Quantum electronics

2

Electrical measurements in nanoelectronics

IFM PAN

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Electrical measurements in nanoelectronics

- Basics of resistance measurements
- Resistance measurements in thin films

Why do we care about resistance measurements?

- device testing (continuity of a conductor,...)
- direct measurement of conductivity (versus temperature, pressure etc.) - material characterization
- indirect measurement of non-electrical quantities – electrical transducers (temperature, pressure, …)

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- inferring intrinsic properties of materials

Dependence of resistivity on temperature in magnetic metals:

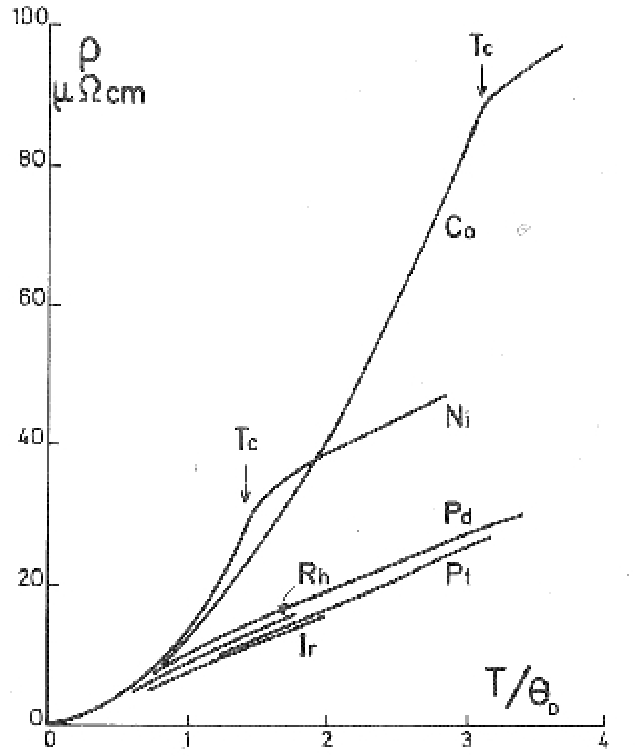

Fig. 10. Resistivity of several transition metals as a function of T/θ_D. θ_D is the Debye temperature.

image from: I.A. Campbell, A. Fert, in “Ferromagnetic Materials” 1982

- below Curie temperature T_c resistivity of a magnetic metals increases with temperature faster than above it

- below T_c temperature increase leads to increased magnetic disorder

- resistivity and magnetic order correlate

T_{Curie}:
Fe 1044 K
Co 1388 K
Ni 627 K

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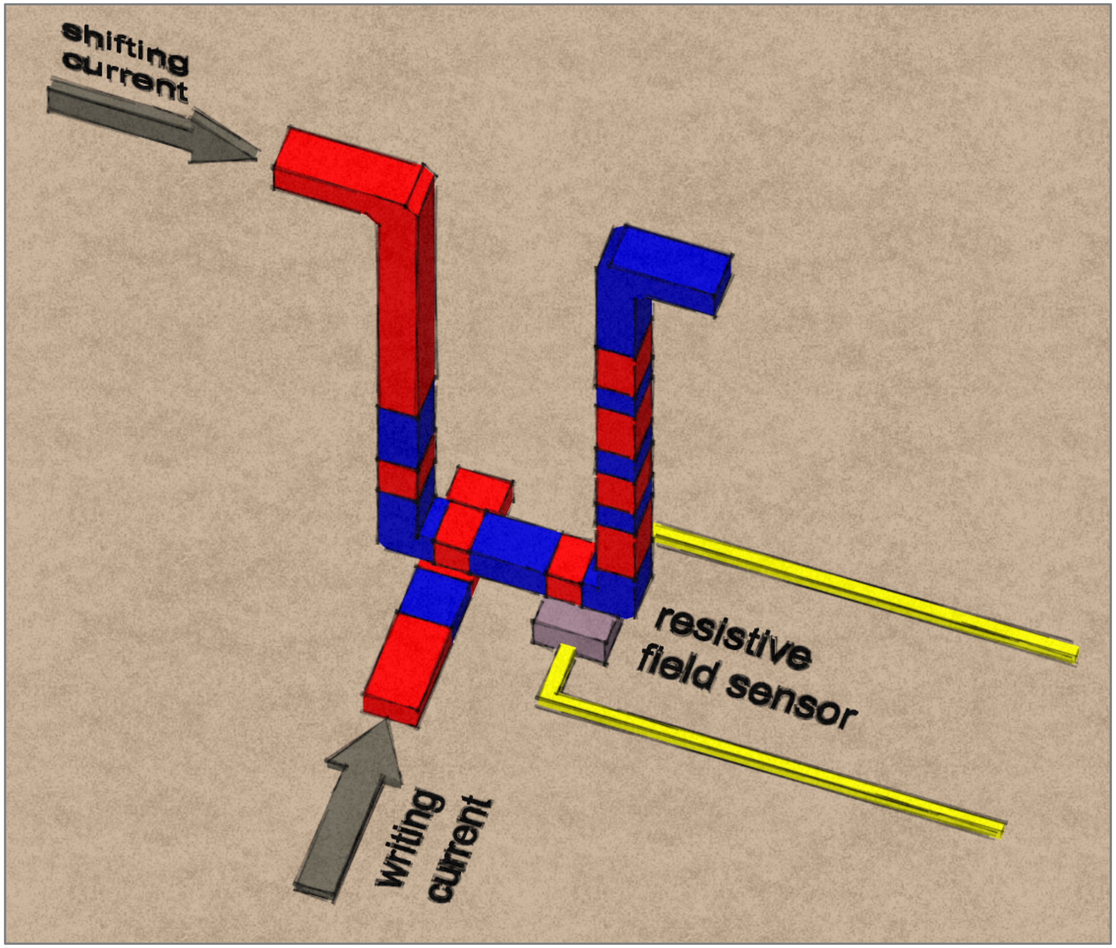
- indirect measurement of non-electrical quantities – electrical transducers (temperature, pressure, …)
- Inferring intrinsic properties of materials
- read out of information in devices

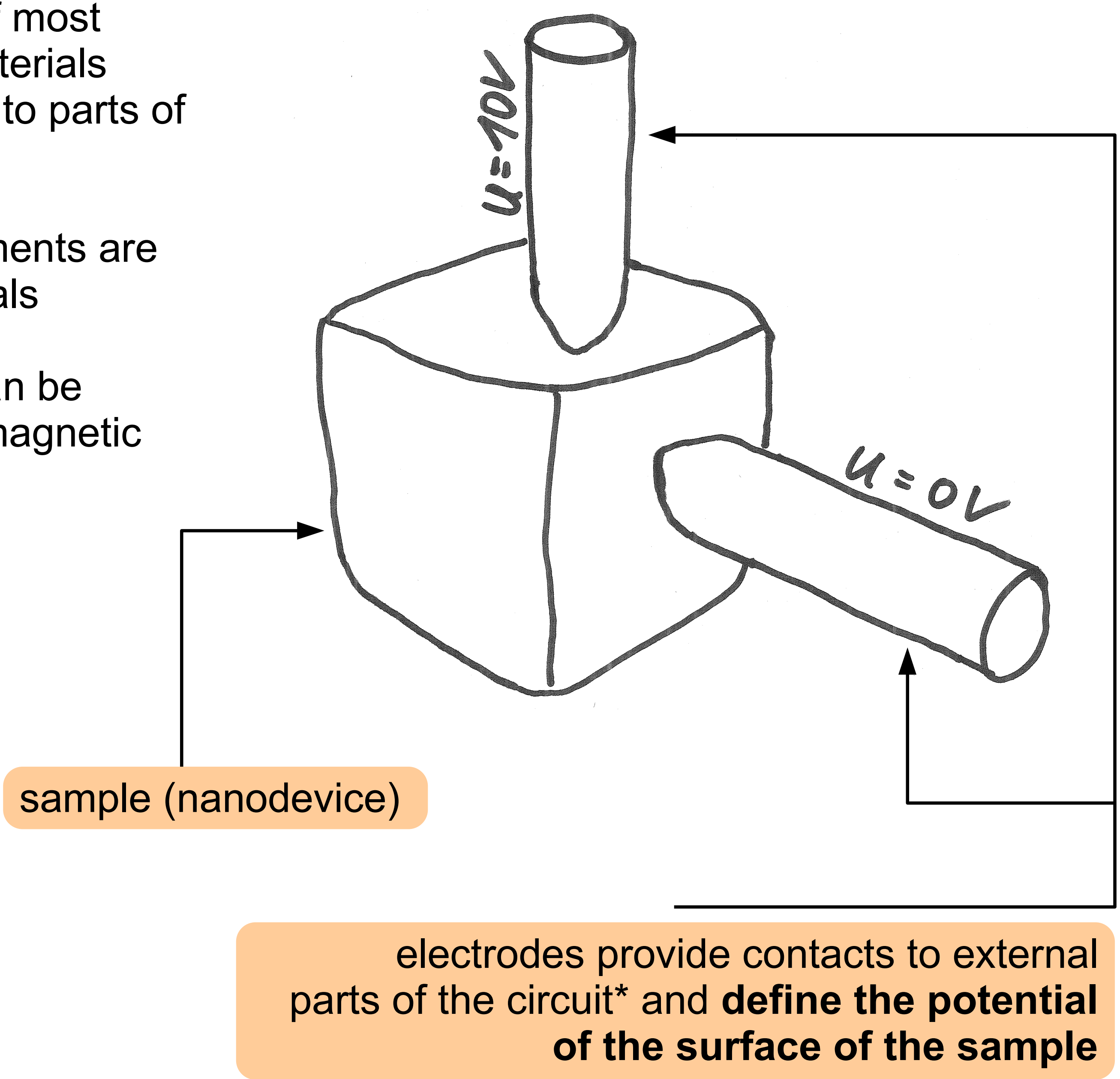


racetrack memory – S.S. Parkin, IBM

- the memory is of the shift register type – no direct access to bit
- the information is written into the “racetrack” by the stray fields of the domains controlled by the writing current
- to read the data the consecutive bits are moved by the shifting current to be detected by the resistive sensor (GMR, TMR)
- the resistive sensor detects stray fields of the domain walls
- the domain walls are pinned with the notches in the track (the wall stays preferentially between the notches because its length, and energy, is lower there)
- the memory needs one transistor for some 100 bits [22]

Basics of resistance measurements

- The practical applications of most nanoelectronics devices/materials require electric connections to parts of external circuits
- Electric conductivity experiments are used to characterize materials
- Many magnetic materials can be investigated using galvanomagnetic phenomena or with giant magnetoresistance



sample (nanodevice)

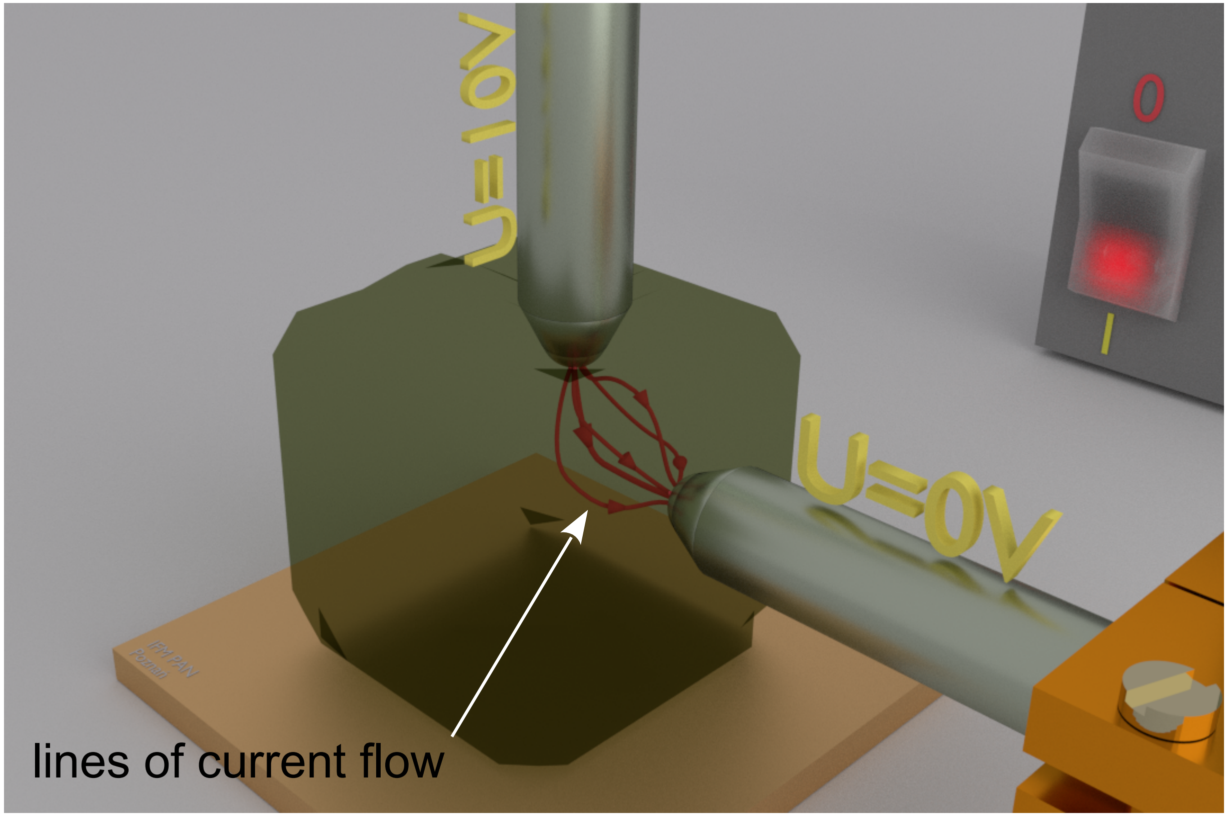
electrodes provide contacts to external parts of the circuit* and **define the potential of the surface of the sample**

