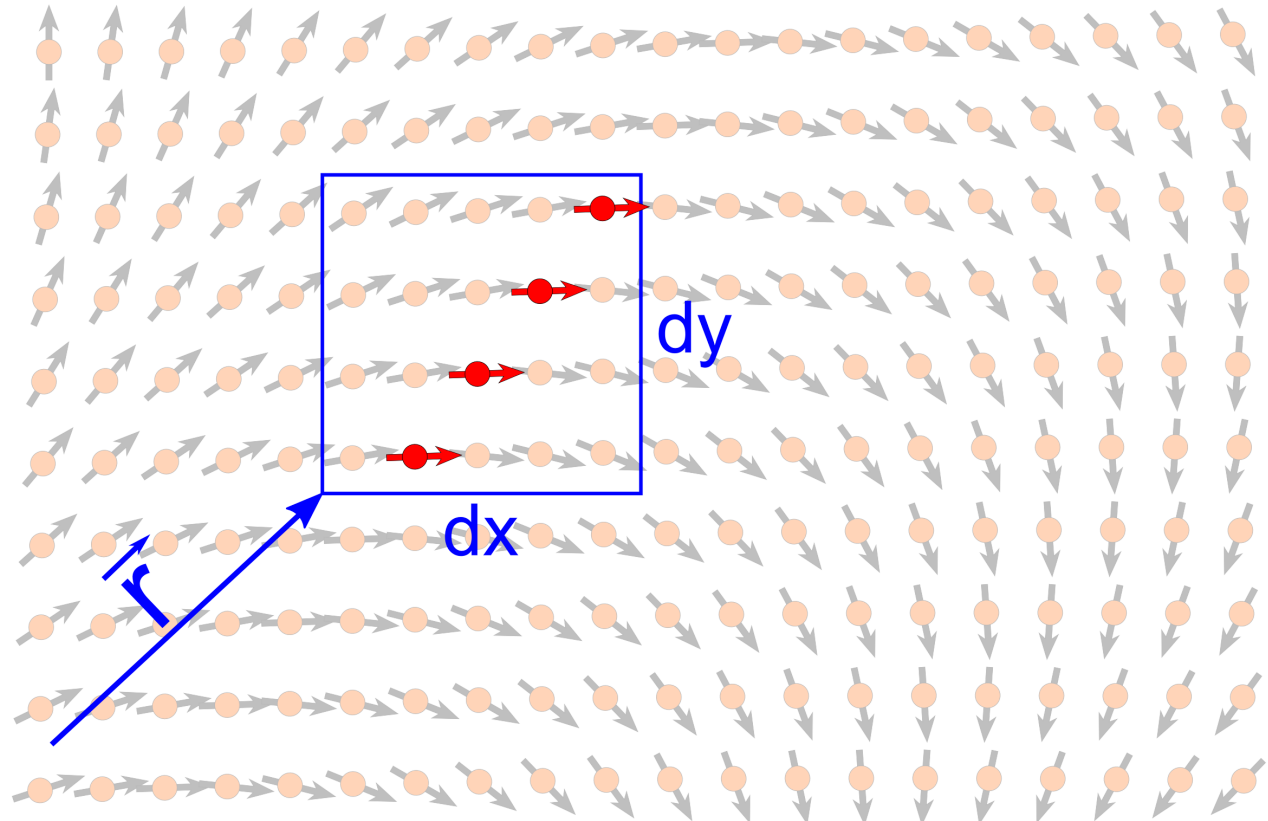
Magnetoresistance

Boltzmann transport equation [9,15,16]

The classical theory of transport is based on a **statistical distribution function f** that specifies the probability of finding a particle with its position and momentum within a small range [15] or, which is equivalent*, concentration of carriers with a given momentum in the neighborhood of the given point in space [9].

$$n(r_x, r_x+dx, r_y, r_y+dy, r_z, r_z+dz, v_x, v_x+dv_x, v_y, v_y+dv_y, v_z, v_z+dv_z) = f(\vec{r}, \vec{v}) dx dy dz dv_x dv_y dv_z$$

number of particles in $dr \cdot dv$ volume at r, v position in six-dimensional space



in the 2-D example to the right four particles in the $dx \cdot dy$ volume have v_y velocities in the range from $-dv_y$ to $+dv_y$

*after introducing normalizing factor

Magnetoresistance

Boltzmann transport equation [9,15,16]

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$$n(r_x, r_x+dx, r_y, r_y+dy, r_z, r_z+dz, v_x, v_x+dv_x, v_y, v_y+dv_y, v_z, v_z+dv_z) = f(\vec{r}, \vec{v}) dx dy dz dv_x dv_y dv_z$$

- In equilibrium the transition rates between any two states exactly balance [15]
- In the presence of external fields the equilibrium state is disturbed and the scattering tends to return the system to equilibrium
- Some **steady state** is attained in which the effect of external fields is balanced by scattering events
- Scattering “*has the important effect of limiting the extent of the response*” - A.C. Smith et al. [15]

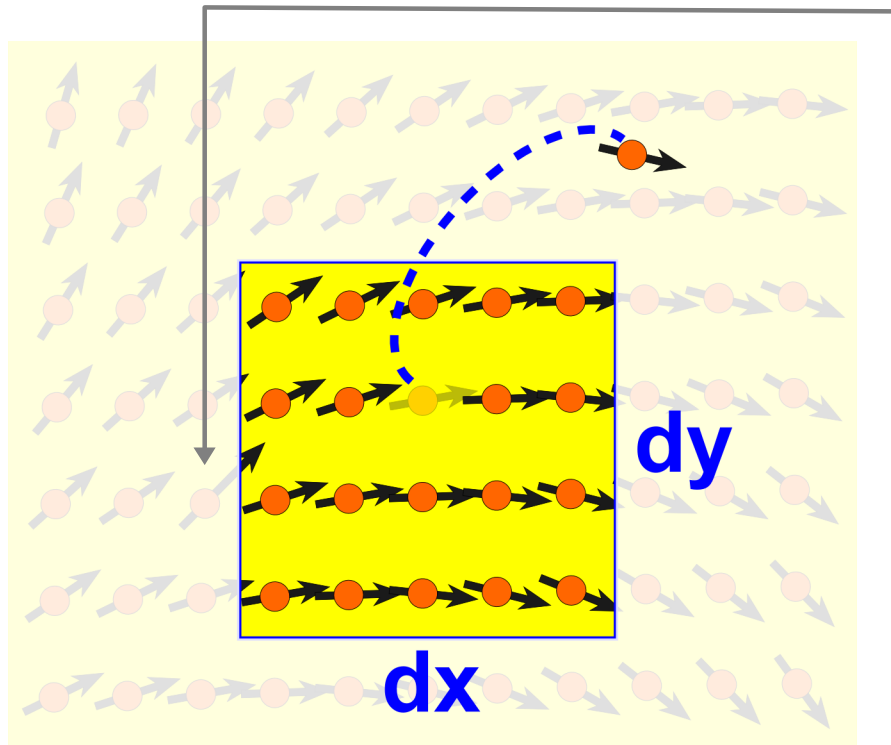
*after introducing normalizing factor

Magnetoresistance

Rate of change of the distribution function $f(r, k, t)$ * [from A.C. Smith *et al.*, 15]:

- the density described by f may change because of scattering of phase points into or out of the volume cell $dr \cdot dv$
- the phase points may flow in or out of the cell due to their spatial velocity and because the velocity changes under the influence of external field (streaming or drift terms)

The carrier (electron) is scattered out of the $dx dy$ volume and it may change its momentum



The carrier (electron) is about to enter $dx dy$ volume but his momentum is different than that of carriers in cell; it will not be in the same $dx dy dv_x dv_y$ volume as neighboring (in real space) electrons

view in real space

*not that the function depends on t now.

Magnetoresistance

Rate of change of the distribution function $f(r, k, t)$ * [from A.C. Smith *et al.*, 15]:

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- the phase points may flow in or out of the cell due to their spatial velocity and because the velocity changes under the influence of external field (streaming or drift terms)

Consider the 1-D movement of a phase points along x-direction. The number of points crossing a $dy dz$ area normal to x is given by:

$$\delta n(x) = \int f(x, k) dk \dot{x} dy dz = f(x) \dot{x} dy dz$$

↑
k disappears from here because we integrate over all velocities
↑
note the dot

The net flow into the region between x and $x+dx$ is given by:

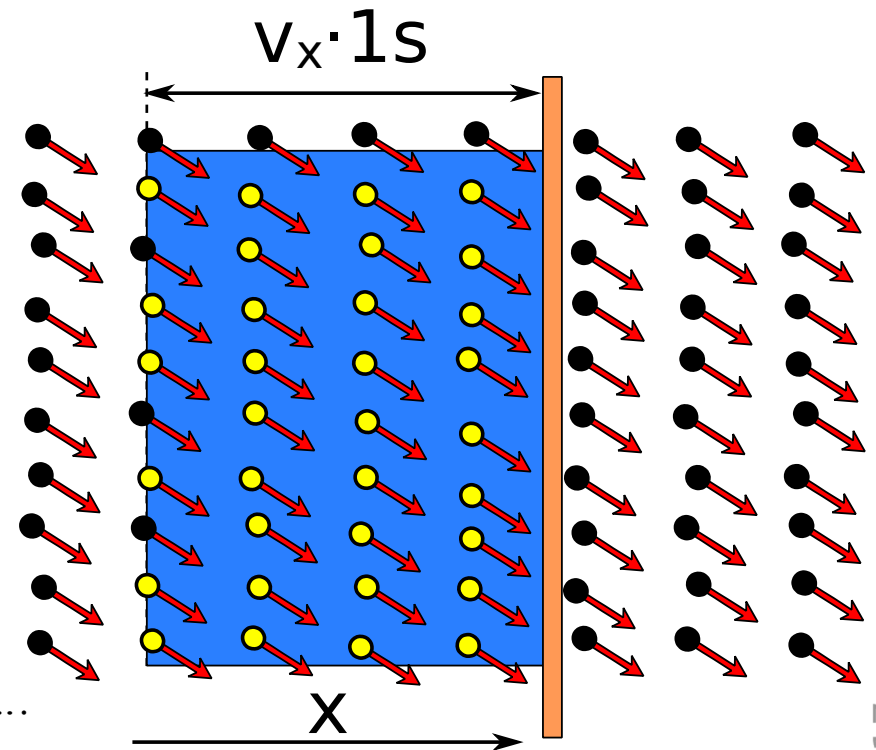
$$\delta n = [\delta n(x) - \delta n(x+dx)] dy dz = [f(x) \dot{x}(x) - f(x+dx) \dot{x}(x+dx)] dy dz$$

Which for $dx \rightarrow 0$ is:

$$\delta n = - \left[\frac{\partial}{\partial x} (f(x) \dot{x}) \right]_{x_0} dx dy dz$$

In 3-D (change of the number of phase points in $dx dy dz$ volume) we have:

$$\delta n = \left[\frac{\partial}{\partial x} (f(x) \dot{x}) \right]_{x_0} dx dy dz + \left[\frac{\partial}{\partial y} (f(y) \dot{y}) \right]_{y_0} dy dx dz + \dots$$



*note that the function depends on t now.

Magnetoresistance

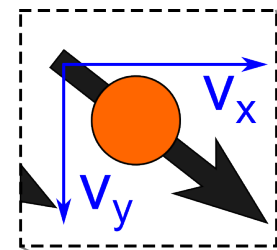
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↑ note the dot



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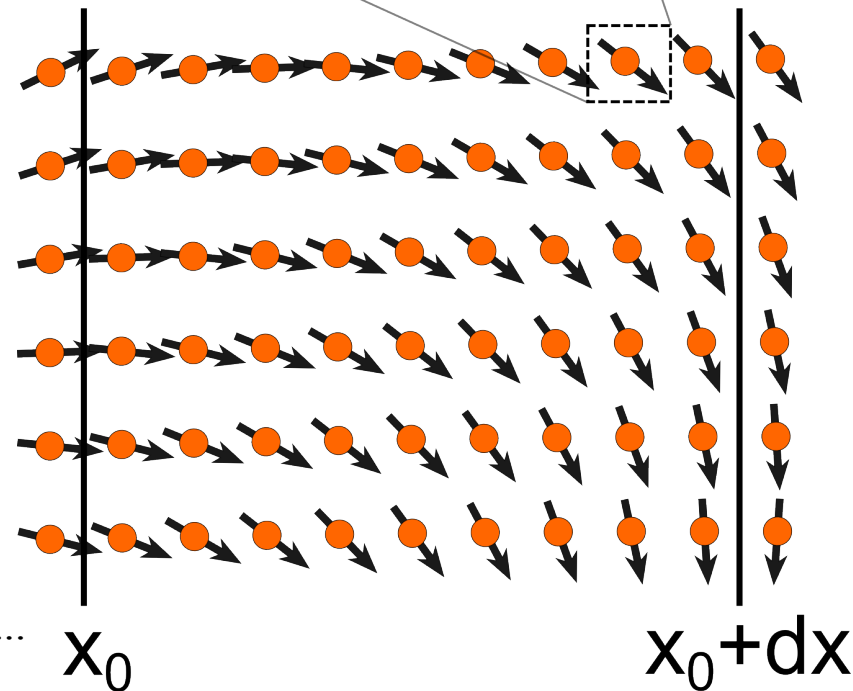
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*note that the function depends on t now.

Magnetoresistance

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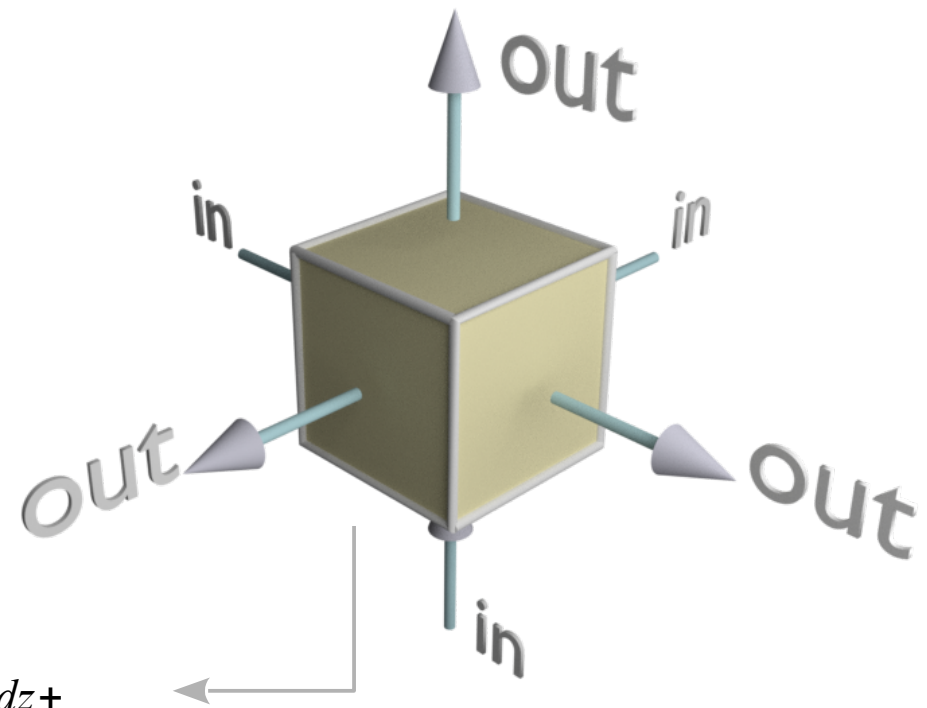
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*note that the function depends on t now.