*there are devices having more electrodes- for example transistor

Basics of resistance measurements

- The definition of Ohm involves knowledge of Volt

Volt (unit of potential difference and of electromotive force) — The volt is the potential difference between two points of a conducting wire carrying a constant current of 1 ampere, when the power dissipated between these points is equal to 1 watt. [1]



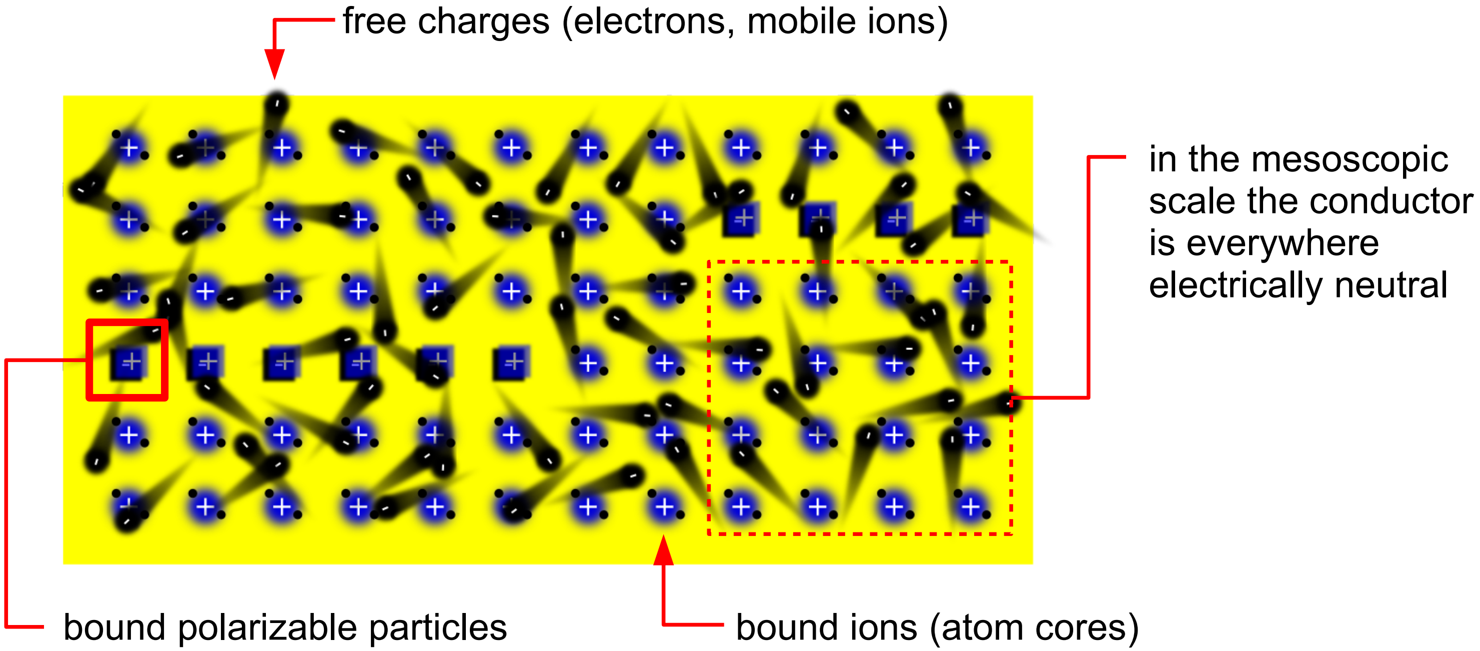
lines of current flow

Ohm (unit of electric resistance) — The ohm is the electric resistance between two points of a conductor when a constant potential difference of **1 volt**, applied to these points, produces in the conductor a current of 1 ampere, the conductor not being the seat of any electromotive force. [1]

[1] The International System of Units (SI) NIST SPECIAL PUBLICATION 330 2008 EDITION

Basics of resistance measurements

Field free electric conductor:

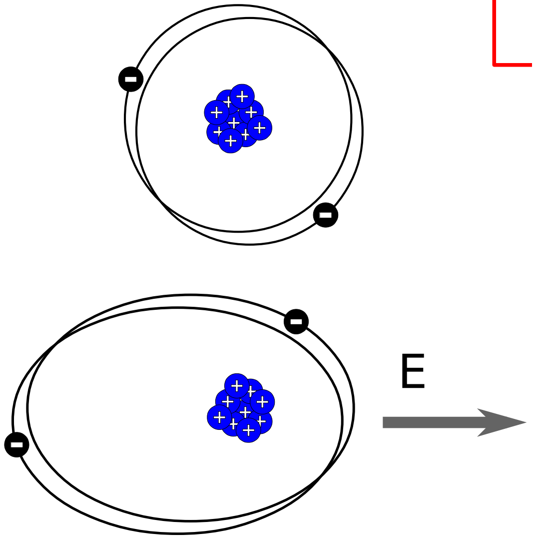


free charges (electrons, mobile ions)

in the mesoscopic scale the conductor is everywhere electrically neutral

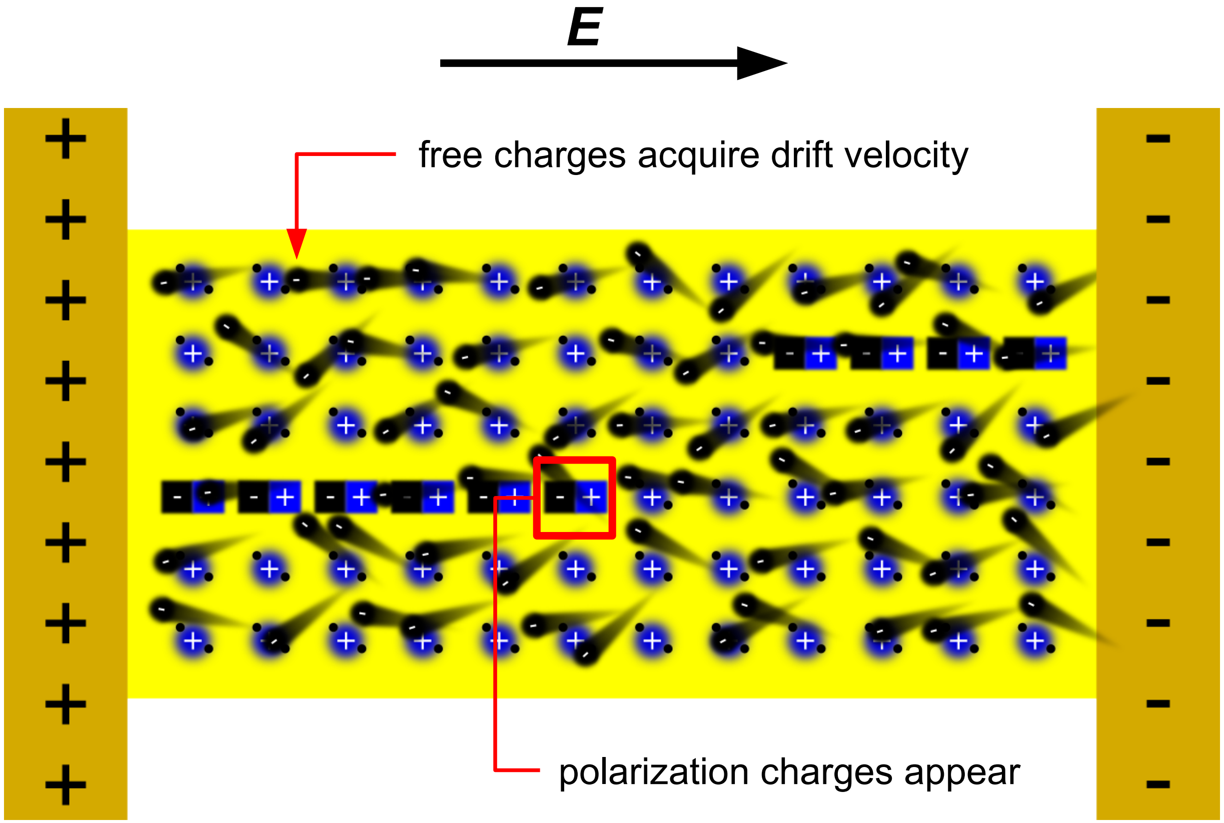
bound polarizable particles

bound ions (atom cores)



Basics of resistance measurements

If the externally imposed electric field is present:



free charges acquire drift velocity

polarization charges appear

$\nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}(\delta_0+\delta_b)$ δ_0 -free charges δ_b -bound charges

$\vec{P}=\alpha\vec{E}$ $\delta_0=\nabla\cdot\vec{D}$ $\delta_b=-\nabla\cdot\vec{P}$

*A.A. Kaufman, B.I. Anderson,….. [5], p. 73

Basics of resistance measurements

- Electric charge is conserved and current density satisfies charge conservation equation*:

$$\frac{\partial \rho}{\partial t}+\nabla \cdot \vec{j}=0$$

- We assume the steady state (long time after the the electrodes established definite potential on the conductor surface – charge density everywhere is constant). We have thus**:

$$\nabla \cdot \vec{j}=0$$

i.e. divergence is zero

Current density ***j***:

$$\vec{j}=\sum_i n_i q_i \vec{v}_i$$

, where n_i is the number of charges q_i in the unit volume that move with velocity v_i. The total charge crossing oriented surface element **ds** in unit time is given by:

$$I=\frac{dQ}{dt}=\vec{ds}\cdot\vec{j}$$

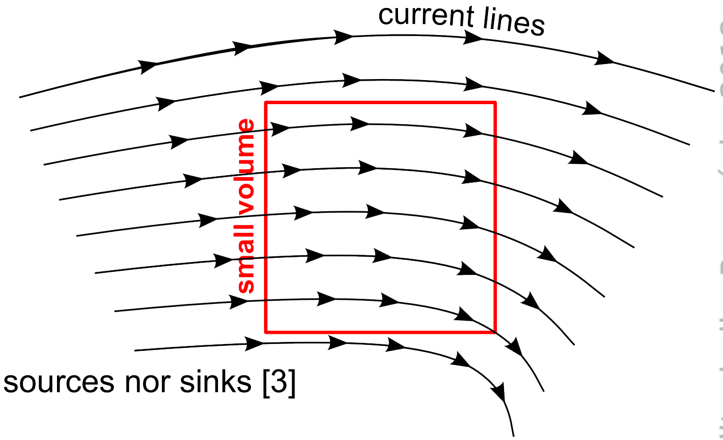
- Assuming for now that the conductor is **isotropic** we can obtain *j* knowing conductivity σ and electric field *E*:

$$\vec{j}=\sigma\vec{E}=-\sigma\nabla\Phi$$

$\vec{E}=-\nabla\Phi$, where Φ is a electric potential

- Combining the above two equations gives [2]:

$$\nabla\cdot\sigma\nabla\Phi=0$$



**fields with a property that divergence vanishes are called solenoidal, they have neither sources nor sinks [3]

* this part taken from: 6.013 Electromagnetism, H. A. Haus and J. R. Melcher, Massachusetts Institute of Technology, 1998 [2]

Basics of resistance measurements

- If conductivity is constant in the whole conductor we have:

$$\nabla^2 \Phi = 0$$

electric potential in a uniform conductor satisfies Laplace's equation

$$\nabla \cdot \sigma \nabla \Phi = 0$$

- Case of nonuniform, isotropic conductivity ($\sigma(\vec{r}) \neq const$)

$$\nabla \cdot \vec{j} = 0$$

$$\frac{\partial \rho}{\partial t} \text{ is still zero}$$

$$\nabla \cdot \sigma \vec{E} = 0$$

conductivity does depend on position and thus cannot be moved before nabla, we use formula for the derivative of the product of two functions [4]:

$$\nabla \cdot \sigma \vec{E} = (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial y}) \cdot \sigma (i E_x + j E_y + k E_z) = \frac{\partial}{\partial x}\sigma E_x + \frac{\partial}{\partial y}\sigma E_y + \frac{\partial}{\partial z}\sigma E_z =$$
$$\sigma\frac{\partial}{\partial x}E_x + E_x\frac{\partial}{\partial x}\sigma + \sigma\frac{\partial}{\partial y}E_y + E_y\frac{\partial}{\partial y}\sigma + \sigma\frac{\partial}{\partial z}E_z + E_z\frac{\partial}{\partial z}\sigma =$$
$$\sigma(\frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z) + E_x\frac{\partial}{\partial x}\sigma + E_y\frac{\partial}{\partial y}\sigma + E_z\frac{\partial}{\partial z}\sigma = \sigma \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \sigma$$

$$\sigma \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \sigma = 0 \qquad (1)$$

- We have Gauss' law for polarizable media:

$$\nabla \cdot \vec{D} = d_0$$

d_0 - free charges

, where d_0 is charge density and ***D*** is electric displacement vector

Basics of resistance measurements

- Recalling that ***D***=ε$\varepsilon_r\varepsilon_0$***E*** we get (transforming in the same way as $\nabla\cdot\sigma\vec{E}$ on the previous slide) for the density of unpaired charges [2]:

$$d_0=\varepsilon_r\varepsilon_0\nabla\cdot\vec{E}+\vec{E}\cdot\nabla\varepsilon_r\varepsilon_0 \longleftarrow \nabla\cdot\sigma\vec{E}=\sigma\nabla\cdot\vec{E}+\vec{E}\cdot\nabla\sigma \qquad (2)$$

- Substituting divergence of E from eq.(1) into eq.(2) we get for the density of unpaired charges [2, 5]:

$$d_0=-\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma+\vec{E}\cdot\nabla\varepsilon \qquad (2\mathrm{a}) \qquad \nabla\cdot\vec{E}=-\frac{1}{\sigma}\vec{E}\cdot\nabla\sigma \qquad \varepsilon=\varepsilon_r\varepsilon_0$$

- In an electrically uniform conductor (σ,ε =const) there are no unpaired free charges [2].
- We can rewrite the above equation (assuming constant ε_r) using the resistivity ρ [5]:

$$\rho=\frac{1}{\sigma}$$

$$d_0=-\varepsilon\rho\vec{E}\cdot\nabla\frac{1}{\rho}=-\varepsilon\rho\vec{E}\cdot\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial y}\right)\frac{1}{\rho}=-\varepsilon\rho\vec{E}\cdot\frac{-1}{\rho^2}\left(i\frac{\partial}{\partial x}\rho+j\frac{\partial}{\partial y}\rho+k\frac{\partial}{\partial y}\rho\right)$$

$$\boxed{d_0=\ \varepsilon\vec{E}\cdot\frac{1}{\rho}\nabla\rho}$$

note the dot product

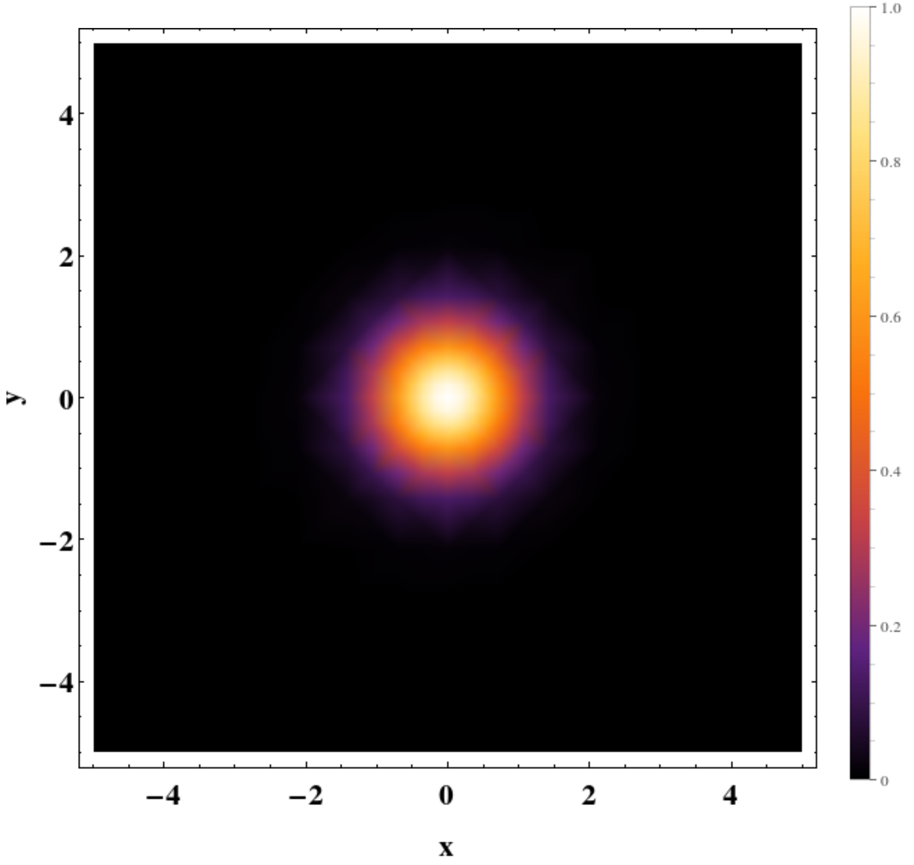
Basics of resistance measurements

- Volume charges arise in places where resistivity/conductivity gradient **is not perpendicular** to the electric field [5, p.102]:

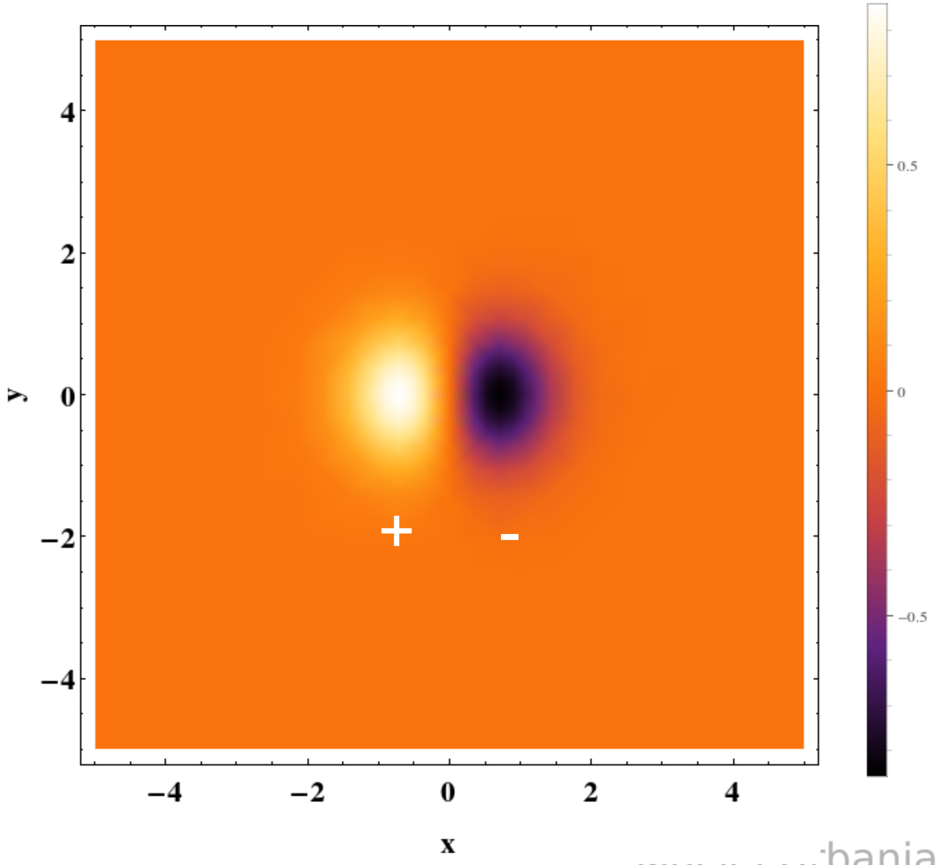
$$d_0 = \varepsilon \vec{E} \cdot \frac{\nabla \rho}{\rho} \tag{3}$$

Examples – no z-dependence:

Point defect – increased resistivity



Point defect – induced charge



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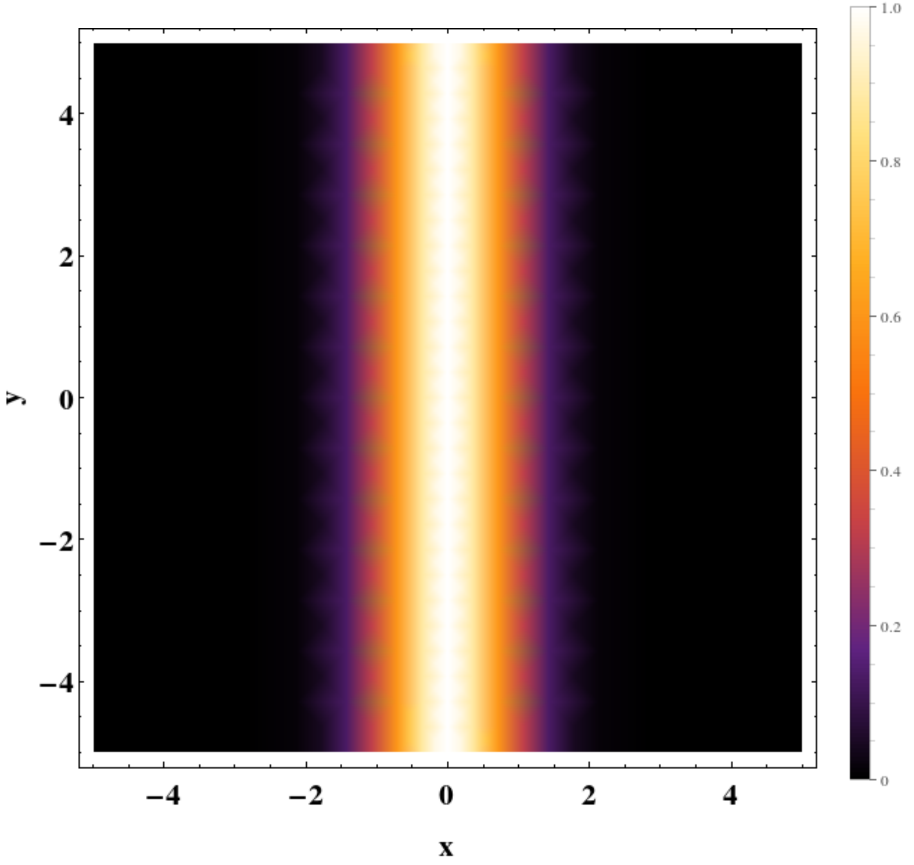
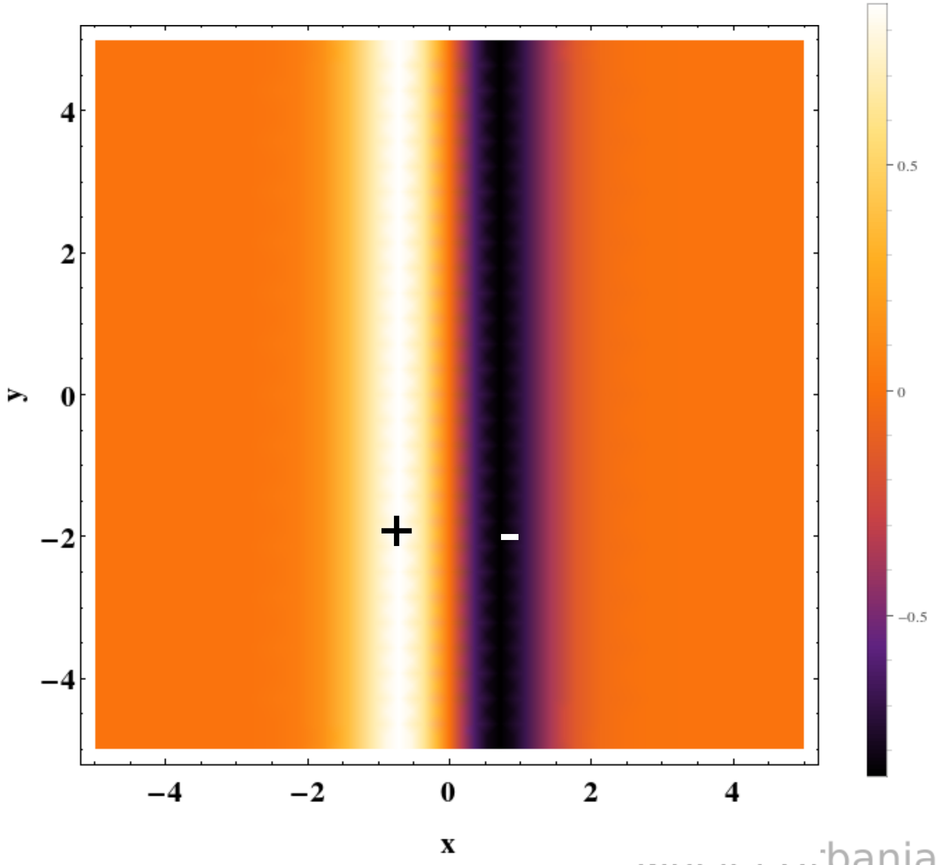
Examples – no z-dependence:

Linear defect – increased resistivity



Linear defect – induced charge



Basics of resistance measurements

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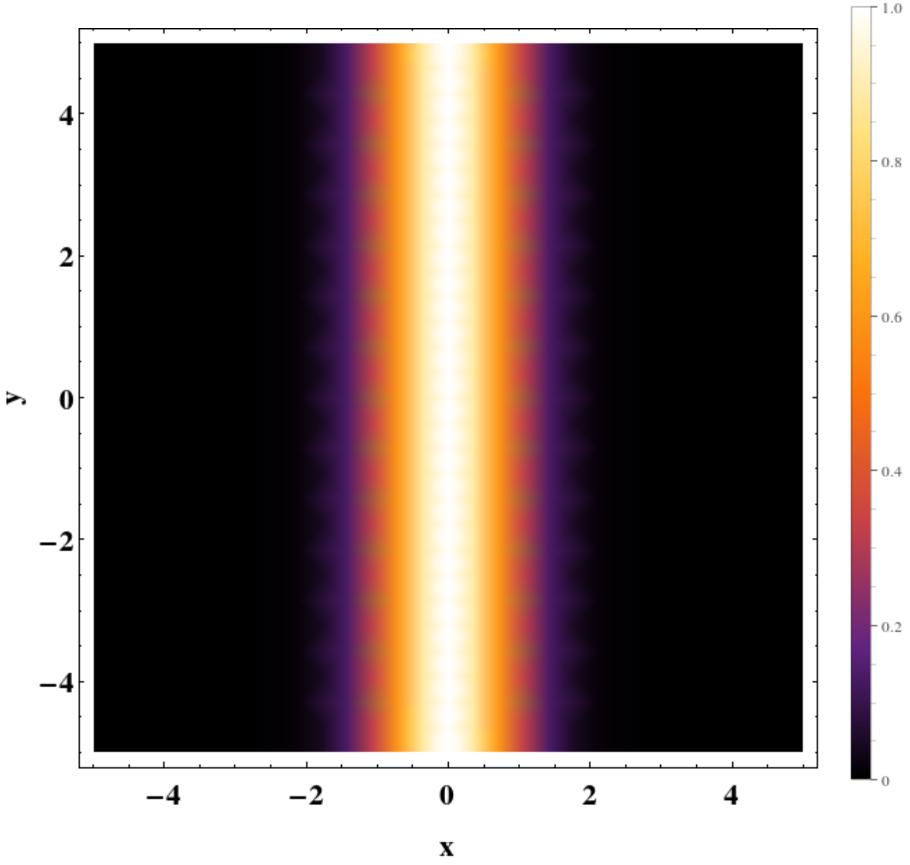
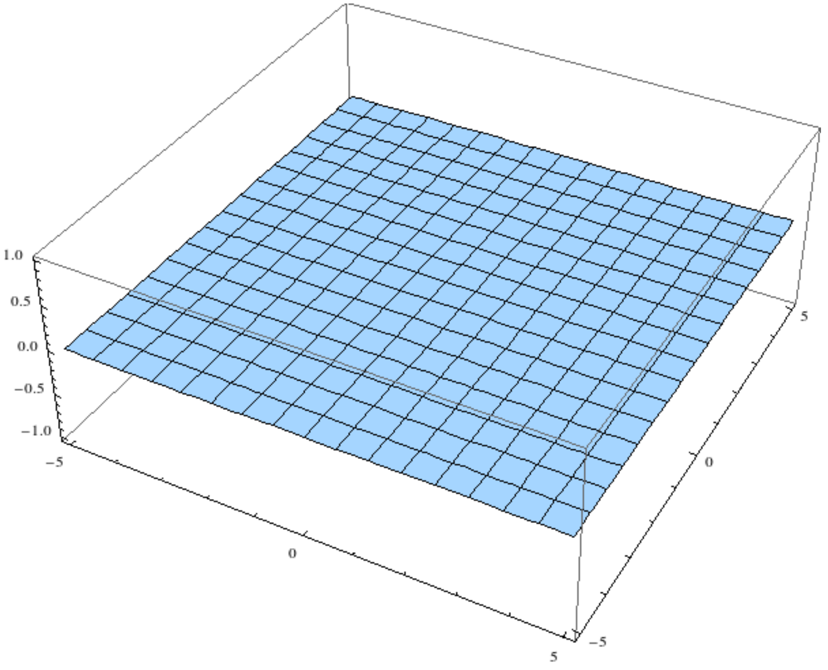


electric field is now perpendicular to resistivity gradient

Linear defect – **no induced charge**



Basics of resistance measurements

Surface charges at the interfaces where the conductivity changes [5]:

- In steady state (some time after the electric field was switched on) the normal component of current across any surface is continuous

$$j_n^{(2)} - j_n^{(1)} = \sigma_2 E_n^{(2)} - \sigma_1 E_n^{(1)} = 0$$

- From Gauss law we have:

$$\vec{E}^{(2)} - \vec{E}^{(1)} = \frac{d_s}{\varepsilon_0}$$

- Rewriting the first equation from the page:

$$\sigma_2 E_n^{(2)} - \sigma_1 E_n^{(1)} = \frac{1}{2}\left[(\sigma_1 + \sigma_2)\left(E_n^{(2)} - E_n^{(1)}\right) + (\sigma_2 - \sigma_1)\left(E_n^{(2)} + E_n^{(1)}\right)\right] = 0$$

and substituting the second one yields:

$$\frac{1}{2}\left[(\sigma_1 + \sigma_2)\frac{d_s}{\varepsilon_0} + (\sigma_2 - \sigma_1)\left(E_n^{(2)} + E_n^{(1)}\right)\right] = 0$$

$$\boxed{d_s = -\varepsilon_0 \frac{(\sigma_2 - \sigma_1)}{(\sigma_1 + \sigma_2)}\left(E_n^{(2)} + E_n^{(1)}\right)}$$

sum of unpaired free and bound (polarization) charges

Basics of resistance measurements

- The surface charge density can be calculated knowing the current density crossing the boundary:

$$\boxed{d_s=-\varepsilon_0\frac{(\sigma_2-\sigma_1)}{(\sigma_1+\sigma_2)}(E_n^{(2)}+E_n^{(1)})}=-\varepsilon_0\frac{(\sigma_2-\sigma_1)}{(\sigma_1+\sigma_2)}\left(\frac{j_n^{(2)}}{\sigma_2}+\frac{j_n^{(1)}}{\sigma_1}\right)=$$

continuity of the current through the interface:

$$j_n^{(2)}=j_n^{(1)}:=j_n$$

$$-\varepsilon_0\frac{(\sigma_2-\sigma_1)}{(\sigma_1+\sigma_2)}j_n\left(\frac{1}{\sigma_2}+\frac{1}{\sigma_1}\right)$$

$$\boxed{d_s=\varepsilon_0 j_n\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_1}\right)}$$

sum of unpaired free and bound (polarization) charges

Basics of resistance measurements

- **Total charge in steady flow** [5, p. 113]*. In polarizable medium we have (eq. (2)):

$$d_0 = \varepsilon \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \varepsilon$$

and for electric field vector:

$$\nabla \cdot \vec{E} = \frac{(d_0 + d_b)}{\varepsilon_0}$$

- d_b denotes density of bound charges, i.e., those originating from polarization (not the ones of resting atom ions providing conduction electrons)
- d_0 denotes the sum of local charge density of conducting particles (in our case usually electrons) and resting atom ions

- Substituting the second equation into the first one we get:

$$d_0 = \varepsilon \frac{(d_0 + d_b)}{\varepsilon_0} + \vec{E} \cdot \nabla \varepsilon \quad \rightarrow \quad d_b = d_0 \left(\frac{1 - \varepsilon_r}{\varepsilon_r} \right) - \frac{1}{\varepsilon_r} \vec{E} \cdot \nabla \varepsilon \qquad (4)$$

these expressions give the density of bound and free charges in polarizable conducting medium

- Equation (2a) reads:

$$d_0 = -\frac{\varepsilon}{\sigma} \vec{E} \cdot \nabla \sigma + \vec{E} \cdot \nabla \varepsilon \qquad (2a)$$

shown previously

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* this part taken almost literally from: Methods in Geochemistry and Geophysics, A.A. Kaufman, B.I. Anderson, Elsevier 2010 [5]

Basics of resistance measurements

- We collect volume charge expressions from previous slide

$$d_0 = -\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma + \vec{E}\cdot\nabla\varepsilon \qquad (2\mathrm{a}) \qquad\qquad d_b = d_0\left(\frac{1-\varepsilon_r}{\varepsilon_r}\right) - \frac{1}{\varepsilon_r}\vec{E}\cdot\nabla\varepsilon \qquad (4)$$

and consider three cases [5, p.113]:

1. Medium is homogeneous (both permittivity and conductivity are constant):

$$d_0 = -\frac{\varepsilon}{\sigma}\vec{E}\cdot\cancelto{0}{\nabla\sigma} + \vec{E}\cdot\cancelto{0}{\nabla\varepsilon} = 0 \qquad d_b = d_0\left(\frac{1-\varepsilon_r}{\varepsilon_r}\right) - \frac{1}{\varepsilon_r}\vec{E}\cdot\cancelto{0}{\nabla\varepsilon} = 0$$

There are neither free nor bound charges

2. Permittivity (polarizability) varies and conductivity is constant:

$$d_0 = -\frac{\varepsilon}{\sigma}\vec{E}\cdot\cancelto{0}{\nabla\sigma} + \vec{E}\cdot\nabla\varepsilon = \vec{E}\cdot\nabla\varepsilon \qquad d_b = d_0\left(\frac{1-\varepsilon_r}{\varepsilon_r}\right) - \frac{1}{\varepsilon_r}\vec{E}\cdot\nabla\varepsilon = -\vec{E}\cdot\nabla\varepsilon$$

the sum is zero

Both free and bound charges appear and **they compensate each other** ($d_0+d_b=0$).