Magnetoresistance

Further we have:

$$\nabla \cdot (f \vec{v}) = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot (i f v_x + j f v_y + k f v_z) = \frac{\partial}{\partial x} f v_x + \frac{\partial}{\partial y} f v_y + \frac{\partial}{\partial z} f v_z$$

fragment of the last expression of the previous page:

...and for the change of n:

$$\delta n = -\nabla \cdot (f(\vec{r}) \vec{v}) d^3 r \quad \leftarrow \delta n = \left[\frac{\partial}{\partial x} (f(x) \dot{x}) \right]_{x_0} dx dy dz + \left[\frac{\partial}{\partial y} (f(y) \dot{y}) \right]_{y_0} dy dx dz + \dots$$

To get the complete expression (drift of position and momentum/velocity points into 6-dimensional phase space volume $d^3r d^3k$) we add together six terms [15]:

$$\delta f(\vec{r}, \vec{v}, t) = -\left[\nabla_r \cdot (f(\vec{r}, \vec{k}, t) \vec{v}) + \nabla_k \cdot (f(\vec{r}, \vec{k}, t) \dot{\vec{k}}) \right] d^3 r d^3 k$$

The continuity equation for f^* reads (we cancel $d^3 r d^3 k$ everywhere and work with density of phase points instead of their numbers):

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t} \right)_{scatt} - \nabla_r \cdot (f \vec{v}) - \nabla_k \cdot (f \dot{\vec{k}})$$

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t} \right)_{scatt} - \vec{v} \cdot \nabla_r f - \dot{\vec{k}} \cdot \nabla_k f$$

$\vec{v} = \hbar^{-1} \nabla_k E_n(k)$

$\nabla \cdot (a\vec{b}) = \vec{b} \cdot \nabla a + a \nabla \cdot \vec{b}$

$\nabla_r \cdot (f \vec{v}) + \nabla_k \cdot (f \dot{\vec{k}}) = \vec{v} \cdot \nabla_r f + f \nabla_r \cdot \vec{v} + \dot{\vec{k}} \cdot \nabla_k f + f \nabla_k \cdot \dot{\vec{k}}$

$\nabla_r \cdot \vec{v} = \hbar^{-1} \nabla_r \cdot [\nabla_k E_n(k)] = 0$ ← since energy E_n does not depend on position r

$\nabla_k \cdot \dot{\vec{k}} = \nabla_k \cdot \frac{q}{\hbar} (\vec{E} + \vec{v} \times \vec{B}) = \nabla_k \cdot \left(\frac{q}{\hbar} \vec{v} \times \vec{B} \right) =$ electric field \vec{E} does not depend on position r

$\frac{q}{\hbar} \nabla_k \cdot (\hat{i} (B_z v_y - B_y v_z) + \hat{j} (-B_z v_x + B_x v_z) + \hat{k} (B_y v_x - B_x v_y)) =$

$\frac{q}{\hbar^2} \left(\hat{i} \frac{\partial}{\partial k_x} + \hat{j} \frac{\partial}{\partial k_y} + \hat{k} \frac{\partial}{\partial k_z} \right) \cdot$

$\left(\hat{i} (B_z \frac{\partial E_n(k)}{\partial k_y} - B_y \frac{\partial E_n(k)}{\partial k_z}) + \hat{j} (-B_z \frac{\partial E_n(k)}{\partial k_x} + B_x \frac{\partial E_n(k)}{\partial k_z}) + \hat{k} (B_y \frac{\partial E_n(k)}{\partial k_x} - B_x \frac{\partial E_n(k)}{\partial k_y}) \right) =$

$\frac{q}{\hbar^2} \left(B_z \frac{\partial^2 E_n(k)}{\partial k_y \partial k_x} - B_y \frac{\partial^2 E_n(k)}{\partial k_z \partial k_x} - B_z \frac{\partial^2 E_n(k)}{\partial k_x \partial k_y} + B_x \frac{\partial^2 E_n(k)}{\partial k_z \partial k_y} + B_y \frac{\partial^2 E_n(k)}{\partial k_x \partial k_z} - B_x \frac{\partial^2 E_n(k)}{\partial k_y \partial k_z} \right) = 0$

0
0
0

*we do not write further the explicit dependence of f on r , v , and t .

Magnetoresistance

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t} \right)_{scatt} - \vec{v} \cdot \nabla_r f - \dot{\vec{k}} \cdot \nabla_k f$$

Boltzmann transport equation

- “derived under the condition that the fictitious particle representing electron executes a classical motion” [15]
- not valid for large external fields and when band-to-band transitions occur

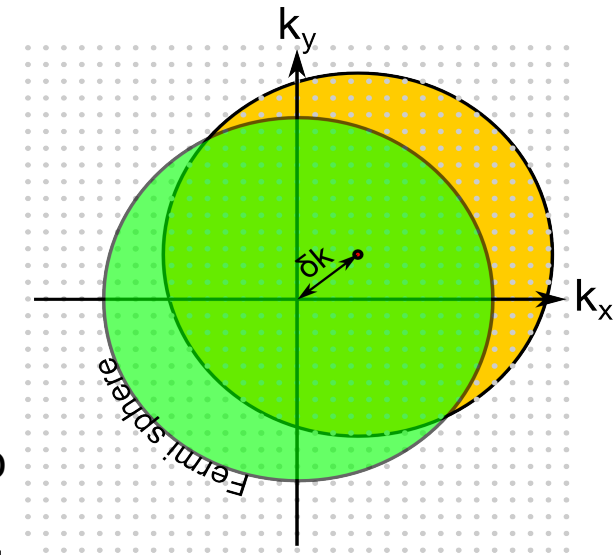
The scattering term is due to: lattice vibrations, impurities, electron-electron scattering, electron-magnon scattering, vacancies, grain boundaries dislocations etc.

In steady state f is constant and we have:

$$\left(\frac{\partial f}{\partial t} \right)_{scatt} - \underbrace{\vec{v} \cdot \nabla_r f}_{\text{diffusion}} - \underbrace{\dot{\vec{k}} \cdot \nabla_k f}_{\text{effect of fields}} = 0$$

The relaxation time approximation:

In general it is not possible to calculate scattering contribution to the change of f . In many applications it is useful to **assume that disturbed system returns to equilibrium exponentially in time*** [9]:



$$\left(\frac{\partial f}{\partial t} \right)_{scatt} = - \frac{f_k - f_k^0}{\tau_k}$$

↑
relaxation time (in general different for each Bloch state)

$$f_k(t) = f_k^0 + \Delta f_k \exp(-t/\tau_k)$$

*this approximation is valid for pure metals for temperatures exceeding Debye temperature and for contaminated metals (or those with defects) for all temperatures [9]

Magnetoresistance

Linearized Boltzmann equation

The distribution function can in general be expanded in powers of the driving field [18]:

$$f = f^0 + \left(\frac{\partial f}{\partial E} \right) \vec{E} + \left(\frac{\partial^2 f}{\partial E^2} \right) \vec{E}^2 + \dots$$

, with f^0 given for electrons by Fermi-Dirac statistics [9]:

$$f_k^0 = \frac{1}{\exp((E_k - E_F)/k_b T) + 1}$$

“When the electric field is small, only a small amount of current flows. The system is only slightly out of equilibrium.” [19]. The distribution function can be written as:

$$f = f^0 + f^1 \quad f^1 \text{ is a small change}$$

Magnetoresistance

Giant magnetoresistance from Boltzmann equation*

Investigating electron transport in thin films one can assume that the system is infinitely extended in xy-plane so that the distribution function depends only on z-coordinate (perpendicular to the film plane).

Using two-channel model of Mott the distribution function is decomposed into two parts:
 -equilibrium distribution function $f_0(z, \vec{v})$ – in zero electric field

-small change g , induced by external field, that depends on electrons spin

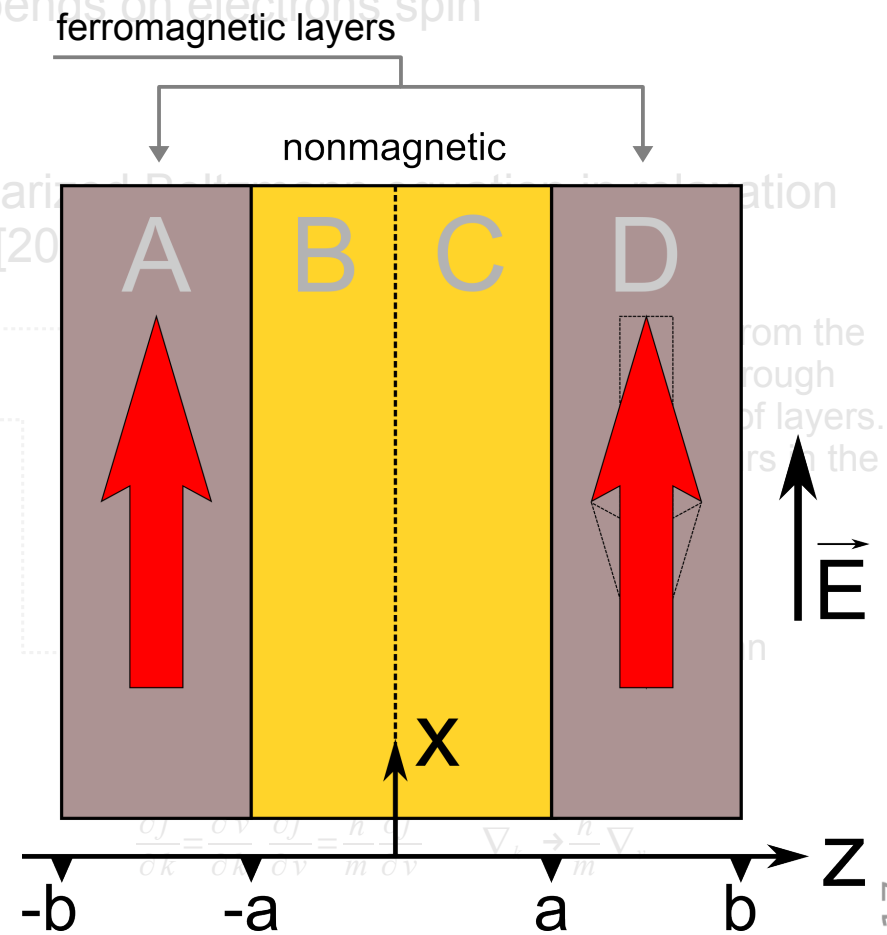
$$f^{\uparrow(\downarrow)}(z, \vec{v}) = f_0(\vec{v}) + g^{\uparrow(\downarrow)}(z, \vec{v})$$

Substituting the above distribution function into linearized Boltzmann equation in steady state approximation we obtain the expression for g [20]

$$\frac{\partial g^{\uparrow(\downarrow)}(z, \vec{v})}{\partial z} + \frac{g^{\uparrow(\downarrow)}(z, \vec{v})}{\tau^{\uparrow(\downarrow)} v_z} = \frac{e \vec{E}}{m v_z} \frac{\partial f_0(\vec{v})}{\partial v_x}$$

m, e – electron effective mass and charge

- relaxation times are spin dependent (spin channels)
- g is divided into two parts depending on the sign of v_z component of velocity



*theory developed by J. Barnaś and coworkers [20]

Magnetoresistance

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m, e – electron effective mass and charge

Note that magnetoresistance results from the presence of external magnetic field through the orientation of magnetic moments of layers. Magnetic field does not explicitly occurs in the model.

Note that previously we had Boltzmann equation in r, k -space

$$p = m v = \hbar k$$

$$\frac{\partial f}{\partial k} = \frac{\partial v}{\partial k} \frac{\partial f}{\partial v} = \frac{\hbar}{m} \frac{\partial f}{\partial v} \quad \nabla_k \rightarrow \frac{\hbar}{m} \nabla_v$$

- relaxation times are spin dependent (spin channels)
- **g is divided into two parts** depending on the sign of v_z component of velocity

*theory developed by J. Barnaś and coworkers [20]

Magnetoresistance

The general solution can be written as [20]:

$$g_{\pm}^{\uparrow(\downarrow)}(z, \vec{v}) = \frac{e\vec{E}\tau^{\uparrow(\downarrow)}}{m} \frac{\partial f_o(\vec{v})}{\partial v_x} \times \left[1 + F_{\pm}^{\uparrow(\downarrow)}(v) \exp\left(\frac{\mp z}{\tau^{\uparrow(\downarrow)}|v_z|}\right) \right]$$

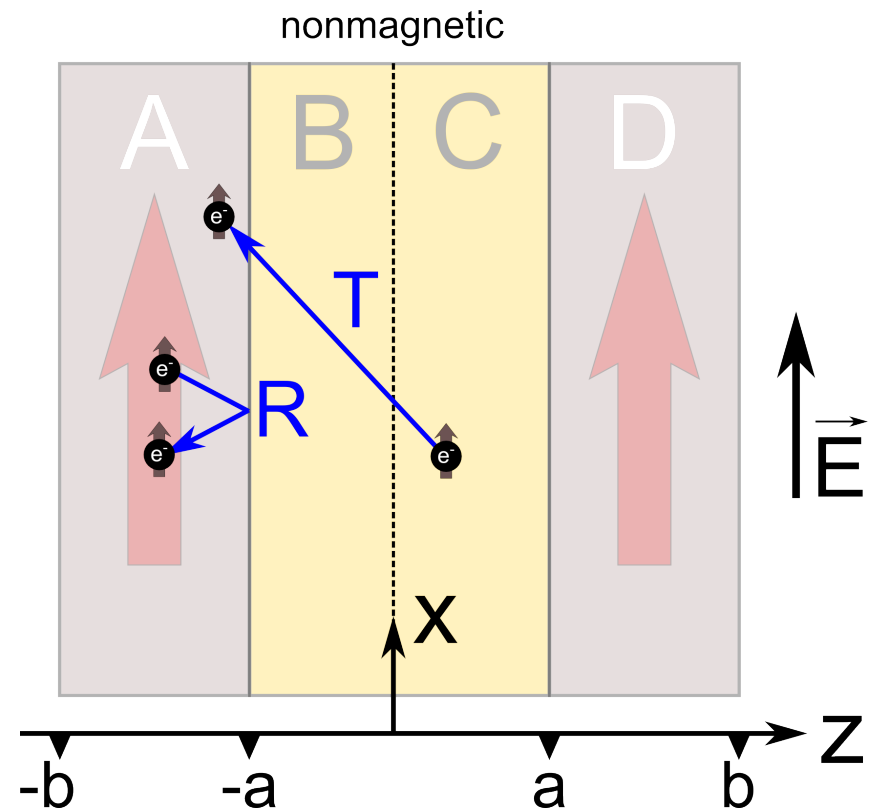
4 arbitrary functions to be determined from boundary conditions

Boundary conditions (BCs) – in steady state the current flowing in the conductor is continuous
 At the interface at $z=-a$ between the ferromagnetic film and the spacer BC can be written in the form:

$$g_{A-}^{\uparrow(\downarrow)}(z, \vec{v}) = T^{\uparrow(\downarrow)} g_{B-}^{\uparrow(\downarrow)}(z, \vec{v}) + R^{\uparrow(\downarrow)} g_{A+}^{\uparrow(\downarrow)}(z, \vec{v}),$$

where T and R are coefficients of a non-diffusive transmission and a reflection (conserving momentum, specular) of electrons.

- the above condition states that in the vicinity of interface the current of electrons flowing in $-z$ direction consists of electrons that came from region B and those which were heading in $+z$ direction and were reflected back to region B
- fraction $(1-T)$ of electrons is scattered diffusely



Magnetoresistance

The general solution can be written as [20]:

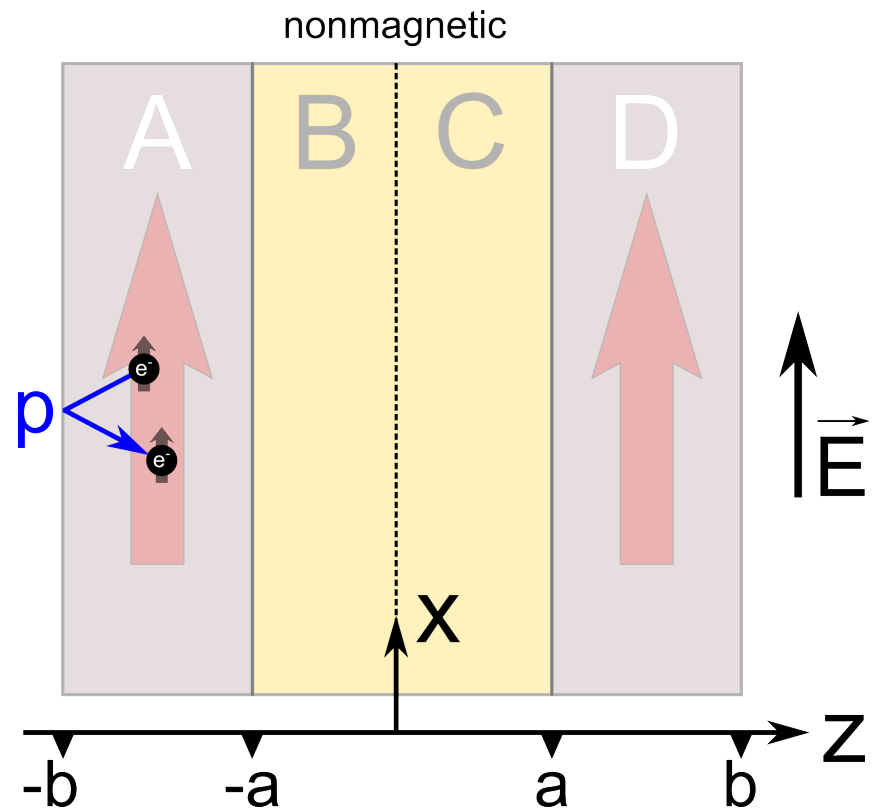
$$g_{\pm}^{\uparrow(\downarrow)}(z, \vec{v}) = \frac{e \vec{E} \tau^{\uparrow(\downarrow)}}{m} \frac{\partial f_o(\vec{v})}{\partial v_x} \times \left[1 + F_{\pm}^{\uparrow(\downarrow)}(v) \exp\left(\frac{\mp z}{\tau^{\uparrow(\downarrow)} |v_z|}\right) \right]$$

4 arbitrary functions to be determined from boundary conditions

Similar Fuchs* BCs are introduced for outer interfaces with specularity factors depending generally on spin:

$$g_{A+}^{\uparrow(\downarrow)}(z, \vec{v}) = p_A^{\uparrow(\downarrow)} g_{A-}^{\uparrow(\downarrow)}(z, \vec{v}) \quad z = -b$$

- the condition states that in the vicinity of the outer interface electrons traveling into the multilayer are those reflected from the interface
- the electrons which are diffusely reflected do not contribute to conductivity along the film (their net contribution is zero)
- any angle dependence of the specularity is neglected for simplicity [20]



*remember Fuchs-Sondheimer theory of the resistivity of thin films

Magnetoresistance

At the fictitious interface at $z=0$ one can write [20]:

$$g_{C^+}^{\uparrow(\downarrow)}(z, \vec{v}) = \cos^2(\theta/2) g_{B^+}^{\uparrow(\downarrow)}(z, \vec{v}) + \sin^2(\theta/2) g_{B^+}^{\downarrow(\uparrow)}(z, \vec{v})$$

$$g_{B^-}^{\uparrow(\downarrow)}(z, \vec{v}) = \cos^2(\theta/2) g_{C^-}^{\uparrow(\downarrow)}(z, \vec{v}) + \sin^2(\theta/2) g_{C^-}^{\downarrow(\uparrow)}(z, \vec{v}),$$

Note that all terms are proportional to $\cos(\theta)$

$$2 \cos^2(\theta/2) - 1 = \cos(\theta)$$

$$1 - 2 \sin^2(\theta/2) = \cos(\theta)$$

which assures an agreement with observed resistance changes versus θ .

The total current (per unit length along the y axis; electric field is along x axis) is given by:

$$I = e \int dz \int d^3v [g^{\uparrow}(\vec{v}, z) + g^{\downarrow}(\vec{v}, z)] \quad \text{a sum of two spin-channels currents}$$

The equations were solved numerically

- the amplitude of magnetoresistance is given by

$$GMR = \frac{\rho^{\uparrow\downarrow} - \rho^{\uparrow\uparrow}}{\rho^{\uparrow\uparrow}}$$

To better analyze the results additional parameters are introduced to describe:

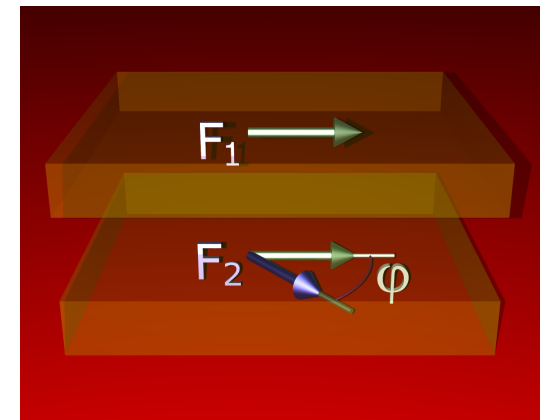
- the spin asymmetry of the diffusive scattering of electrons at interfaces

$$N_s = (1 - T^{\uparrow}) / (1 - T^{\downarrow})$$

- spin asymmetry of the bulk scattering rate in the ferromagnetic material

$$N_b = \lambda^{\uparrow} / \lambda^{\downarrow}$$

λ - electron mean free paths



Magnetoresistance

The main results of the analysis of Barnaś *et al.* [20] can be summarized as follows [21]:

- GMR increases monotonically with mean free path λ if the interface scattering dominates
- GMR displays clear maximum versus λ if the bulk scattering dominates

Assumptions:

- $p_A^\uparrow = p_A^\downarrow = p_D^\uparrow = p_D^\downarrow = p$
- $r = s = t = 1$ conductivities of all layers are equal
- $D^{\uparrow(\downarrow)} = (1 - T^{\uparrow(\downarrow)}) = 0$ - no scattering at interfaces

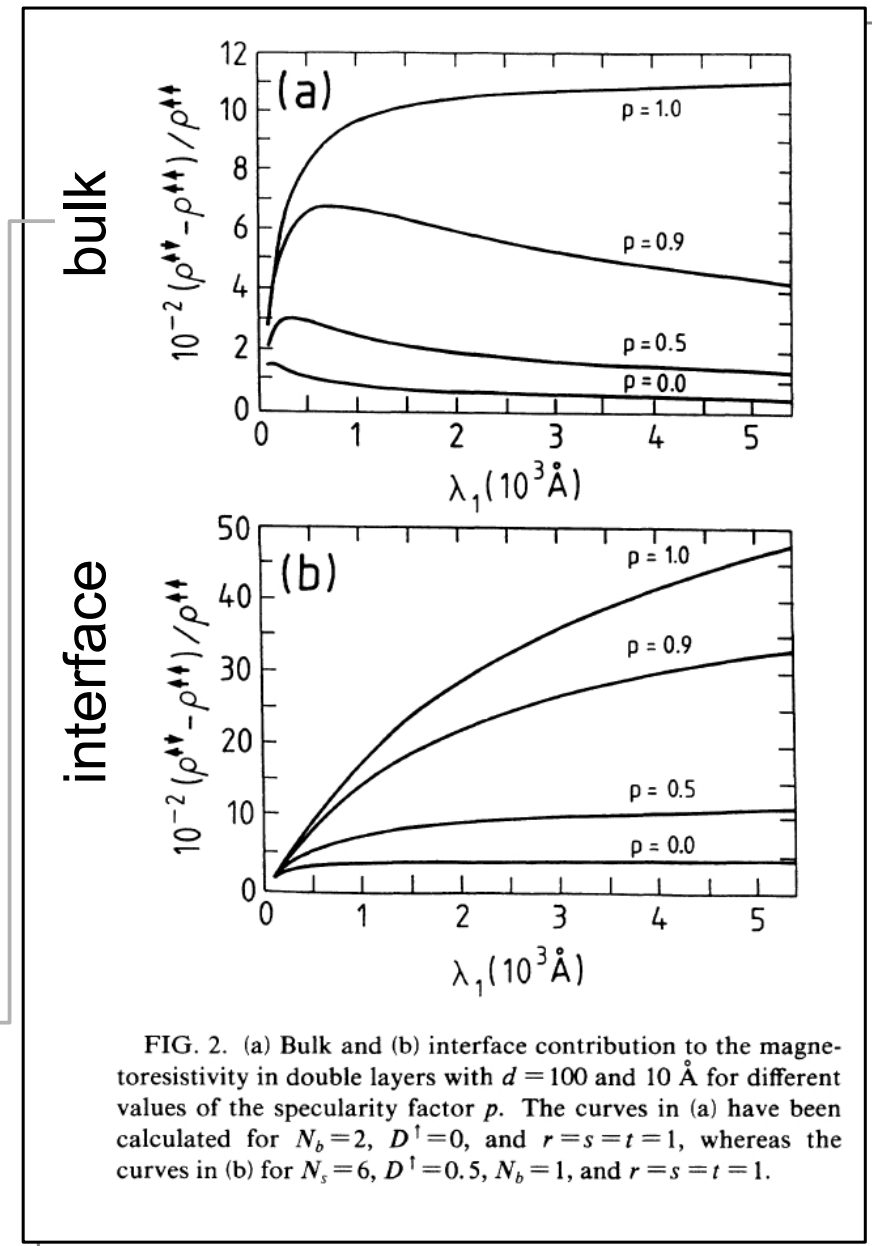


image from: J. Barnaś, A. Fuss, R.E. Camley, P. Grunberg, W. Zinn, Phys. Rev. B **42**, 8110 (1990)

Magnetoresistance

The main results of the analysis of Barnaś *et al.* [20] can be summarized as follows [21]:

- GMR decreases monotonically with the **thickness of ferromagnetic layer** if the interface scattering dominates
- GMR shows a distinct maximum versus **thickness of the ferromagnetic layers** if the bulk scattering dominates (if the thickness of the ferromagnetic layer exceeds λ part of it becomes inactive in GMR but still contributes to conductivity [21])

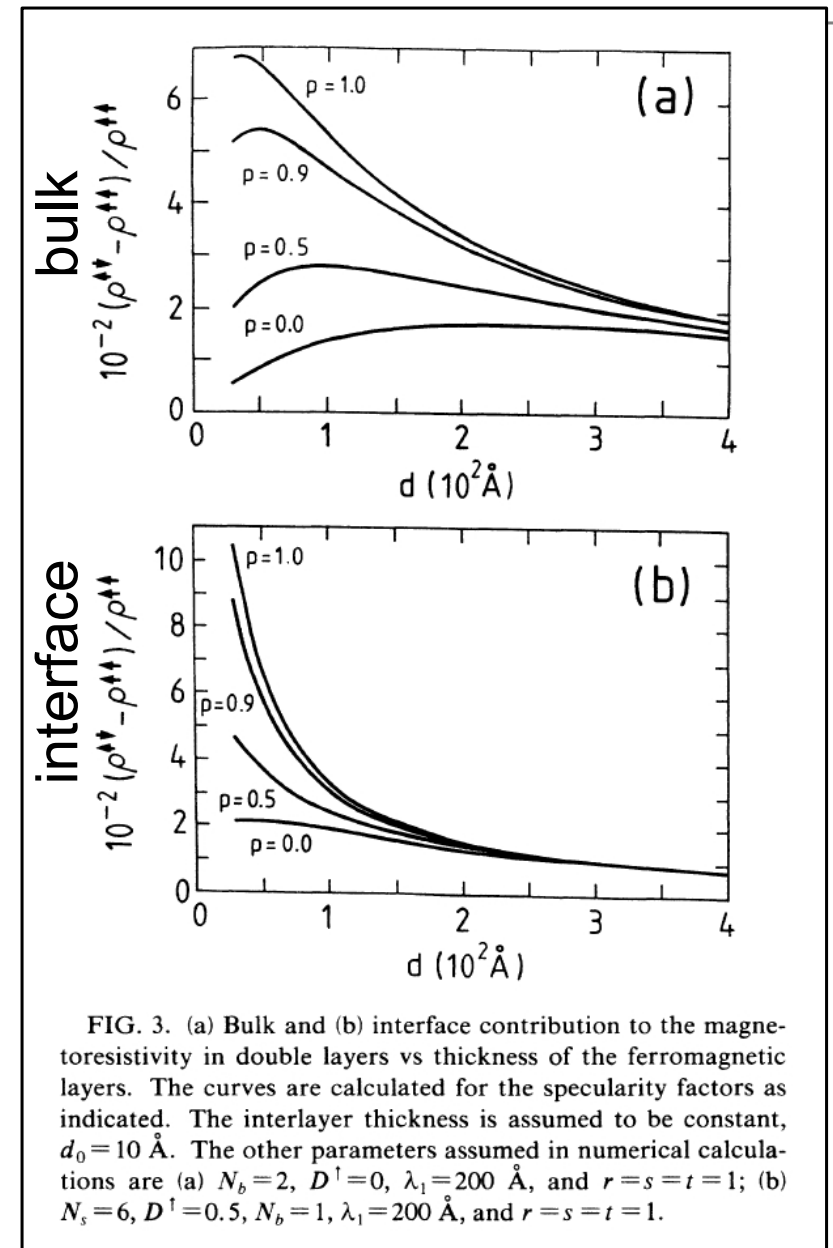


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Magnetoresistance

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- GMR decreases monotonically with the thickness of ferromagnetic layer if the interface scattering dominates
- GMR shows a distinct maximum versus thickness of the ferromagnetic layers if the bulk scattering dominates (if the thickness of the ferromagnetic layer exceeds λ part of it becomes inactive in GMR but still contributes to conductivity [21])
- GMR increases with the increase of repetition number of basic bilayers (ferromagnet/nonmagnetic spacer) – number of GMR active interfaces within λ increases (additionally in thick multilayers the influence of outer surfaces of the system decreases)

Magnetoresistance

Angular dependence of GMR

$$\Delta R \propto \cos(\theta)$$

- in the limit of quantum transport (QT) deviations from the dependence occur due to interference of electron waves reflected from interfaces and/or surfaces [23]
- in QT limit the dependence is proportional to cosine if the structure is symmetrical and the crystal potential is independent of spin
- in the case of current perpendicular to plane geometry (CPP) the significant deviations were observed too [23]