Basics of resistance measurements

- We collect volume charge expressions from previous slide

$$d_0=-\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma+\vec{E}\cdot\nabla\varepsilon \qquad (2a)$$

$$d_b=d_0\left(\frac{1-\varepsilon_r}{\varepsilon_r}\right)-\frac{1}{\varepsilon_r}\vec{E}\cdot\nabla\varepsilon \qquad (4)$$

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$$d_b=d_0\left(\frac{1-\varepsilon_r}{\varepsilon_r}\right)-\frac{1}{\varepsilon_r}\vec{E}\cdot\nabla\varepsilon=-\vec{E}\cdot\nabla\varepsilon$$

the sum is zero

Both free and bound charges appear but **they compensate each other** ($d_0+d_b=0$).

3. Polarizability is constant and conductivity varies:

$$d_0=-\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma+\vec{E}\cdot\cancelto{0}{\nabla\varepsilon}=-\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma$$

$$d_b=d_0\left(\frac{1-\varepsilon_r}{\varepsilon_r}\right)-\frac{1}{\varepsilon_r}\vec{E}\cdot\cancelto{0}{\nabla\varepsilon}=\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma\left(1-\frac{1}{\varepsilon_r}\right)$$

$$d_0+d_b=-\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma+\frac{\varepsilon}{\sigma}\vec{E}\cdot\nabla\sigma\left(1-\frac{1}{\varepsilon_r}\right)=\varepsilon_0\frac{\vec{E}\cdot\nabla\sigma}{\sigma}$$

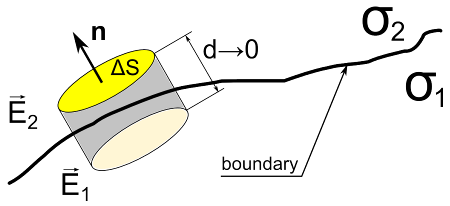
Both free and bound charges appear but **their sum does not depend on permittivity**.

Basics of resistance measurements

- Boundary conditions [2]*

From the solenoidity of the current vector ($\nabla \cdot \vec{j} = 0$) and divergence theorem we have:

$$\nabla \cdot \sigma \vec{E} = 0 \quad \rightarrow \quad \vec{n} \cdot (\sigma_1 \vec{E}_1 - \sigma_2 \vec{E}_2) = 0 \qquad (5)$$



To get eg. (5) we integrate the product σ E only over the yellow surfaces as d tends to zero.

- If one region is insulating (σ≈0) the normal component of ***E*** in the conductor is negligible and **the current flows parallel to the boundary**:

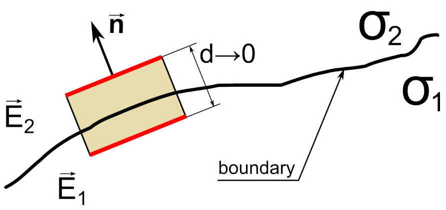
$$\vec{n} \cdot (\sigma_1 \vec{E}_1 - 0) = 0 \quad \rightarrow \quad \vec{E}_1 = 0$$

In quasi-static approximation (no time changing fields) we have from Maxwell's equations:

$$\nabla \times \vec{E} = 0$$

which together with Stokes' theorem leads to:

$$\vec{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$



the integral of E along the border of the rectangle must be zero since the curl of E is zero

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* this part taken from: 6.013 Electromagnetism, Herman A. Haus and James R. Melcher, Massachusetts Institute of Technology, 1998 [2]

Basics of resistance measurements

- One dimensional conductor [2]*. From previous slides we have:

$$\nabla \cdot \sigma \nabla \Phi = 0$$

- Since, as assumed, the properties of the conductor do not depend on y and z coordinates we have:

$$\frac{d}{dx}\left(\sigma \frac{d}{dx}\Phi\right) = 0$$

- It follows from the above equation that (C_1 is some constant):

$$\sigma \frac{d}{dx}\Phi = C_1 \qquad \leftarrow \qquad \frac{d}{dx}C_1 = 0$$

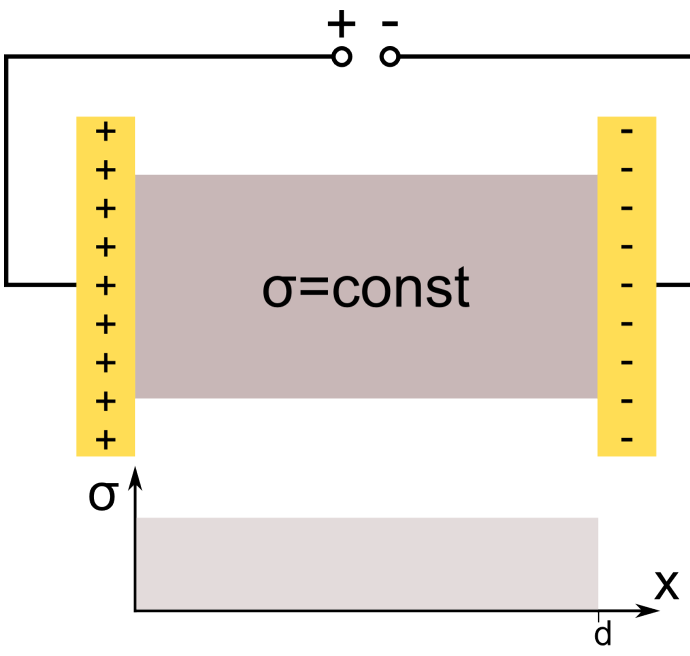
- Integrating we get:

$$\int d\Phi = \int \frac{C_1}{\sigma}dx \quad \rightarrow \quad \Phi + C_2 = \int \frac{C_1}{\sigma}dx \quad \underset{C_3=-C_2}{\rightarrow} \quad \Phi = C_3 + \int \frac{C_1}{\sigma}dx$$

- If conductivity is constant we have:

$$\Phi = C_3 + \frac{C_1}{\sigma}x$$

← potential within the conductor



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* this part taken from: 6.013 Electromagnetism, Herman A. Haus and James R. Melcher, Massachusetts Institute of Technology, 1998 [2]

Basics of resistance measurements

- From boundary conditions (potentials at the electrodes) we have:

$$\Phi(x=0)=U \qquad \Phi(x=d)=0$$

$$\Phi=C_3+\frac{C_1}{\sigma}x: \qquad U=C_3+\frac{C_1}{\sigma}\times 0 \qquad 0=U+\frac{C_1}{\sigma}d$$

- It follows that constants are:

$$C_1=-U\frac{\sigma}{d} \qquad C_3=U$$

and the potential inside the conductor is give by:

$$\Phi=C_3+\frac{C_1}{\sigma}x$$

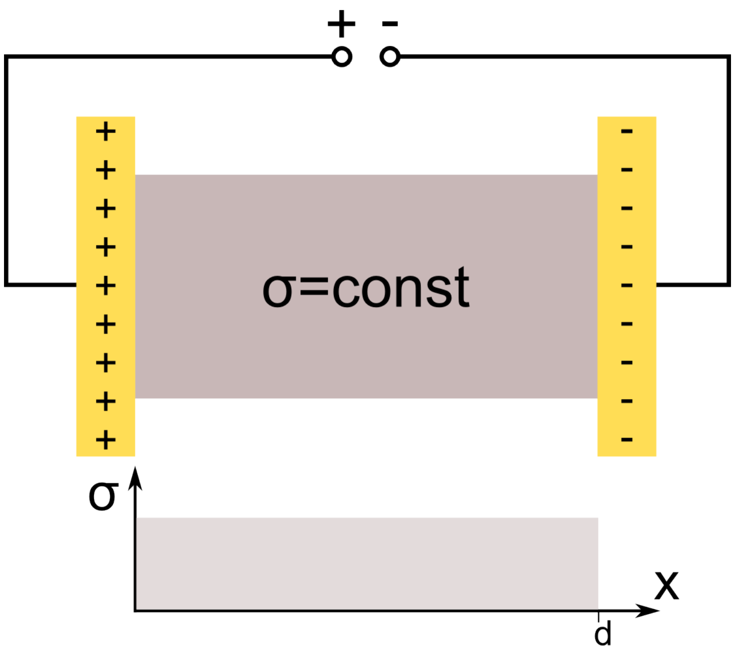
$$\Phi=U\left(1-\frac{x}{d}\right)$$

- Thus **the electric field inside the conductor is constant** and given by:

$$E_x=-\frac{d}{dx}\Phi=\frac{d}{dx}U\left(1-\frac{x}{d}\right)=\frac{U}{d}$$

- The associated current density is:

$$j_x=\sigma E_x=\sigma\frac{U}{d}$$



If cross section area of the conductor is *S* then its resistance is:

$$\boldsymbol{R}=\frac{U}{I}=\frac{U}{j_x S}=\frac{1}{\sigma}\frac{d}{S}=\rho\frac{\boldsymbol{d}}{\boldsymbol{S}}$$

Basics of resistance measurements

- **Piece-wise uniform** one dimensional conductor [2]*

We can write for uniform pieces of the conductor:

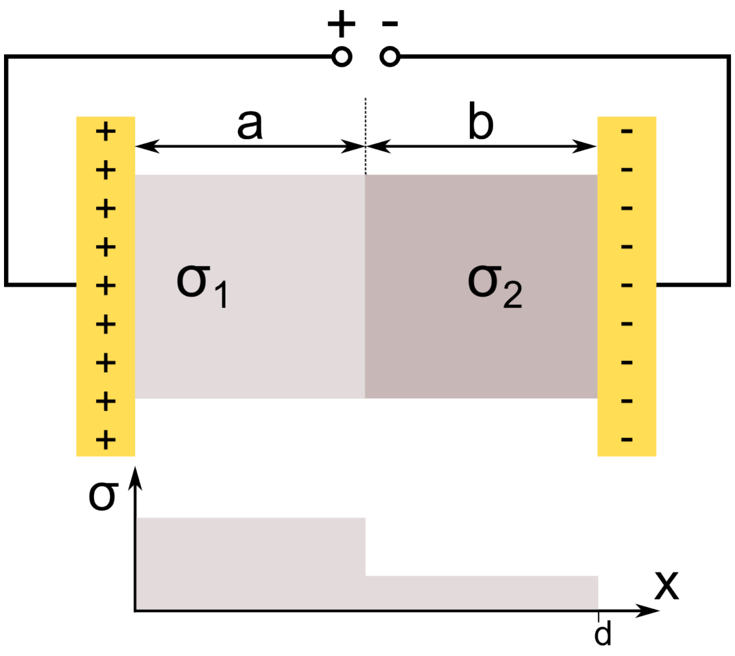
$$j_1=\sigma_1 E_x^{(1)}=\sigma_1\frac{U_1}{a} \qquad j_2=\sigma_2 E_x^{(2)}=\sigma_2\frac{U_2}{b}$$

- In steady state both current densities must be equal and the sum of potential drops within both pieces must be equal to external voltage *U*. We get thus a set of equations:

$$\sigma_1\frac{U_1}{a}=\sigma_2\frac{U_2}{b} \qquad U_1+U_2=U$$

- Solving for U_1 and U_2 we have:

$$U_1=U\frac{a\sigma_2}{b\sigma_1+a\sigma_2} \qquad U_2=U\frac{b\sigma_1}{b\sigma_1+a\sigma_2}$$



and for electric filed within the regions (which is uniform in a uniform conductor):

$$E_1=U\frac{\sigma_2}{b\sigma_1+a\sigma_2} \qquad E_2=U\frac{\sigma_1}{b\sigma_1+a\sigma_2}$$

Note that in the region with higher conductivity the electric field is weaker than in the one with lower conductivity

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* this part taken from: 6.013 Electromagnetism, Herman A. Haus and James R. Melcher, Massachusetts Institute of Technology, 1998 [2]

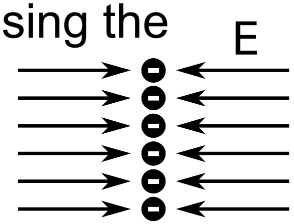
Basics of resistance measurements

- Notice too, that the difference in electric field value on both sides of the interface comes **from the charge accumulation** there.