CPP geometry

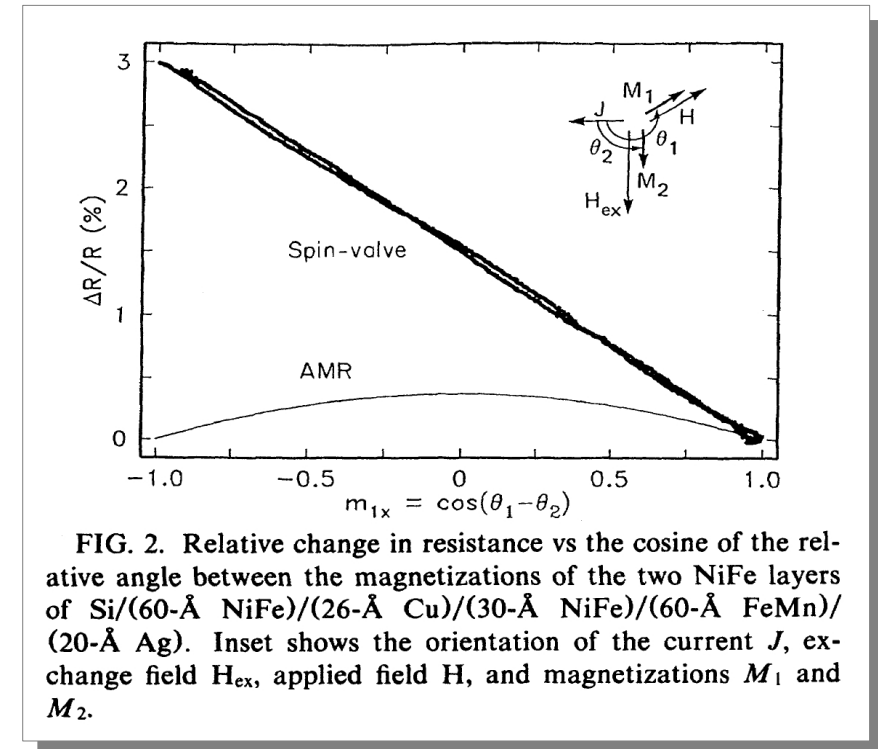
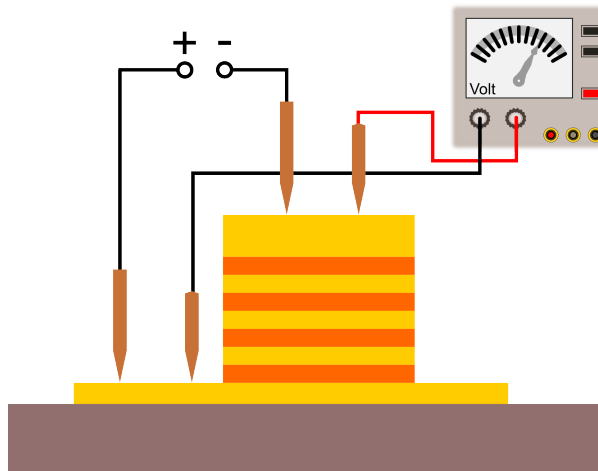
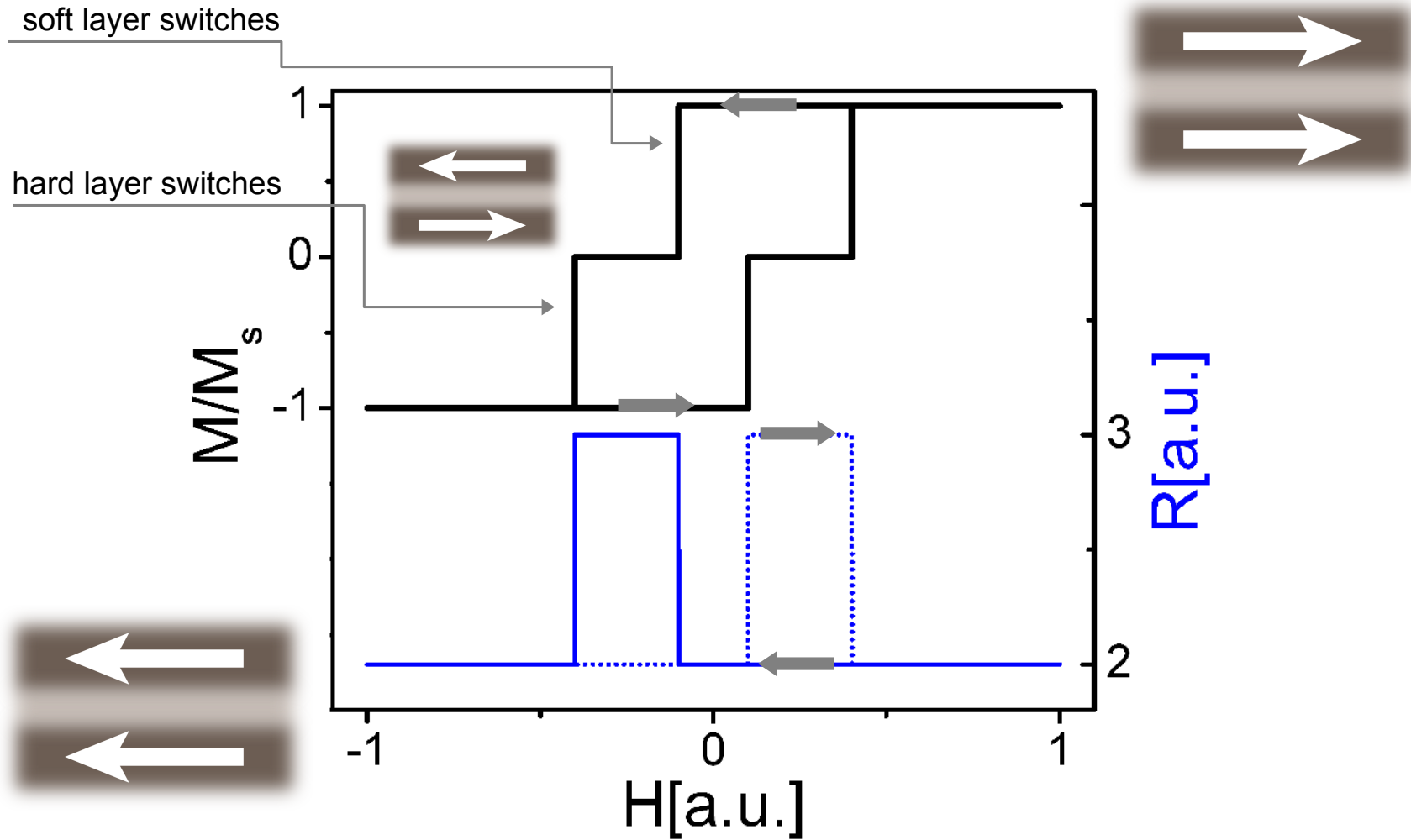


image from B. Dieny, V.S. Speriosu, S.S.P. Parkin, B.A. Gurney, D.R. Wilhoit, D. Mauri, Phys. Rev. B **43**, 1297 (1991)

Magnetoresistance

Angular dependence of GMR

- knowing field dependence of magnetic moments configuration one can approximately predict the shape (*not the amplitude!*) of $R(H)$ dependence



example: GMR of spin valve with two layers of different switching fields

Magnetoresistance

Angular dependence of GMR

- Co(10nm)/Au(6nm)/Co(10nm) magnetic moments configuration predict the shape (not the amplitude!) of $R(H)$ dependence
- note the increase of GMR amplitude with decreasing temperature (increase of mean free path and a decreased phononic contribution) from 1.2 to 1.7%
- magnetic layers have different magnetic moments so after switching of the soft layer the net magnetization is different from zero

A digression:

“All these features can be used for verification of the theoretical predictions with the experimental results. However, the most reliable one seems to be the temperature dependence of the effect. This follows from the fact that the relevant experiments are performed on one single sample. In the case of other features one has to compare data obtained on different samples.” J. Barnaś et al. [20]

image from: J. Barnaś, A. Fuss, R.E. Camley, P. Grunberg, W. Zinn, Phys. Rev. B **42**, 8110 (1990)
example: GMR of spin

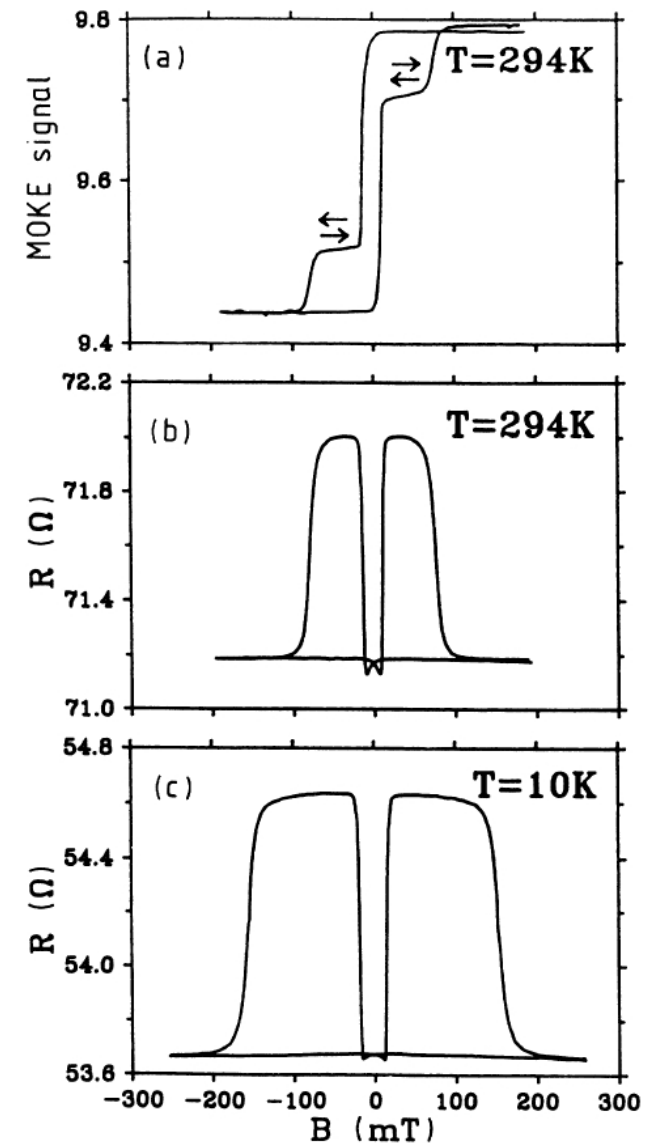


FIG. 9. (a) MOKE-signal and (b) and (c) resistance of the Co/Au/Co structure with the Co-film thickness $d = 100 \text{ \AA}$ and the Au-interlayer thickness $d_0 = 60 \text{ \AA}$.

Spin coupling

The magnetic interactions between magnetic ions in a solid depend on numerous factors (neighboring ions, temperature, external fields etc.)

In some case to describe the system one uses Hamiltonian involving simultaneous interaction between several spins [35,36]:

$$E_{4s} = - \sum_{ijkl} K_{ijkl} [(\vec{S}_i \cdot \vec{S}_j)(\vec{S}_k \cdot \vec{S}_l) + (\vec{S}_i \cdot \vec{S}_l)(\vec{S}_j \cdot \vec{S}_k) - (\vec{S}_i \cdot \vec{S}_k)(\vec{S}_j \cdot \vec{S}_l)]$$

the energy term involves orientations of all four spin

In some other cases it is not enough to use bilinear forms* and biquadratic forms are introduced in addition

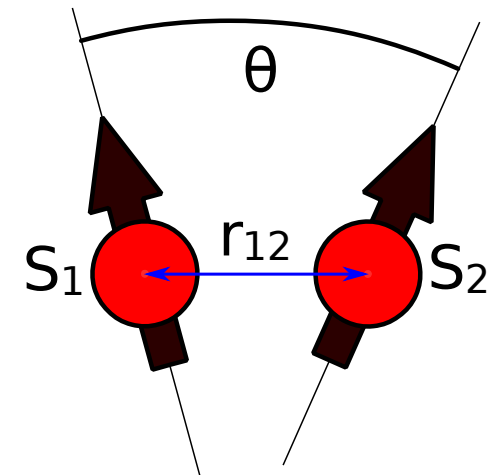
$$E_{4s} = - \sum_{ij} K_{ij} (\vec{S}_i \cdot \vec{S}_j)^2$$

In most relevant cases however it is enough to use only **two spin terms that are bilinear** [38]

$$E_{bilinear} = - \sum_{ij} K_{ij} S_1^i S_2^j = K_{xx} S_1^x S_2^x + K_{xy} S_1^x S_2^y + \dots$$

K_{ij} is a coupling 3×3 matrix, and in matrix notation we have

$$E_{bilinear} = \vec{S}_1 [K] \vec{S}_2$$



*"Form refers to a polynomial function in several variables where each term in the polynomial has the same degree. The degree of the term is the sum of the exponents." - K.C Border [37]

Spin coupling

The interaction matrix, like any 3×3 matrix [38], may be decomposed into a multiple of the identity matrix, an antisymmetric part (three different coefficients), and traceless* symmetric part:

$$K_{ij} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} = J \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & D_1 & D_2 \\ -D_1 & 0 & D_3 \\ -D_2 & -D_3 & 0 \end{bmatrix} + \begin{bmatrix} A_1 & A_4 & A_5 \\ A_4 & A_2 & A_6 \\ A_5 & A_6 & A_3 \end{bmatrix}$$

$$J \vec{S}_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{S}_2 = S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z = J \vec{S}_1 \cdot \vec{S}_2 \quad \text{exchange coupling}$$

$$\leftarrow E_{bilinear} = \vec{S}_1 [K] \vec{S}_2$$

$$\vec{S}_1 \begin{bmatrix} 0 & D_1 & D_2 \\ -D_1 & 0 & D_3 \\ -D_2 & -D_3 & 0 \end{bmatrix} \vec{S}_2 = -D_1 S_1^y S_2^x - D_2 S_1^z S_2^x + D_1 S_1^x S_2^y - D_3 S_1^z S_2^y + D_2 S_1^x S_2^z + D_3 S_1^y S_2^z$$

$$= D_1 (S_1^x S_2^y - S_1^y S_2^x) - D_2 (S_1^z S_2^x - S_1^x S_2^z) + D_3 (S_1^y S_2^z - S_1^z S_2^y)$$

$$= (\hat{i} D_3, -\hat{j} D_2, \hat{k} D_1) \cdot \vec{S}_1 \times \vec{S}_2 = \vec{D} \cdot (\vec{S}_1 \times \vec{S}_2)$$

Dzyaloshinskii-Moriya interaction

*trace of a matrix – a sum of diagonal elements

Spin coupling

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The matrix of the dipole-dipole interaction

$$E_{dipole-dipole} = \frac{-\mu_0}{4\pi|r|^3} [3(\hat{r}_{12} \cdot \vec{S}_1)(\hat{r}_{12} \cdot \vec{S}_2) - \vec{S}_1 \cdot \vec{S}_2], \quad \hat{r}_{12} \text{ - unit vector along the vector connecting two spins}$$

reads

$$M_{dipole-dipole} = \frac{-\mu_0}{4\pi|r|^3} \begin{bmatrix} 3\hat{r}_x^2 - 1 & 3\hat{r}_x\hat{r}_y & 3\hat{r}_x\hat{r}_z \\ 3\hat{r}_x\hat{r}_y & 3\hat{r}_y^2 - 1 & 3\hat{r}_y\hat{r}_z \\ 3\hat{r}_x\hat{r}_z & 3\hat{r}_y\hat{r}_z & 3\hat{r}_z^2 - 1 \end{bmatrix}, \quad \hat{r}_x^2 + \hat{r}_y^2 + \hat{r}_z^2 = 1$$

symmetric, traceless

Mathematica 9.0.1.0 code to get dipole-dipole matrix:

```
n=3;
wer={"x","y","z"};
r=Table[ ToExpression [StringJoin ["r",wer[[ i]]]],{i,1,n}];
S1 =Table[ ToExpression [StringJoin ["S1",wer[[ i]]]],{i,1,n}];
S2 =Table[ ToExpression [StringJoin ["S2",wer[[ i]]]],{i,1,n}];
macierz=Table[ToExpression[StringJoin["S1", wer[[i]],"*S2",wer[[j]]]],{i,1,n},
{j,1,n}];
m= Expand[ 3( r.S1)( r.S2)-S1.S2];(*write in here the spin hamiltonian (two
spin interaction), example dipole-dipole:
m= Expand[ 3( r.S1)( r.S2)-S1.S2];
*)
macierz2 =Table [Coefficient [m,macierz[[ i,j]]],{i,1,n},{j,1,n}];(*macierz2 is
the interaction matrix*)
TraditionalForm[macierz2]
```

*trace of a matrix – a sum of diagonal elements

Spin coupling

Anisotropic spin-spin interactions – those terms of the spin Hamiltonian that are not invariant under rotation in spin space (unaccompanied by rotation in real space) [38]

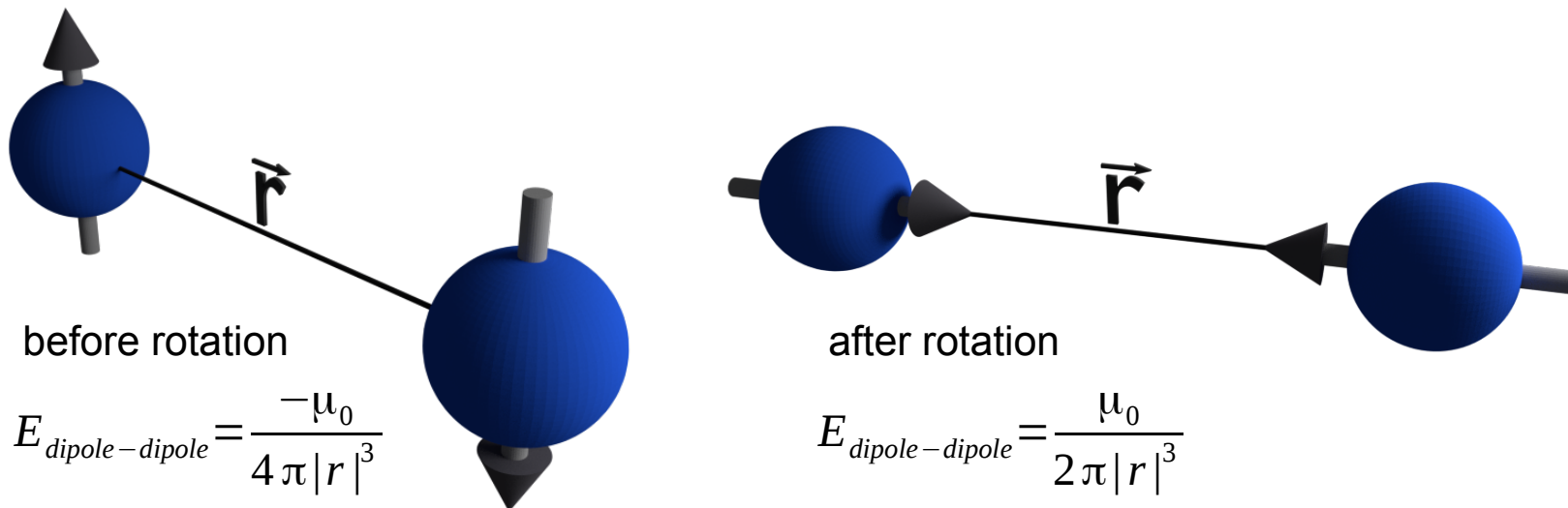
Compare two states:

- one spins point in +z direction and the other one in -z direction; both spins are on y-axis:

$$S_1^x=0, S_1^y=0, S_1^z=1; S_2^x=0, S_2^y=0, S_2^z=-1; \quad \hat{r}_x=0, \hat{r}_y=1, \hat{r}_z=0$$

- as above but both spins (not spinors) are rotated by 90 Deg about x-axis

$$S_1^x=0, S_1^y=1, S_1^z=0; S_2^x=0, S_2^y=-1, S_2^z=0; \quad \hat{r}_x=0, \hat{r}_y=1, \hat{r}_z=0$$

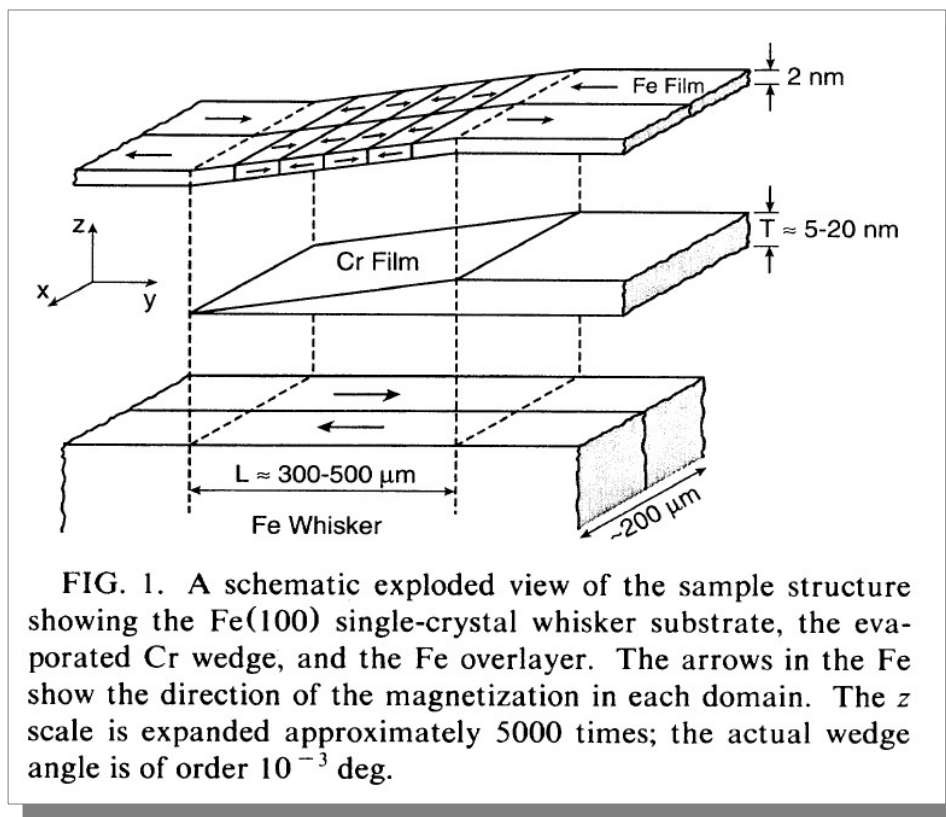


The energies obtained in both cases are different – dipole-dipole interaction is anisotropic

Magnetoresistance

RKKY-like interlayer coupling

- two Fe layers separated by a Cr wedge-shaped spacer; scanning electron microscopy with polarization analysis (SEMPA)
- measurement on a single specimen!
- up to six oscillations in coupling were observed



Obtaining wedge-shaped films:

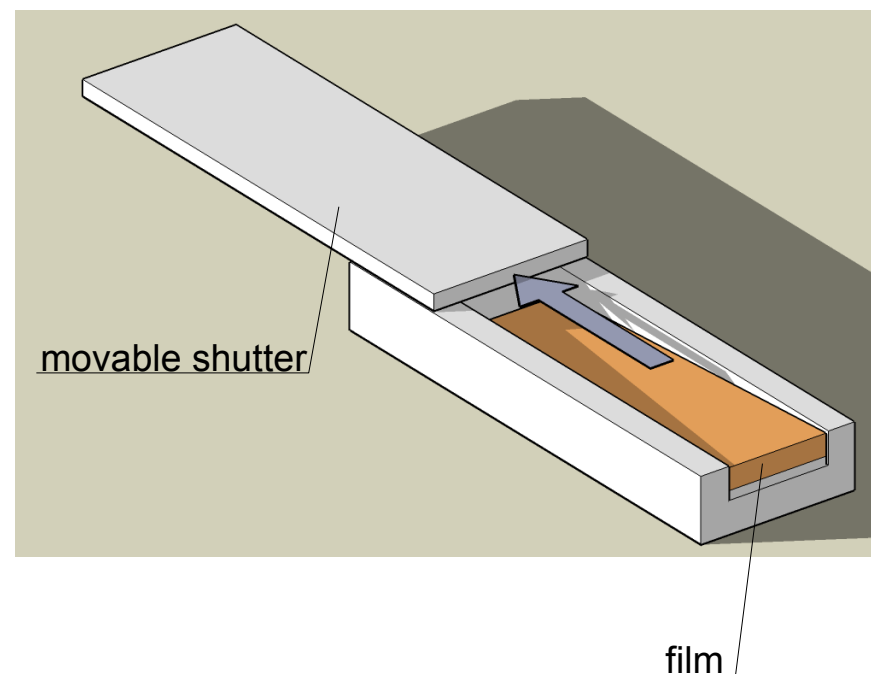


image from J. Unguris, R. J. Celotta, and D. T. Pierce Phys. Rev. Lett. **67**, 140 (1991)

Magnetoresistance

RKKY-like interlayer coupling

- two Fe layers separated by a Cr wedge-shaped spacer; scanning electron microscopy with polarization analysis (SEMPA)
- measurement on a single specimen!
- up to six oscillations in coupling were observed
- different periods of coupling depending on temperature of the substrate during the film growth: samples grown at elevated temperature are of better quality and the magnetization of the upper Fe layer changes with each atomic-layer change in Cr thickness
- **“lower quality” samples display only RKKY-like coupling**

grown at elevated temperatures (200-300°C)

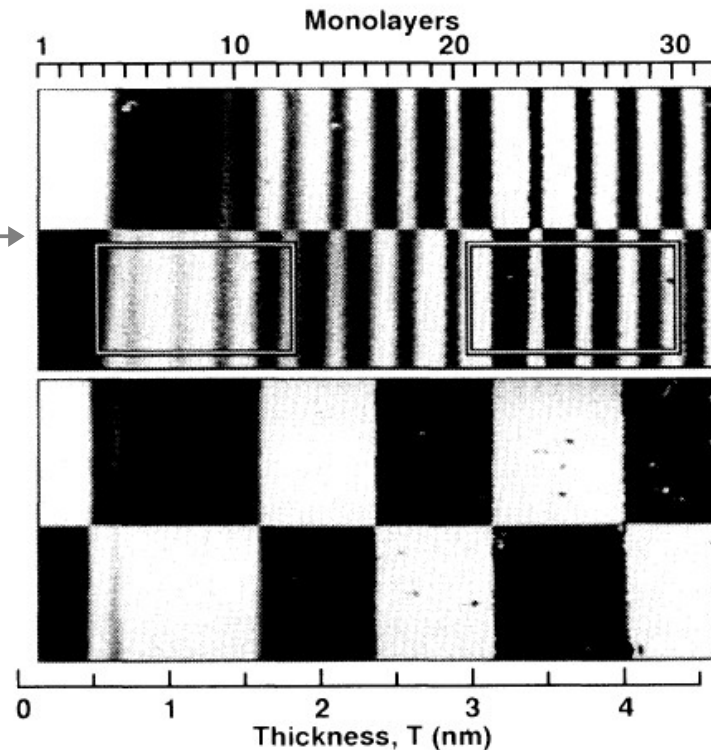


FIG. 3. The difference in the magnetic coupling of the Fe layers in the Fe/Cr/Fe sandwich for the Cr wedge grown (2.7 ML/min) on a substrate at room temperature (lower panel) and grown (7.2 ML/min) on a substrate at elevated temperature (upper panel) is clear in these SEMPA M_y images. The images in the upper and lower panels represent areas 300×280 and $350 \times 290 \mu\text{m}$, respectively.

Magnetoresistance

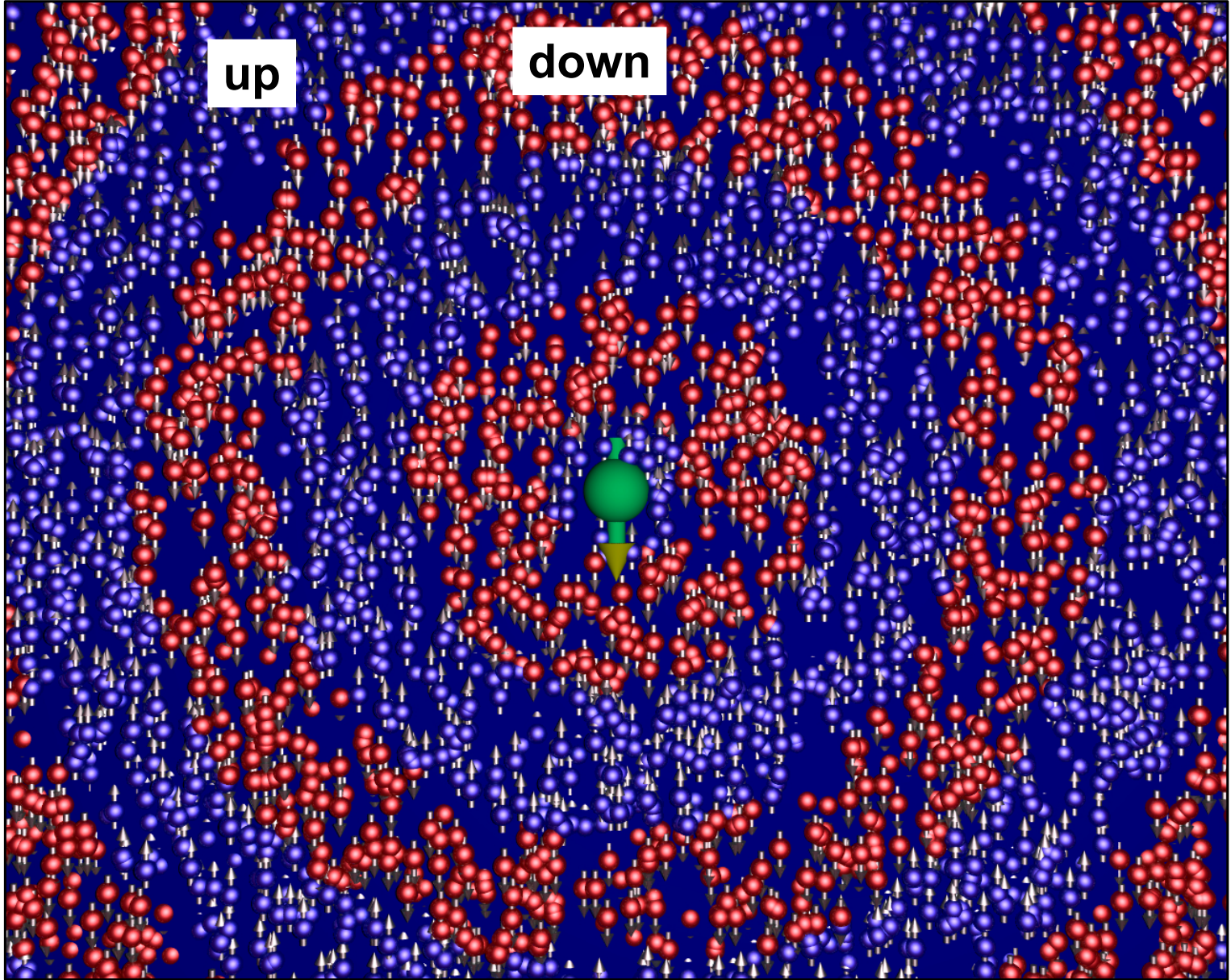
RKKY-like interlayer coupling

Magnetic impurity in a conducting medium induces spatial fluctuations of spin polarization of s-electrons about the impurity [9]

- the oscillatory term of wave number $2 k_F$ falls off like r^{-3} at large distances

impurity

electrons

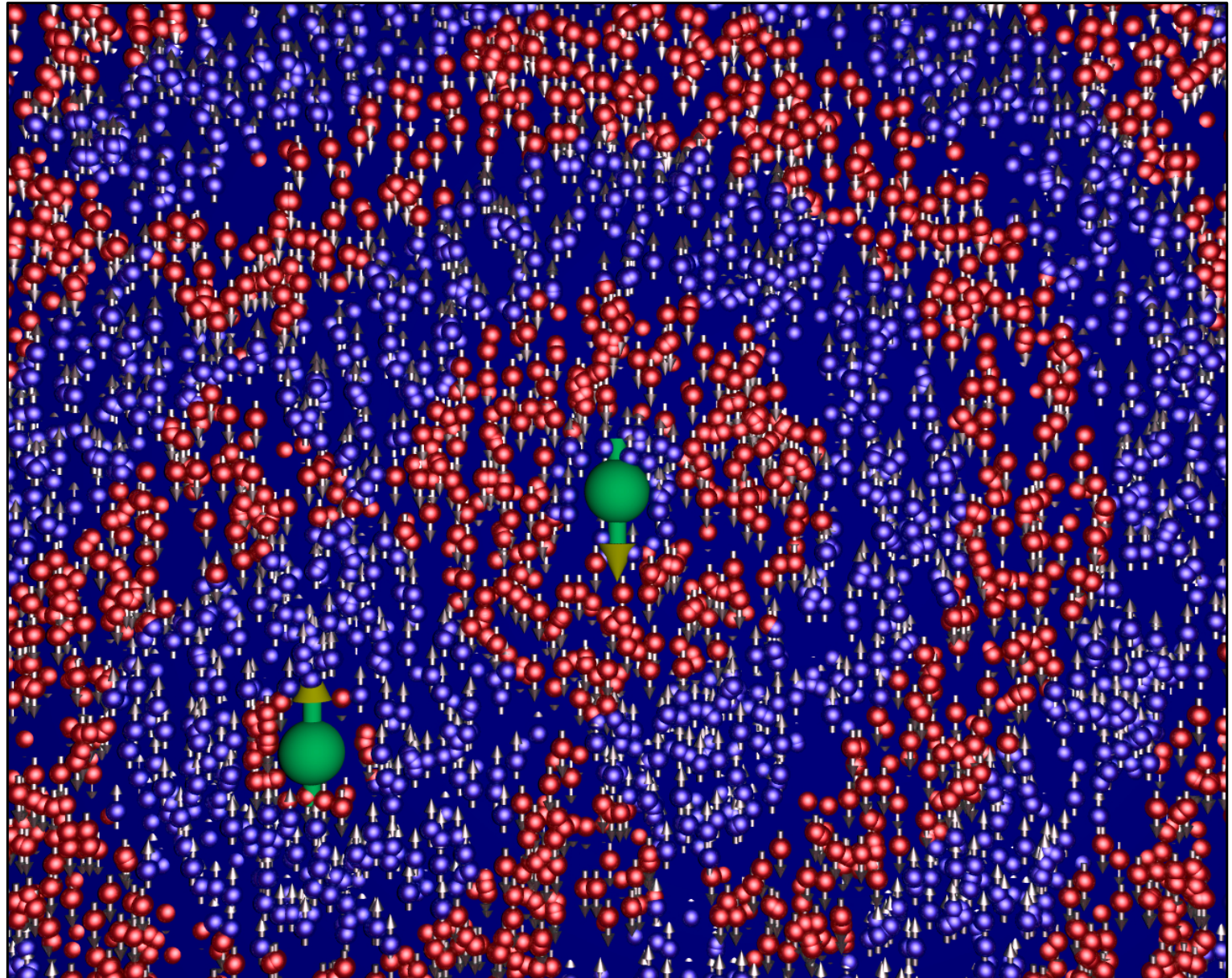


Magnetoresistance

RKKY-like interlayer coupling

Magnetic impurity in a conducting medium induces spatial fluctuations of spin polarization of s-electrons about the impurity [9]