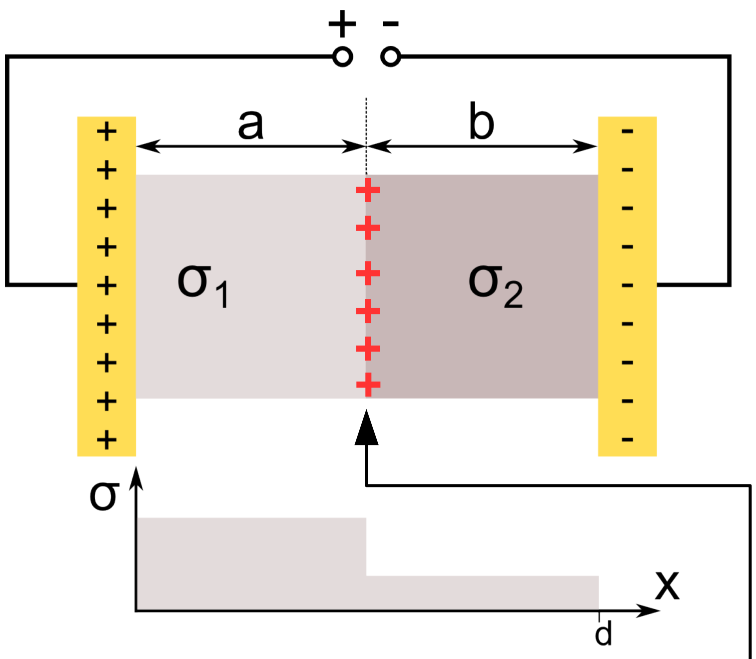
From Gauss law we have, for the electric field of a uniformly charged plane:

$$E=\frac{d}{2\varepsilon_0}$$

And the field jump associated with crossing the interface is:



$$\Delta E=\frac{d}{\varepsilon_0}$$



- Comparing this with the electric field jump in our composite conductor we get:

$$E_2-E_1=U\frac{\sigma_1-\sigma_2}{b\sigma_1+a\sigma_2}=\frac{d}{\varepsilon_0} \rightarrow d=\varepsilon_0 U\frac{\sigma_1-\sigma_2}{b\sigma_1+a\sigma_2}$$

surface density of charges accumulated at the interface

- From the previous slide we have:

$$j=j_2=j_1=\sigma_1 E_1=U\frac{\sigma_1\sigma_2}{b\sigma_1+a\sigma_2}$$

$$\boxed{d=\varepsilon_0 j\frac{\sigma_1-\sigma_2}{\sigma_1\sigma_2}}$$

Basics of resistance measurements

- Let us calculate the surface density of the charges accumulated at the junction between 1mm diameter **copper** (very good conductor) and aluminum* wires (good conductor) if a 1 A current flows through.

$\sigma_{Cu} \approx 6 \times 10^7 \, S/m$ $\sigma_{Al} \approx 3.5 \times 10^7 \, S/m$

constant value from NIST (http://physics.nist.gov/cuu/Constants/index.html)

$\varepsilon_0 = 8.854\,187\,817... \times 10^{-12} \, F\, m^{-1}$

$e^- = 1.602\,176\,565 \times 10^{-19} \, C$

- Current surface density is: $j = I / \pi r^2 \approx 1.27 \times 10^6 \, A/m^2$

  and using the expression from the previous slide we have:

$$d = \varepsilon_0 \, j \frac{\sigma_2 - \sigma_1}{\sigma_1 \sigma_2} \approx -1.342 \times 10^{-13} \;\; C\, m^{-2}$$

, i.e., some 1 million electrons per square meter which really is negligible.

- Even in nanodevices where current densities can be significantly higher (of the order of 10^{14} A/m^2) the charge accumulation plays no important role in the interconnects between conductors.

---

* both copper and aluminum are used in interconnects in integrated circuits

Basics of resistance measurements

- In many practical applications the size and the shape of **the samples** restricts the choice of positions where the electrodes can be placed

sometimes the samples are really biG

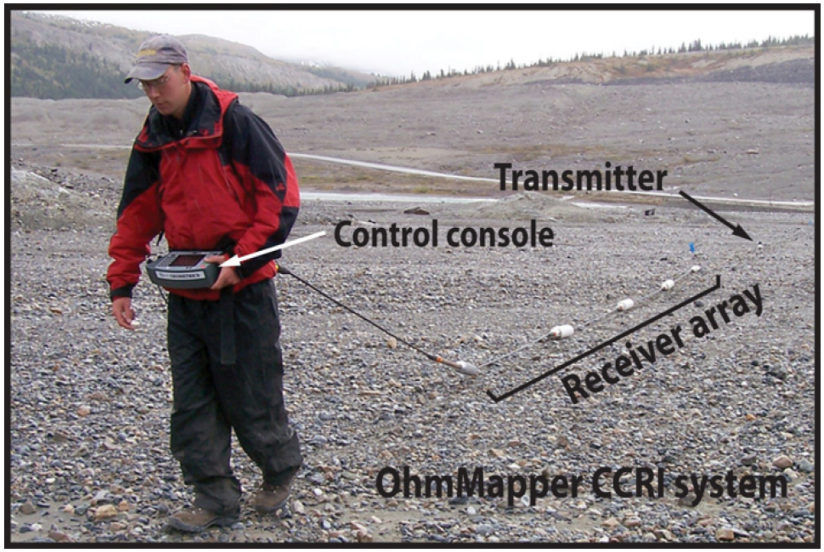


Figure 4. OhmMapper CCRI system being towed while conducting a resistivity survey (figure4.jpg). **

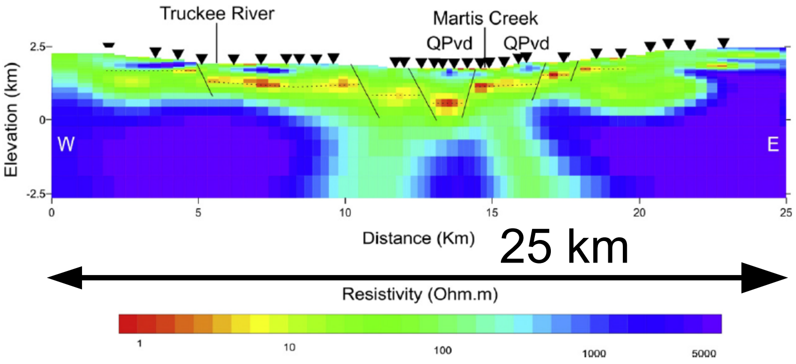


Fig. 7. Crustal resistivity section from MT measurements. Solid and dotted lines denote inferred extensional faults and offset depositional horizon, respectively. The thin low-resistivity surface layer beneath Martis Creek represents the Prosser Creek alluvium, imaged in greater detail by the ERT and TEM measurements. Geology as in Fig. 2.

...and its better to drive over your sample

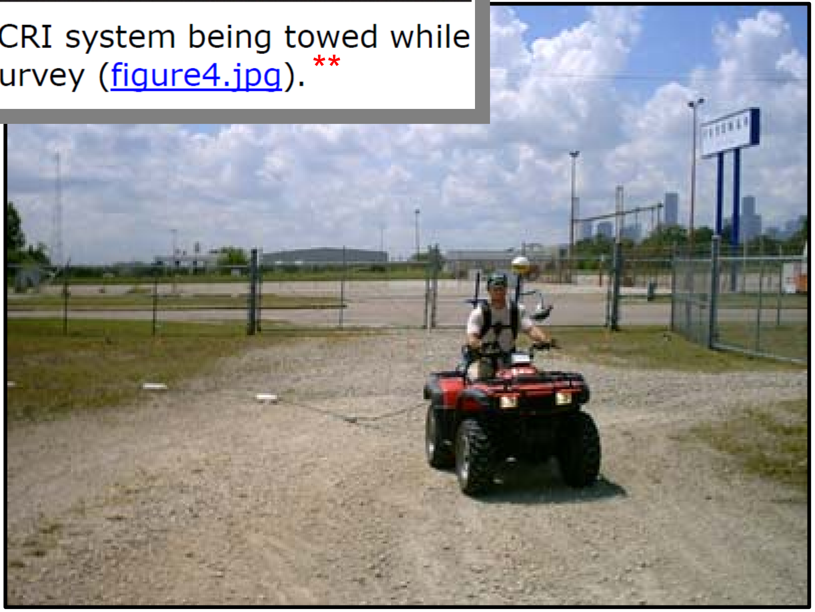


image from US Geological Service: http://water.usgs.gov/ogw/bgas/profiles/Feb2004-NE.html retrieved 2013.08.01

** image from: P.A. Wainstein, J.M. Wan Bun Tseung, B.J. Moorman, C.W. Stevens, Mars 4, 1 (2008) doi: 10.1555/mars.2008.0001

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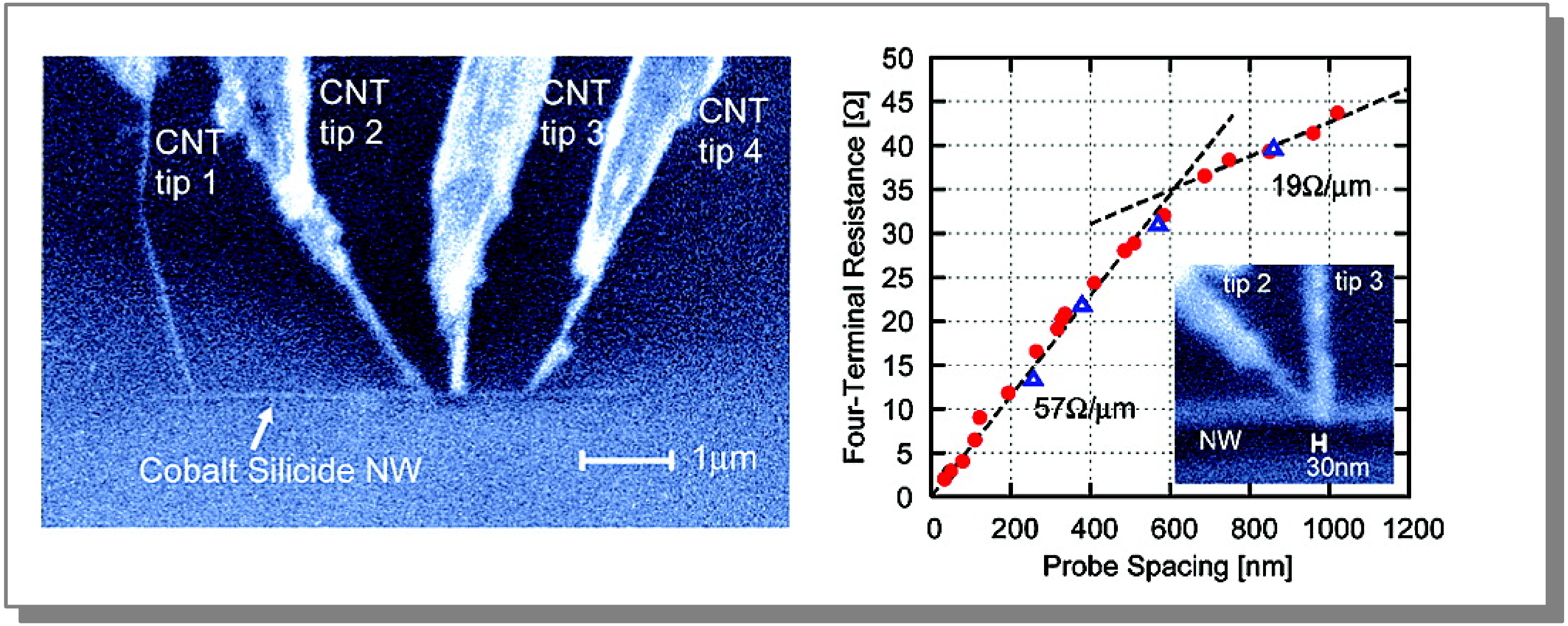
* image from P.A. Bedrosian, B.L. Burton, M.H. Powers, B.J. Minsley,J.D. Phillips, L.E. Hunter, Journal of Applied Geophysics **77**, 7 (2012)

Basics of resistance measurements

- In many practical applications the size and shape of **the samples** restricts the choice of positions where the electrodes can be placed

sometimes the samples are really small



- Electrodes: PtIr-coated carbon nanotube tips, 4 point probe (see later in the lecture)
- Sample: $CoSi_2$ nanowire with width no more than 160±20 nm
- The minimum probe spacing on the NW was **30±20 nm**
- Commercially available probes have about 5 µm probe spacing but the are much more robust (see for example *M4PP Micro Four-Point Probe* from Kleindiek).