- the oscillatory term of wave number $2k_F^*$ falls off like r^{-3} at large distances
- the second impurity placed in the vicinity experiences interaction with the first impurity
- depending on the distance between impurities the interactions may be **ferromagnetic** or **antiferromagnetic**



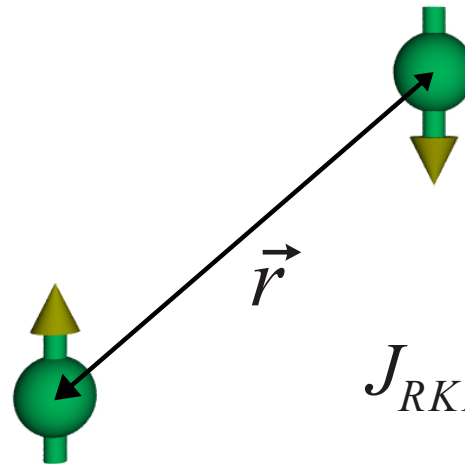
*Fermi wave vector

Magnetoresistance

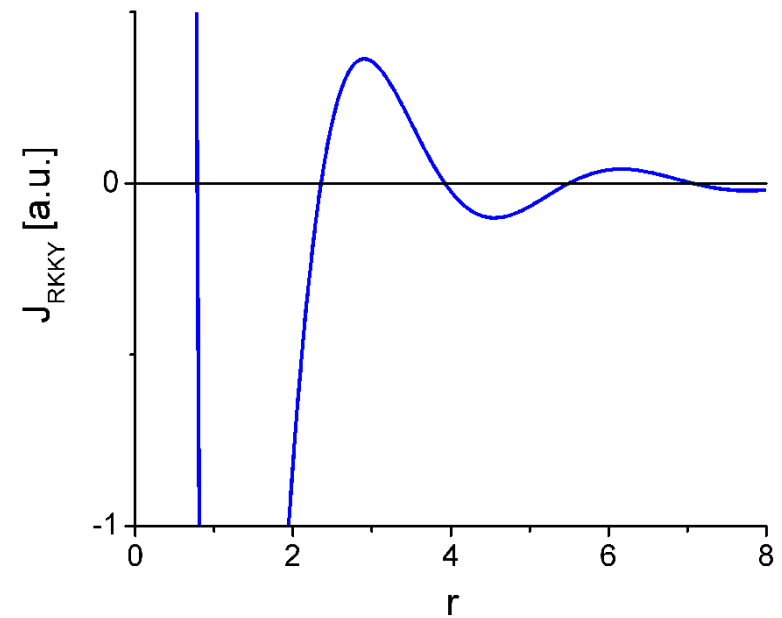
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$$J_{RKKY} \propto \frac{1}{r^3} \cos(2 k_F r)$$



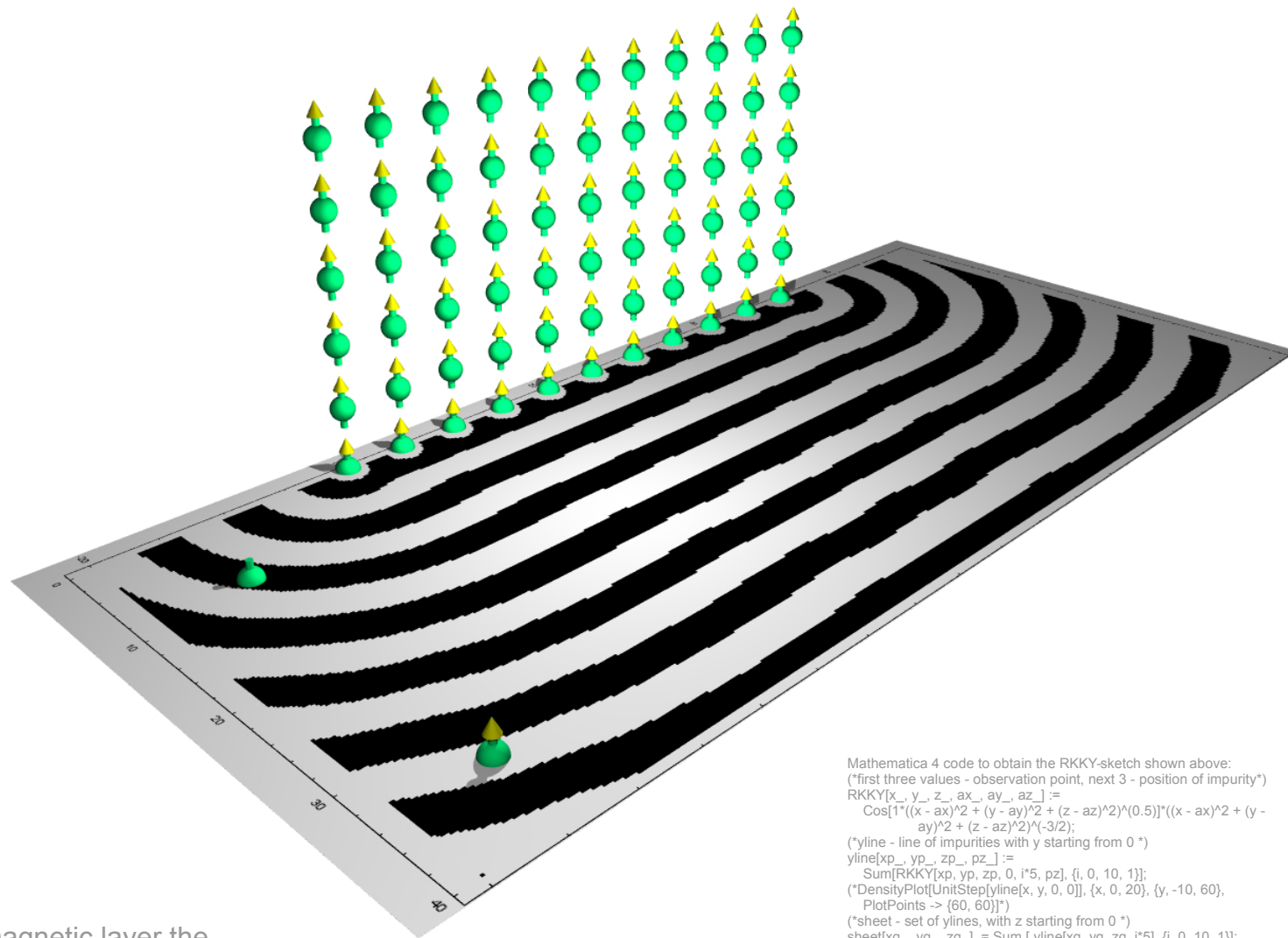
Magnetoresistance

RKKY-like interlayer coupling

A plane composed of exchange coupled impurities creates spatial oscillations of spin polarization in the direction perpendicular to its surface

- if the moments are strongly coupled ferromagnetically they form a ferromagnetic layer
- a similar, parallel, layer or multilayer placed a certain distance away experiences ferromagnetic or antiferromagnetic coupling depending on a distance from the first layer

schematic drawing of a RKKY spin polarization due to single atom thick (11×11 atoms) layer of impurities*



```

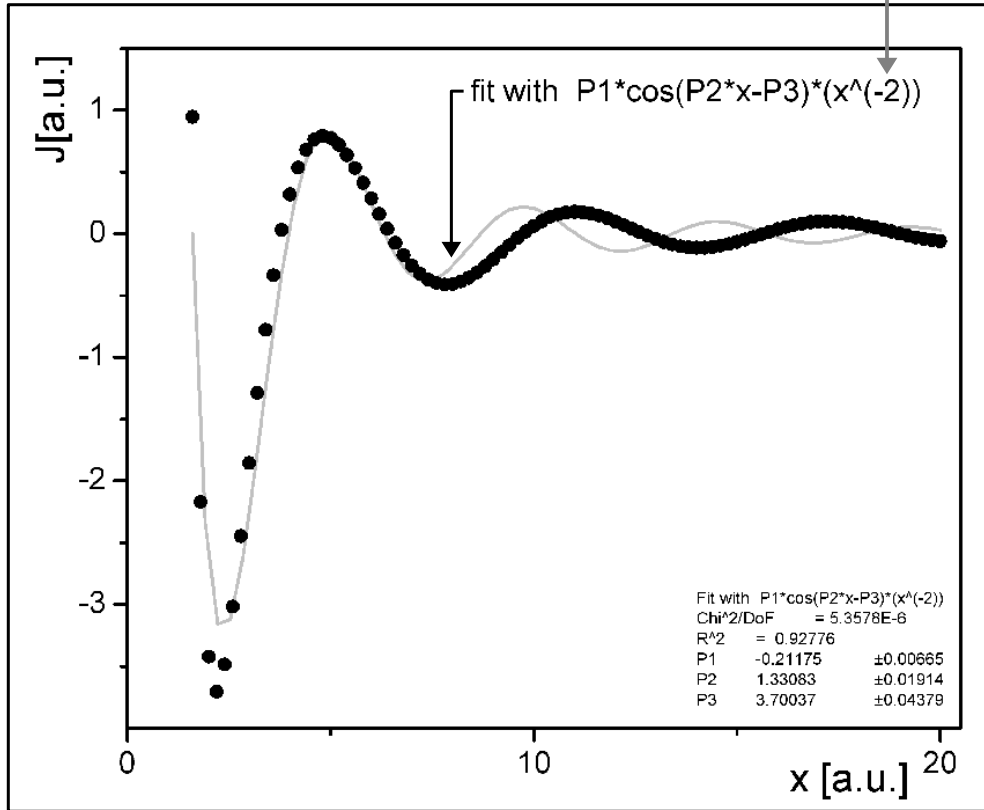
Mathematica 4 code to obtain the RKKY-sketch shown above:
(*first three values - observation point, next 3 - position of impurity*)
RKKY[x_, y_, z_, ax_, ay_, az_] :=
  Cos[1*((x - ax)^2 + (y - ay)^2 + (z - az)^2)^(0.5)]*((x - ax)^2 + (y -
  ay)^2 + (z - az)^2)^(-3/2);
(*yline - line of impurities with y starting from 0 *)
yline[xp_, yp_, zp_, pz_] :=
  Sum[RKKY[xp, yp, zp, 0, i*5, pz], {i, 0, 10, 1}];
(*DensityPlot[UnitStep]yline[x, y, 0]], {x, 0, 20}, {y, -10, 60},
  PlotPoints -> {60, 60}])
(*sheet - set of ylines, with z starting from 0 *)
sheet[xq_, yq_, zq_] = Sum [ yline[xq, yq, zq, i*5], {i, 0, 10, 1}];
DensityPlot[UnitStep[sheet[x, y, 25]], {x, 0, 40}, {y, -20, 70},
  PlotPoints -> {200, 200* 9/4 }, AspectRatio -> 9/4, Mesh -> False,
  ImageSize -> 600]
  
```

in case of quasi-infinite/real ferromagnetic layer the lines delimiting areas of opposite spin polarization would not be bowed except at the ends

*the drawing shows the sign of the coupling (black and gray correspond to positive and negative spin polarization)

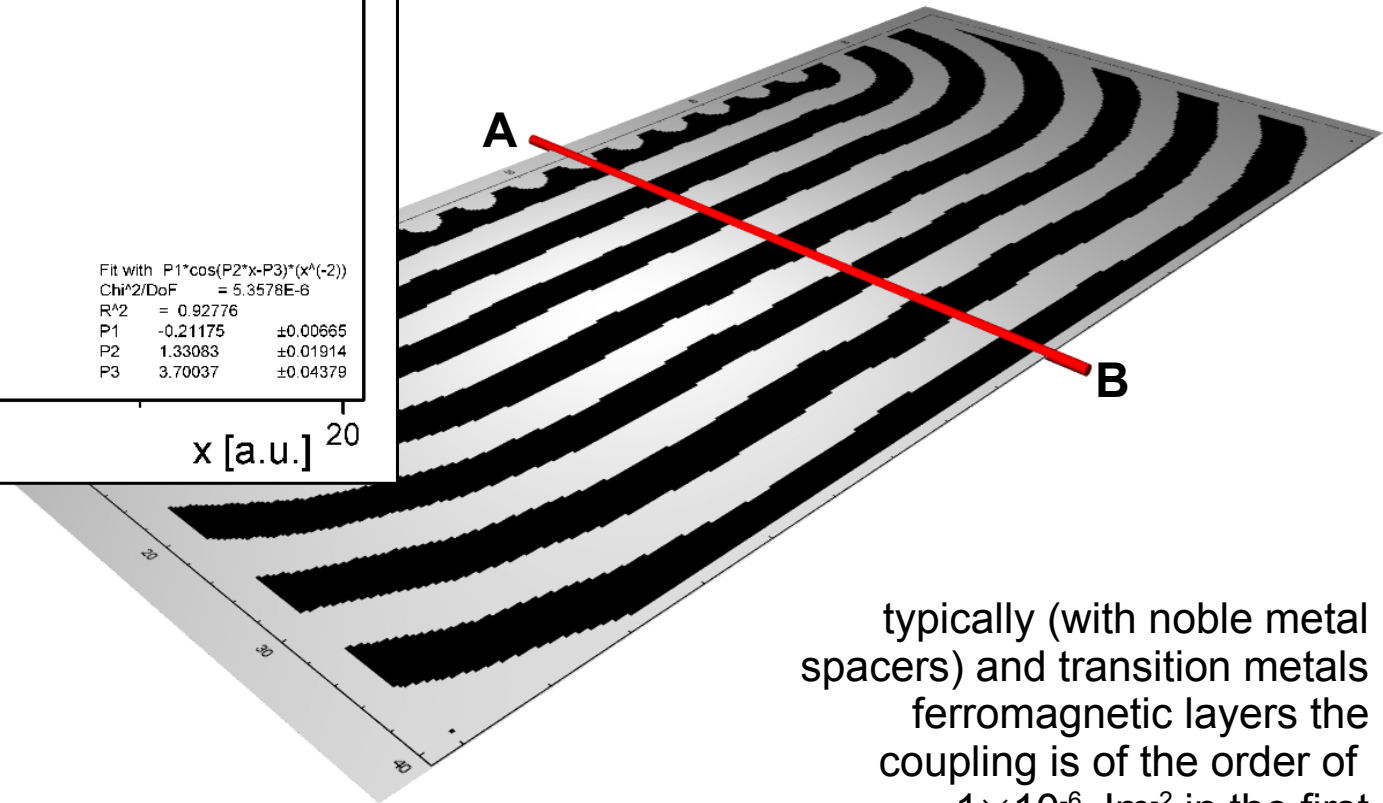
Magnetoresistance

RKKY-like interlayer coupling



Theoretical considerations show that the coupling between **two ferromagnetic layers** is inversely proportional to the square of the spacer thickness [30]

$$J_{RKKY} \propto \frac{1}{r^2}$$



the coupling along AB line

typically (with noble metal spacers) and transition metals ferromagnetic layers the coupling is of the order of $1 \times 10^{-6} \text{ Jm}^{-2}$ in the first antiferromagnetic maximum

RKKY-like interlayer coupling

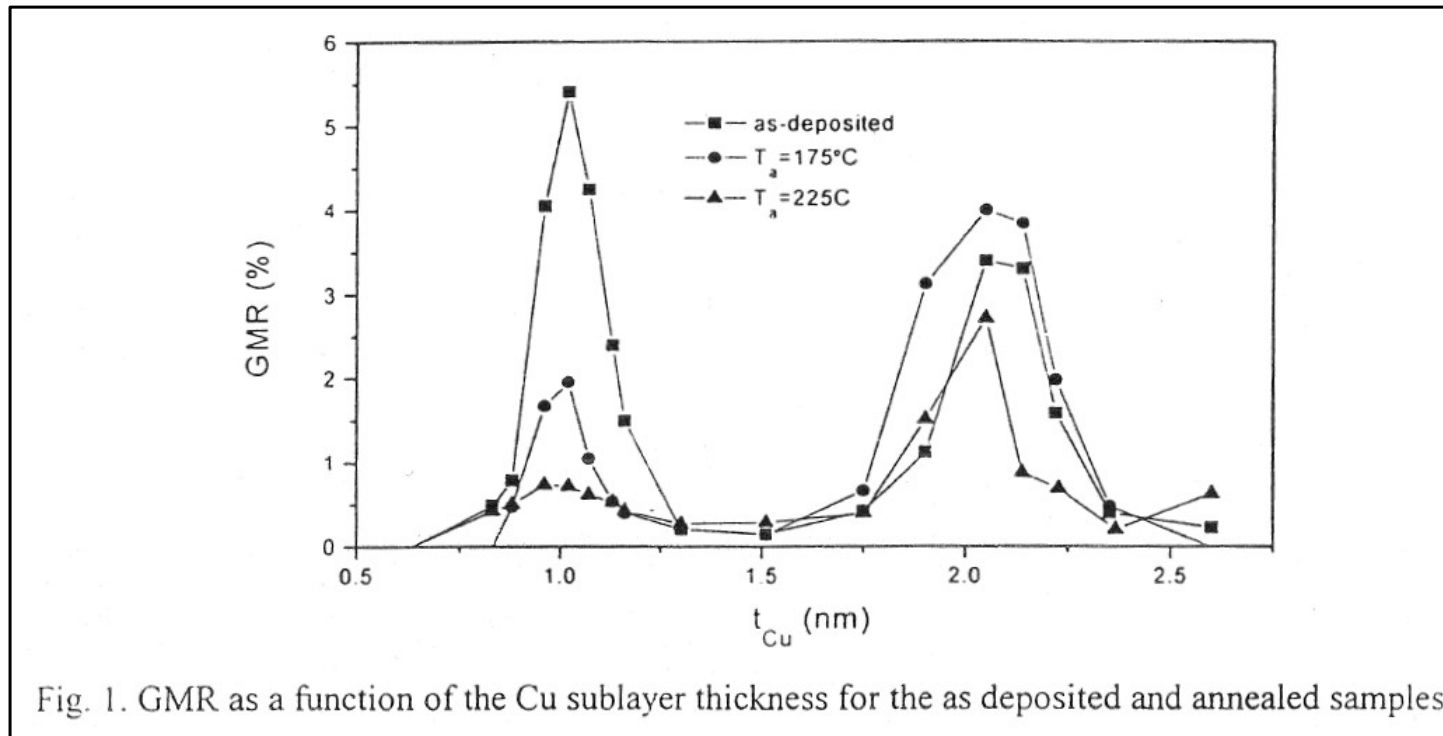


Fig. 1. GMR as a function of the Cu sublayer thickness for the as deposited and annealed samples.