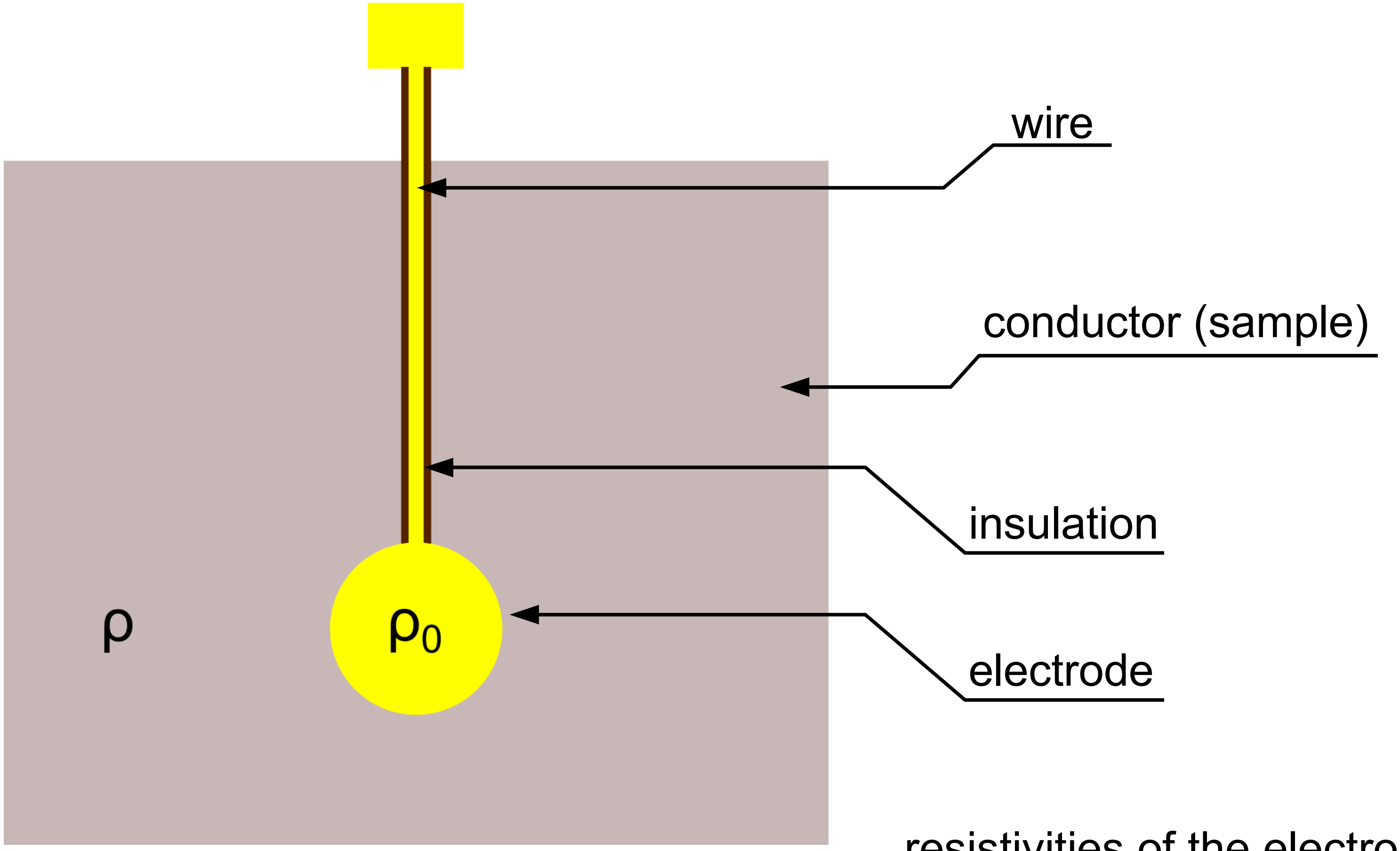
www.nanotechnik.com

S.Yoshimoto *et al.*, NANO LETTERS **7**, 956 (2007) [7]

Basics of resistance measurements

Four point probe

- let us consider the isotropic spherical current electrode in a uniform and isotropic conducting medium (p.104 of [5])
- the electrode is connected with an isolated wire to the outside world (we neglect the contact surface of the wire with the electrode)

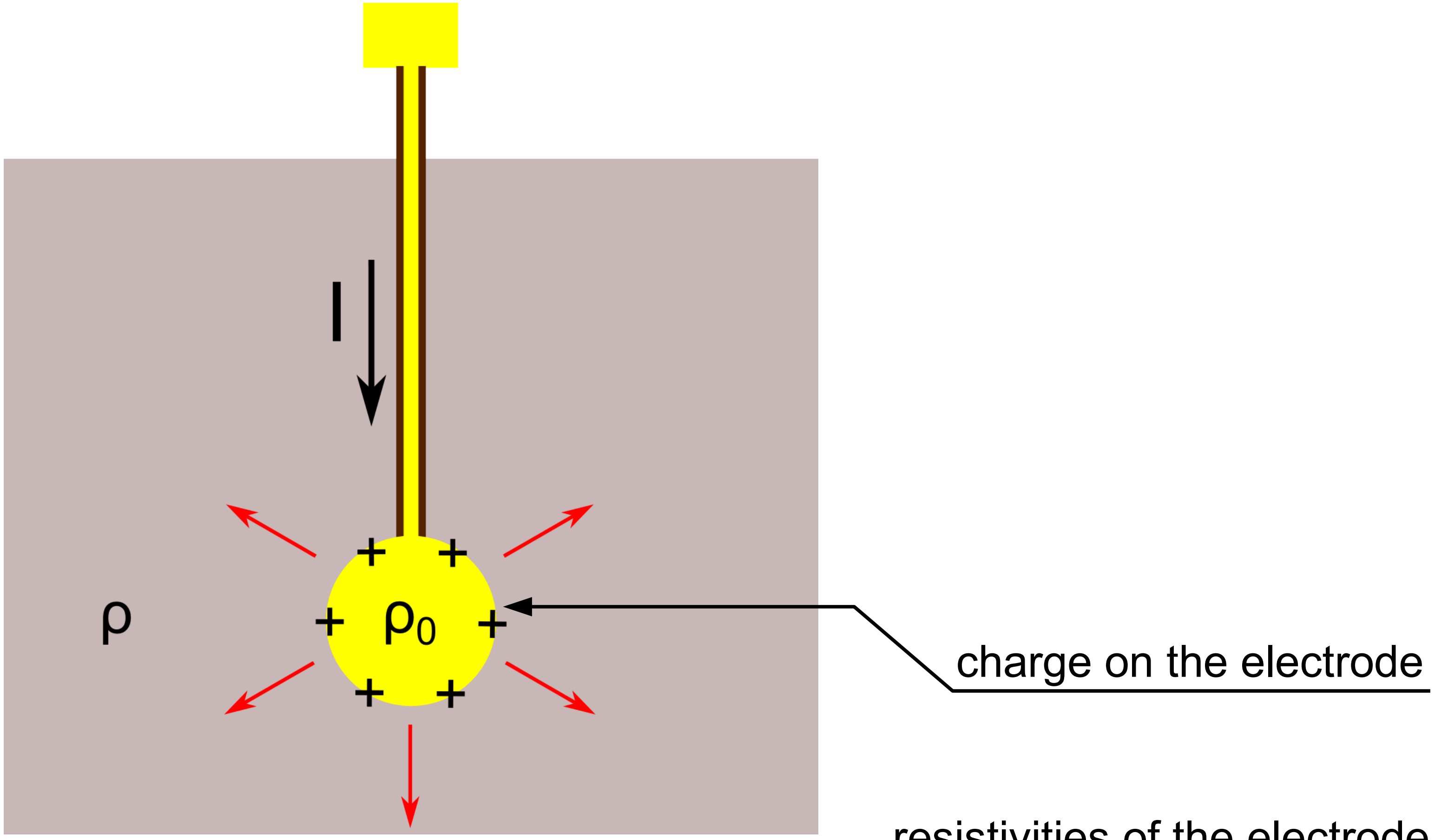


resistivities of the electrode and the sample are different

Basics of resistance measurements

Four point probe

- let us suppose we are feeding a current *I* from the electrode to the conductor
- we know from previous slides that the charges appear where there is a change in conductivity; in our case on the surface of the electrode



charge on the electrode

resistivities of the electrode and the sample are different

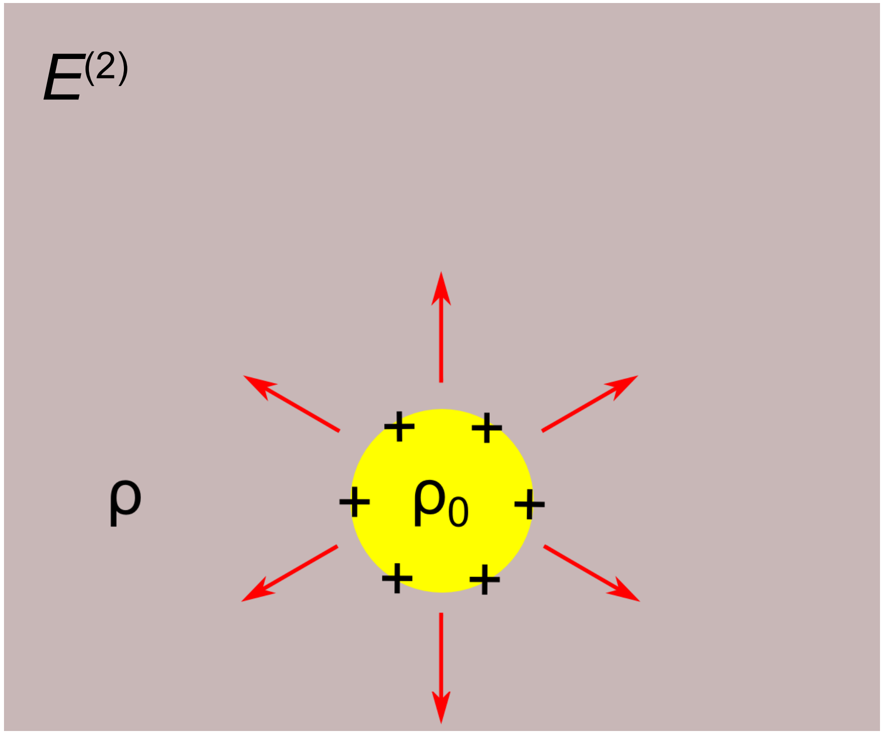
Basics of resistance measurements

Four point probe

- neglecting the cross section of the wire and using a symmetry of the problem we find that electric field has **only radial components** and that its discontinuity at the surface of the electrode is given by:

$$E_r^{(1)} - E_r^{(2)} = \frac{d}{\varepsilon_0}$$

(1) - inside the electrode, (2) - inside the probe



- From the continuity of the current across the surface of the electrode we have:

$$j_r^{(1)} = j_r^{(2)} := j_r$$

$$\vec{j} = \vec{E}\sigma = \frac{\vec{E}}{\rho}$$

$$j_r \rho_1 - j_r \rho_2 = \frac{d}{\varepsilon_0}$$

$$d = \varepsilon_0 j_r (\rho_1 - \rho_2)$$

- The total charge on the electrode is:

$$Q = \oint_s d\, ds = \oint_s \varepsilon_0 j_r (\rho_1 - \rho_2)\, ds$$

$$Q = \varepsilon_0 (\rho_1 - \rho_2) I$$

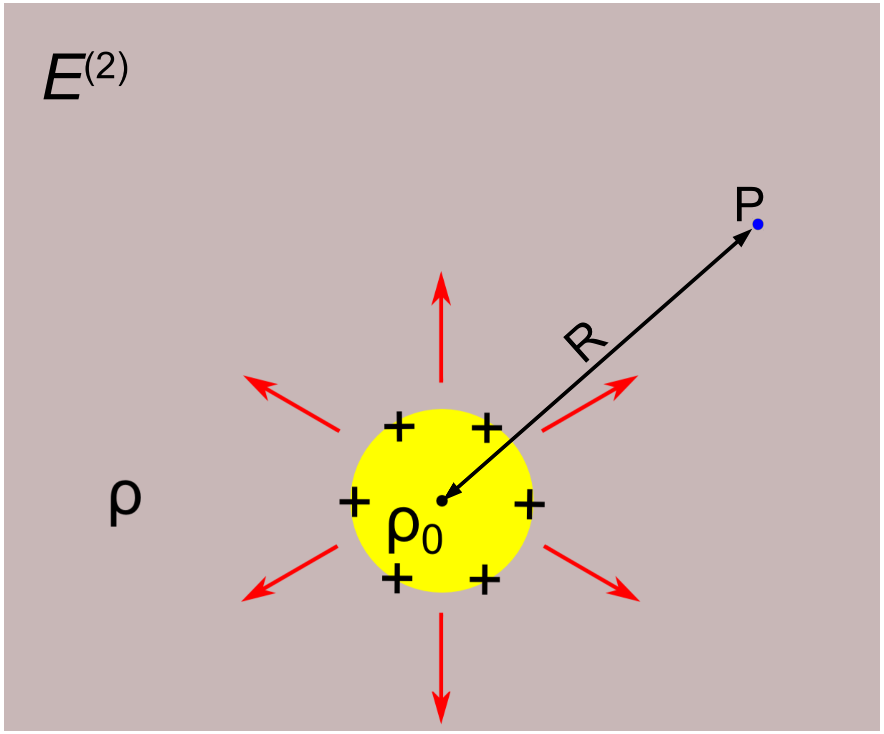
total current

Basics of resistance measurements

Four point probe

- neglecting the cross section of the wire and using a symmetry of the problem we find that electric field has only radial components and that its discontinuity at the surface of the electrode is given by:

$$E_r^{(1)} - E_r^{(2)} = \frac{d}{\varepsilon_0}$$



- From Gauss law we have for the field due to the charge on the electrode:

$$\oint_s \vec{E}\cdot ds = \frac{Q}{\varepsilon_0} \quad \rightarrow \quad 4\pi R^2 E_r = \frac{\varepsilon_0(\rho_1-\rho_2)I}{\varepsilon_0}$$

$$\boxed{E_r = \frac{(\rho_1-\rho_2)}{4\pi R^2} I}$$

additional field due to the current flow

note that ***R*** is the distance from the electrode center to the observation point and not the radius of the electrode