- $\text{Si}(100)/\text{Cu}(20\text{nm})[\text{Ni}_{83}\text{Fe}_{17}(2\text{nm})/\text{Cu}(t_{Cu})]_{100}$
- GMR reflects the oscillatory character of the RKKY-like coupling between permalloy layers
- in MLs with identical magnetic layers (the same switching fields) GMR can be observed only for spacer thicknesses corresponding to antiferromagnetic coupling; otherwise the magnetic field does not change relative orientation of magnetic moments of neighboring layers

Magnetoresistance

RKKY-like interlayer coupling

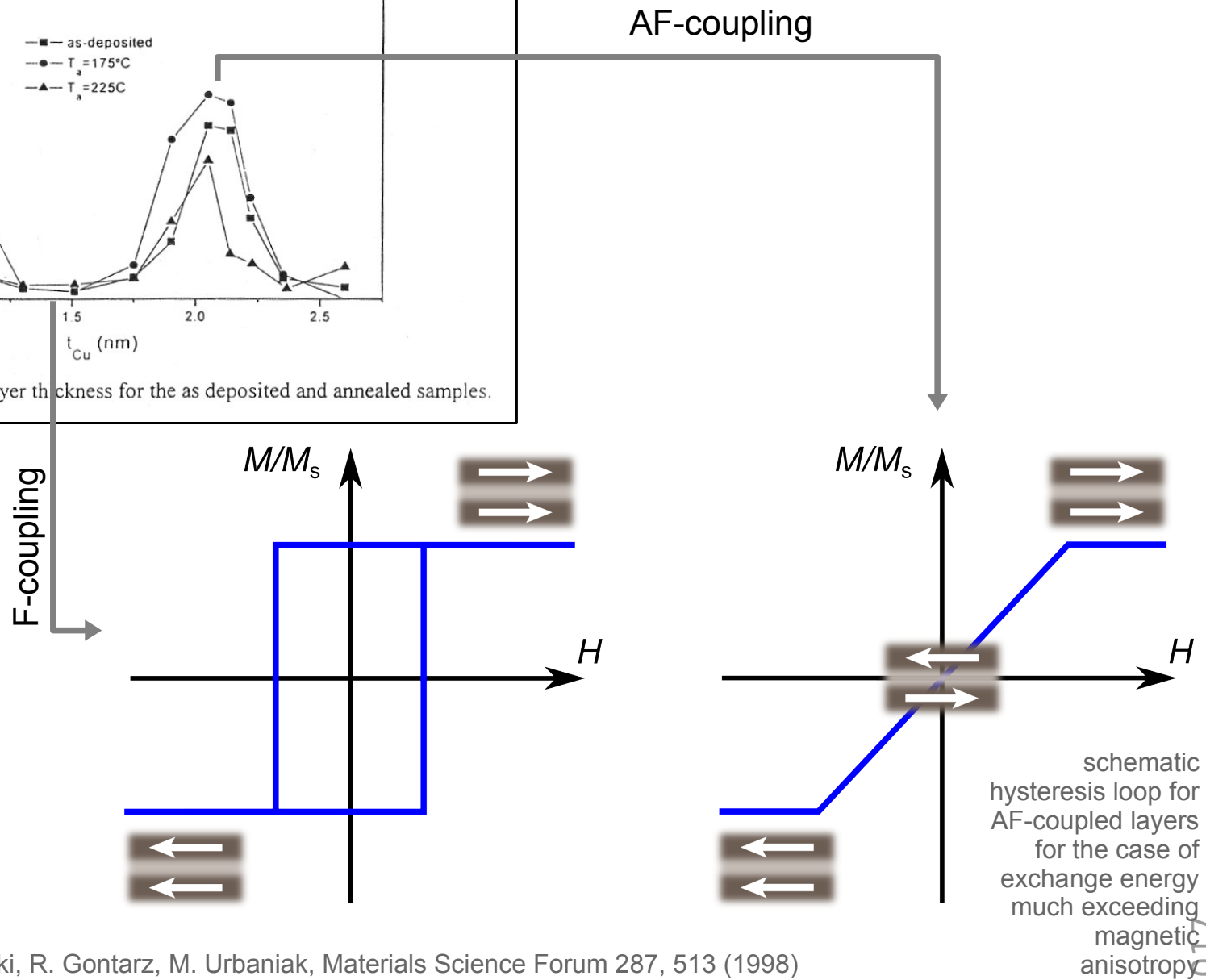
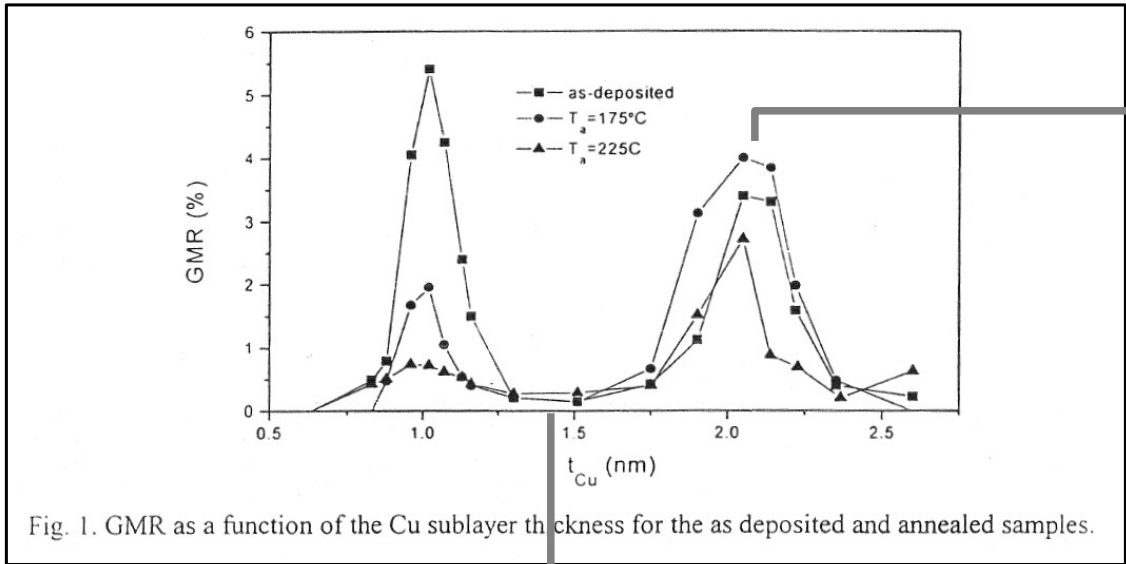


image from: F. Stobiecki, T. Luciński, R. Gontarz, M. Urbaniak, Materials Science Forum 287, 513 (1998)

Magnetoresistance

Inverse CPP GMR [31]

- “*Fe doped with V gains negative spin asymmetry for bulk scattering*” - the up-spin channel is characterized by higher resistivity

$$\rho_{\uparrow(\downarrow)} = \rho_{bulk} (1 \mp \beta)$$

- similarly the interface resistivity depends on spin orientation – factor γ (positive)
- in $(\text{FeV}/\text{Cu}/\text{Co}/\text{Cu})_{20}$ multilayers the resistance of saturated system (all magnetizations pointing in one direction) may be higher than for the case of antiparallel orientation of magnetizations in neighboring magnetic layers
- depending on the FeV layer thickness the GMR can be either normal or inverse; at small FeV layer thicknesses the interface scattering dominates resulting in normal GMR
- the crossover thickness of FeV layers lies between 2 and 3 nm

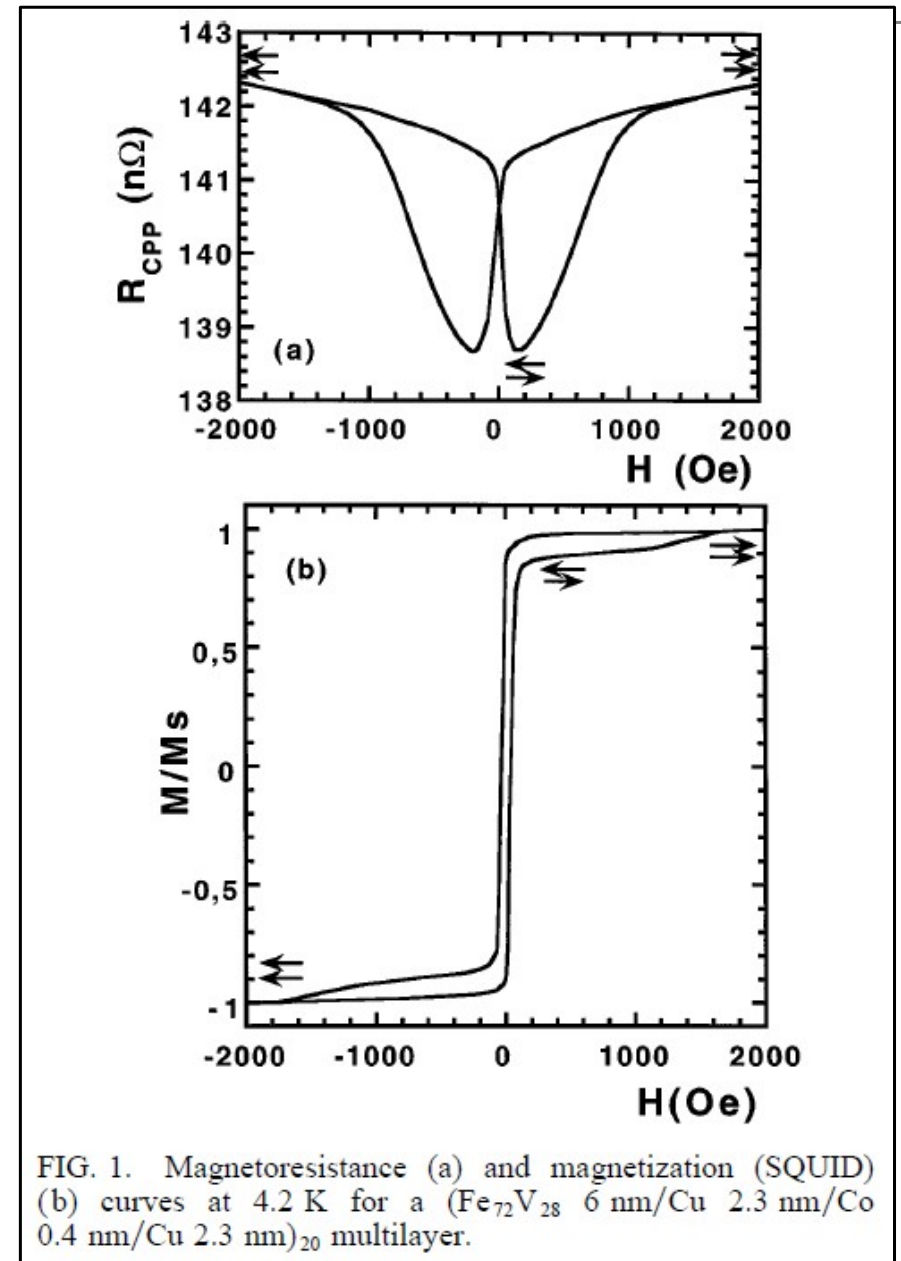
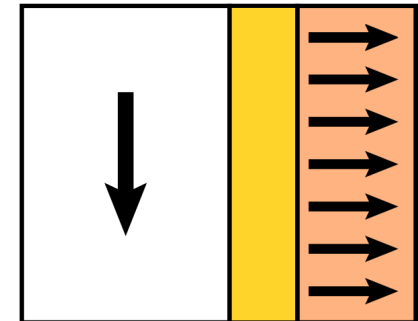
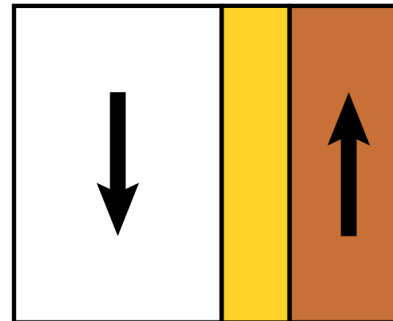
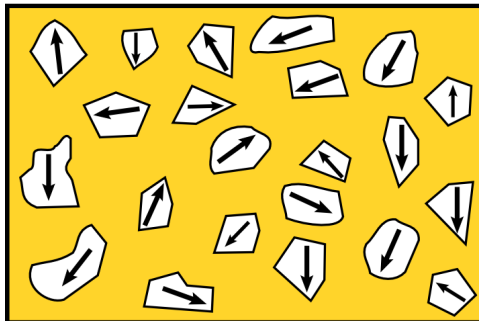
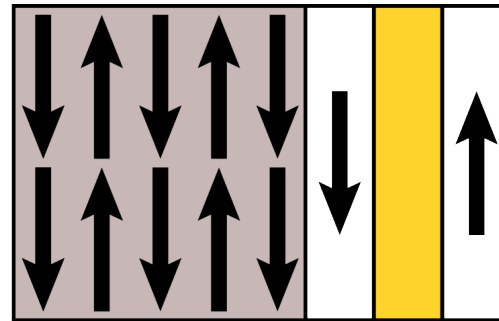
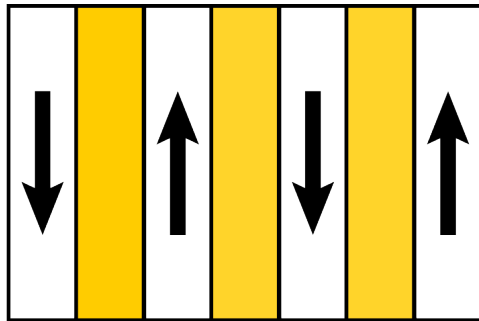







image from: S.Y. Hsu, A. Barthélemy, P. Holody, R. Loloee, P. A. Schroeder, A. Fert, Phys. Rev. Lett. **78**, 2652 (1997)

Magnetoresistance

Typical GMR systems



-  - nonmagnetic conductor
-    - different ferromagnetic conductors
-  - antiferromagnet

Typical GMR systems

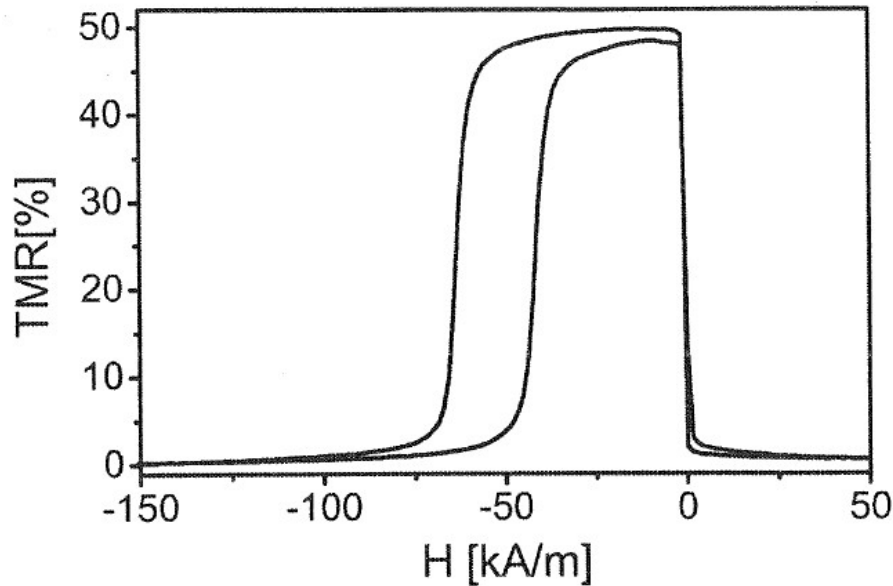
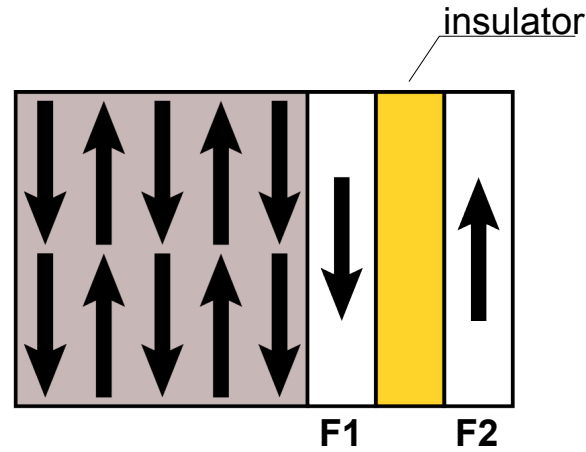
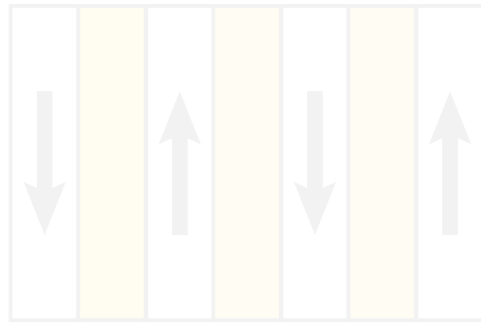


Fig. 6 The room temperature TMR curve of $\text{Cu}(30 \text{ nm})/\text{Ni}_{80}\text{Fe}_{20}(4 \text{ nm})/\text{Mn}_{83}\text{Ir}_{17}(15 \text{ nm})/\text{Co}_{70}\text{Fe}_{30}(2 \text{ nm})/\text{Al}(1.4 \text{ nm}) + \text{Ox}/\text{Ni}_{80}\text{Fe}_{20}(4 \text{ nm})/\text{Ta}(3 \text{ nm})/\text{Cu}(55 \text{ nm})/\text{Au}(20 \text{ nm})$ multilayer.

GMR (or TMR*) in systems with exchange bias

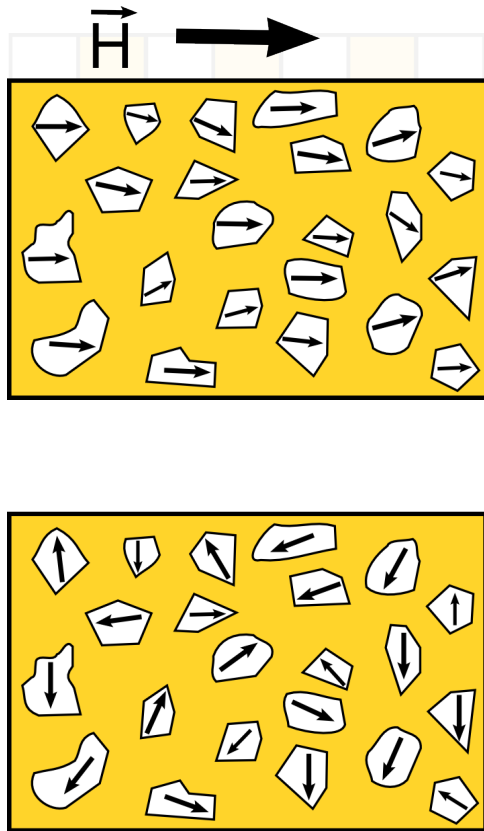
- magnetization of ferromagnetic layer F1 “fixed” by the anisotropy of the antiferromagnetic layer
- magnetization of the F2 layer is “free” to rotate in external magnetic field
- both conducting and insulating spacer may be used
- very high field sensitivities of the effect achievable

image from: M. Urbaniak, J. Schmalhorst, A. Thomas, H. Brückl, G. Reiss, T. Luciński, F. Stobiecki Phys. Stat. Sol. (a) **199**, 284 (2003)

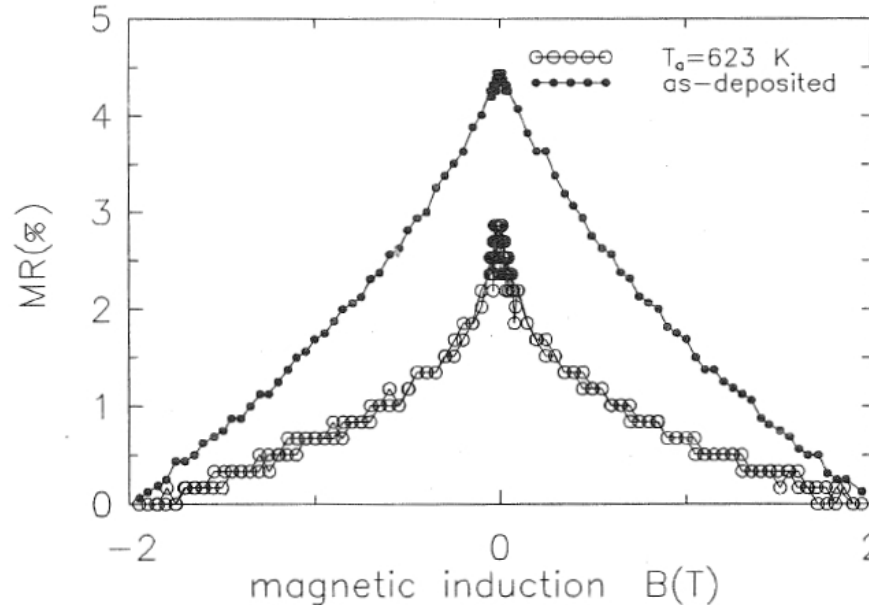
*tunneling magnetoresistance

Magnetoresistance

Typical GMR systems



- nonmagnetic conductor
- different ferromagnetic conductors
- antiferromagnet



directions of the magnetic moments within the grain depend on external field and on the effective magnetic anisotropy of the grain (shape, magnetocrystalline etc.) and on interactions with other grains.

Fig. 1. Exemplary magnetoresistive curves obtained for 76 nm thick $\text{Co}_{20}\text{Ag}_{80}$ film in as-deposited state and after annealing at 623 K

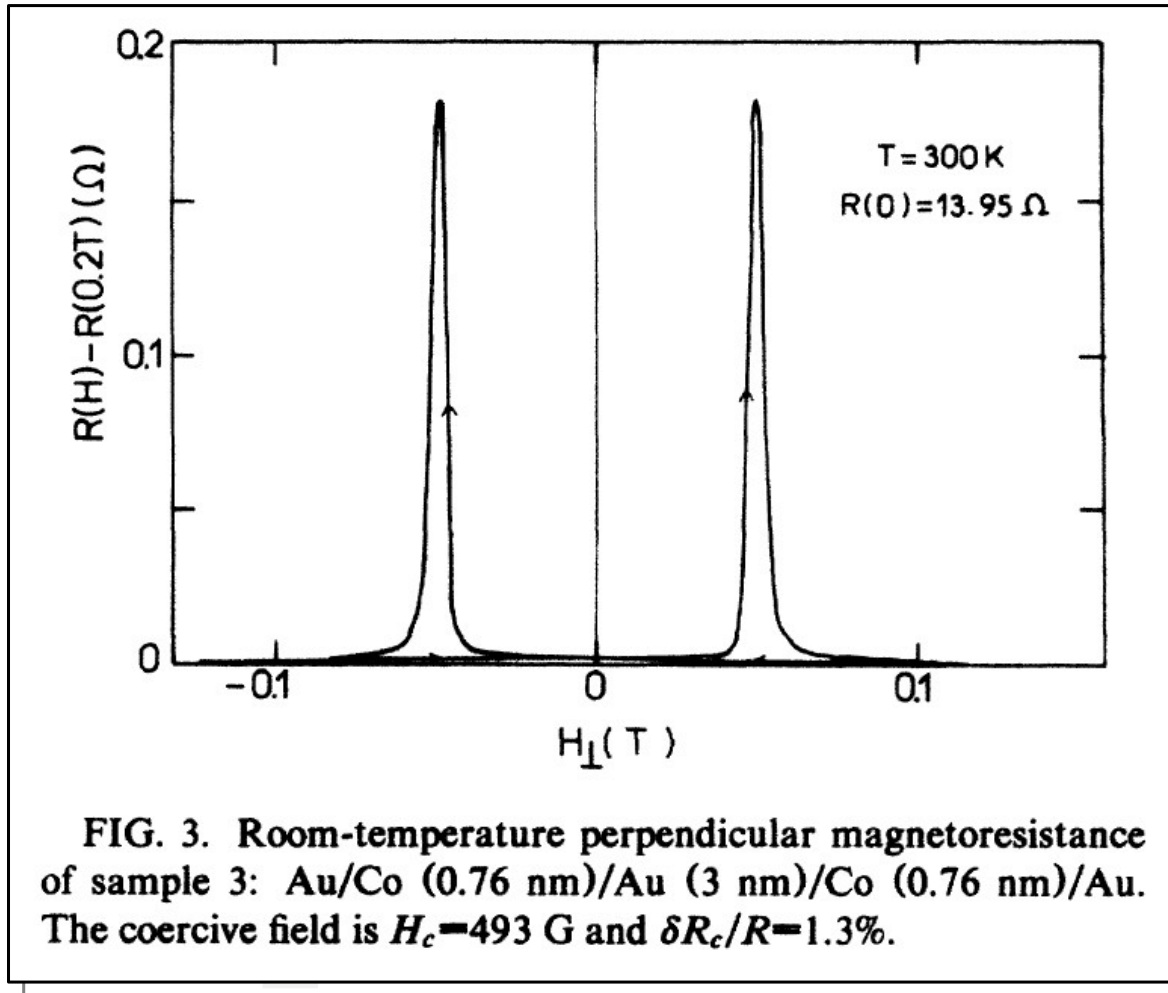
Granular GMR ($G^2\text{MR}$)

- magnetic grains in nonmagnetic matrix (content below percolation threshold)
- resistance saturates in high fields

image from: M. Urbaniak, I. Gościńska, H. Ratajczak, Phys. Stat. Sol. (a) **160**, 121 (1997)

Magnetoresistance

Typical GMR systems



GMR in systems with perpendicular magnetic anisotropy

- two Co layers with slightly different coercive fields
- **first observation of GMR** – before “Nobel papers” by A. Fert and P. Grünberg; the explanation (three different mechanisms proposed) of the effect was not correct

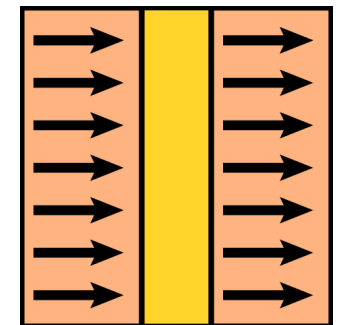


image from: E. Vélú, C. Dupas, D. Renard, J.P. Renard, J. Seiden, Phys. Rev. B 37, 668 (1988)

Magnetoresistance

Typical GMR systems

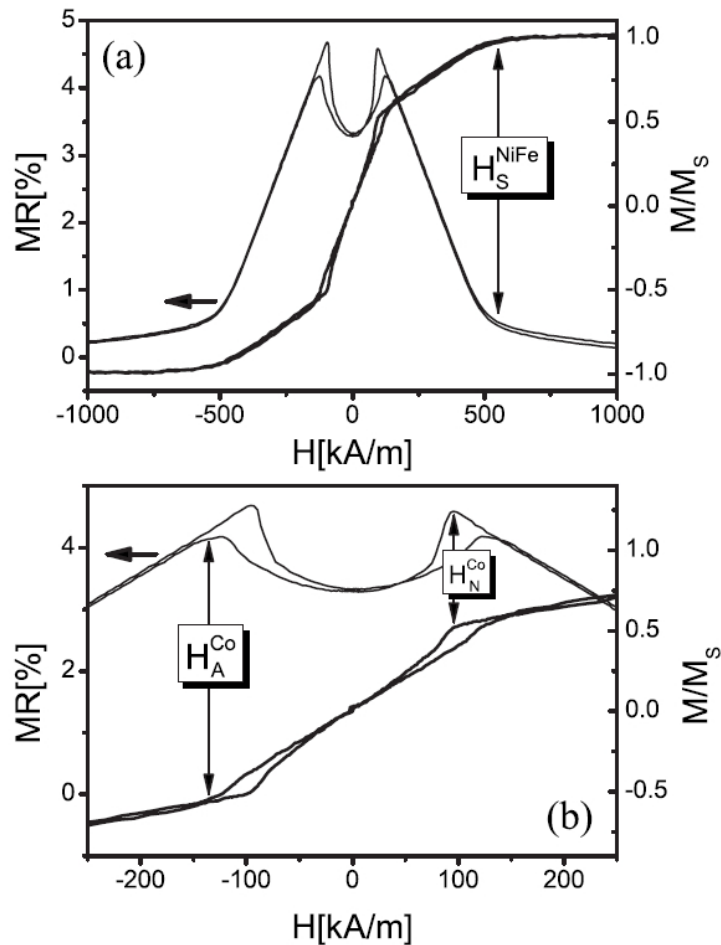


FIG. 1. The magnetization hysteresis and the current-in-plane magnetoresistance of sputtered $[\text{Ni}_{80}\text{Fe}_{20}(2 \text{ nm})/\text{Au}(2.4 \text{ nm})/\text{Co}(1.1 \text{ nm})/\text{Au}(2.4 \text{ nm})]_{10}$ ML measured with the magnetic field applied perpendicularly to the sample plane (source: Ref. 3). Panel (b) shows a small field range. H_S^{NiFe} denotes a saturation field of the NiFe layers; H_N^{Co} and H_A^{Co} denote nucleation and annihilation fields of the Co domain structure, respectively.

GMR in systems with alternating direction of magnetic anisotropy

- maximal angle between magnetic moments of neighboring magnetic layers approx. 90 deg
- in MLs the domain structure of one layer can influence the reversal of the second layer
- approx. linear dependence of resistance on the applied perpendicular field strength – sensor applications

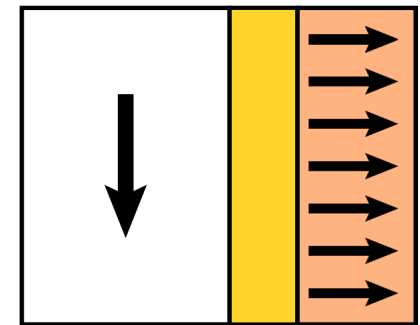
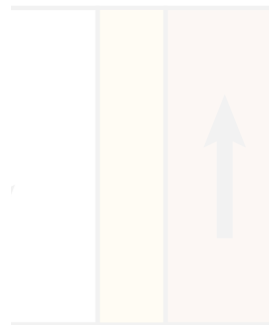


image from: M. Urbaniak, J. Appl. Phys. **104**, 094909 (2008)

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