Basics of resistance measurements

Four point probe

- consider an electrode driven into the conductor, as shown below [8]
- symmetry of the problem and the high resistivity of the upper half-space results in current flowing radially out of the electrode
- Equipotential lines form a set of concentric hemispheres
- We assume the conducting medium to be **isotropic** so the density of a current crossing a hemisphere with radius R is:

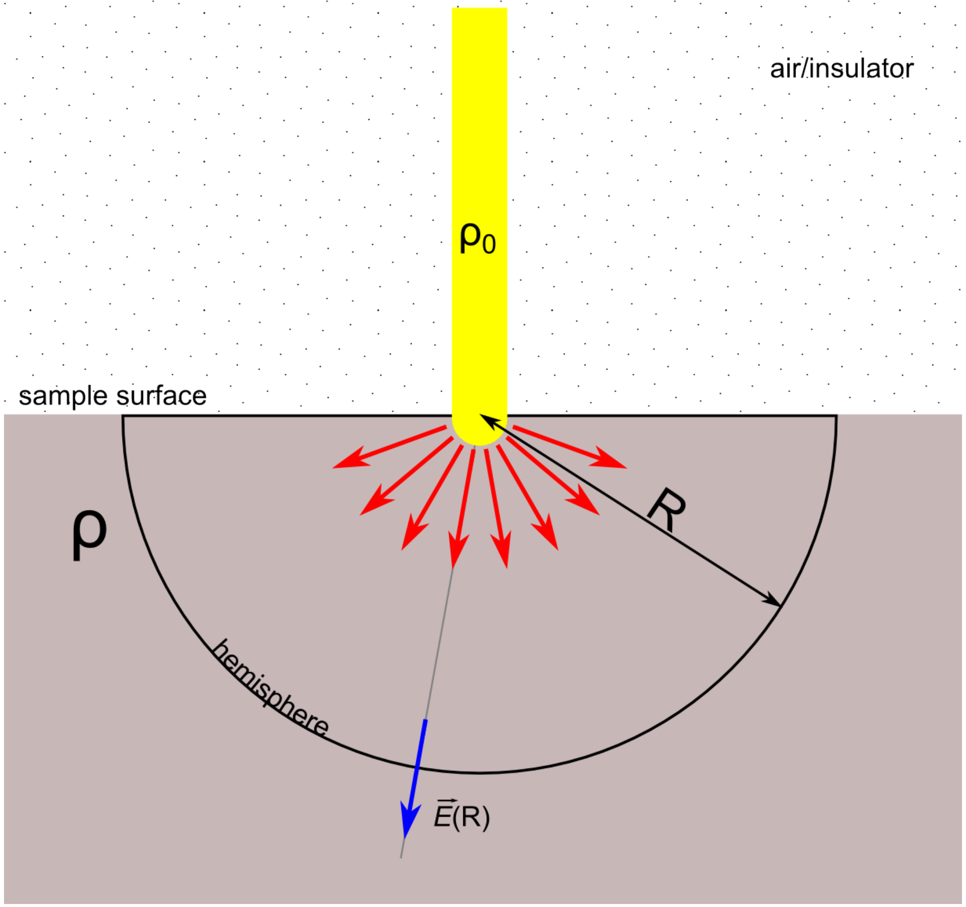
$$\vec{j}=\frac{\vec{E}_r}{\rho}$$

- The total current flowing through the hemisphere is:

$$I=\frac{1}{2}4\pi R^2 j=\frac{1}{2}4\pi R^2\frac{E_r}{\rho}$$

- It follows that the electric field at R is:

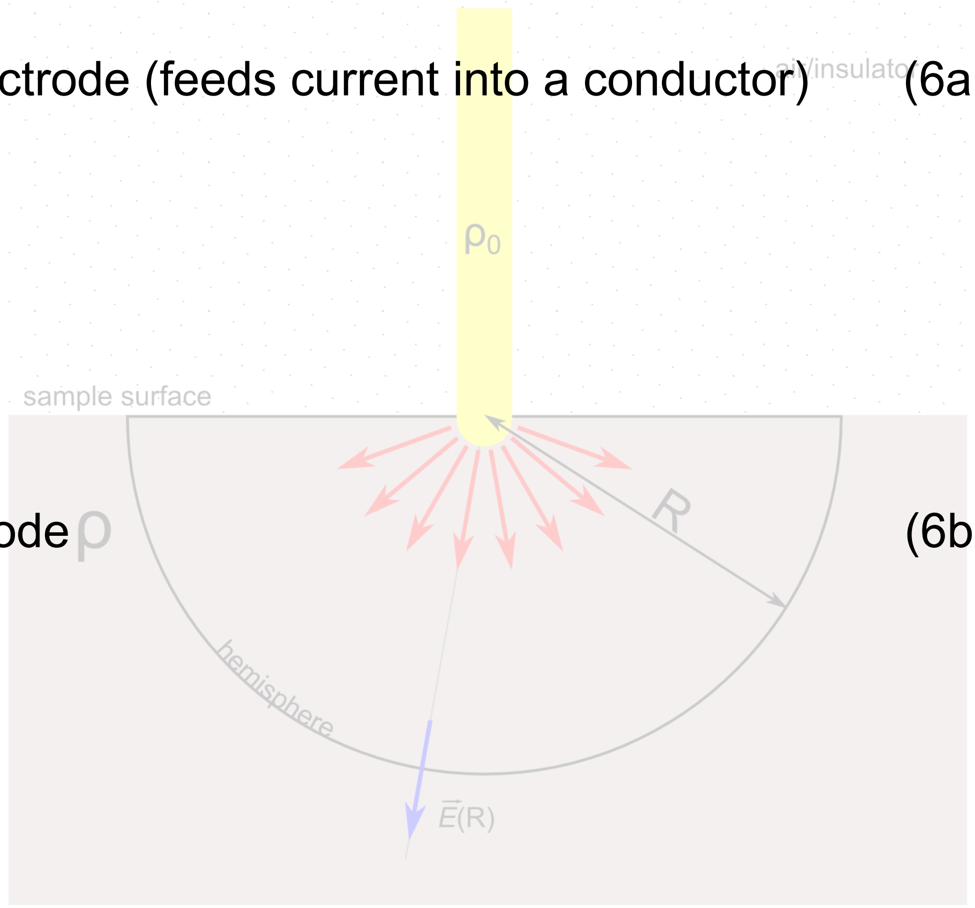
$$E_r=\frac{\rho}{2\pi R^2}I$$



Basics of resistance measurements

- … and potential at R (potential at infinity is assumed to be zero):

$$U=\int_{R}^{\infty} E_r(R)\,dR=\int_{R_0}^{\infty}\frac{\rho}{2\pi R^2}I\,dR=\frac{\rho}{2\pi R_0}I$$

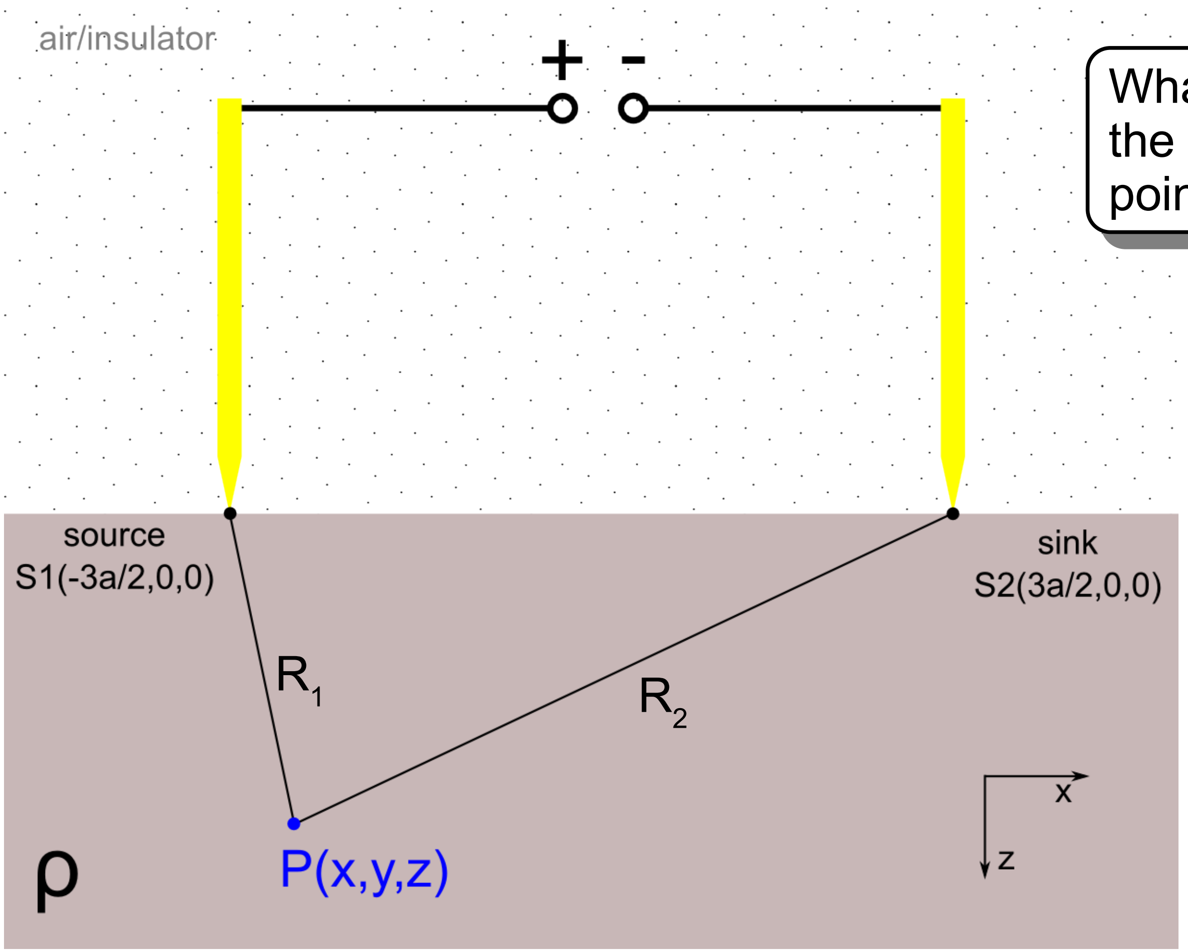


$$\boxed{U=\frac{\rho}{2\pi R}I}$$ potential of a source electrode (feeds current into a conductor) (6a)

$$\boxed{U=-\frac{\rho}{2\pi R}I}$$ potential of a sink electrode (6b)

Basics of resistance measurements

- We have now two **point electrodes*** resting on a homogeneous and isotropic conductor occupying a half-space (the rest is insulating) [8]:



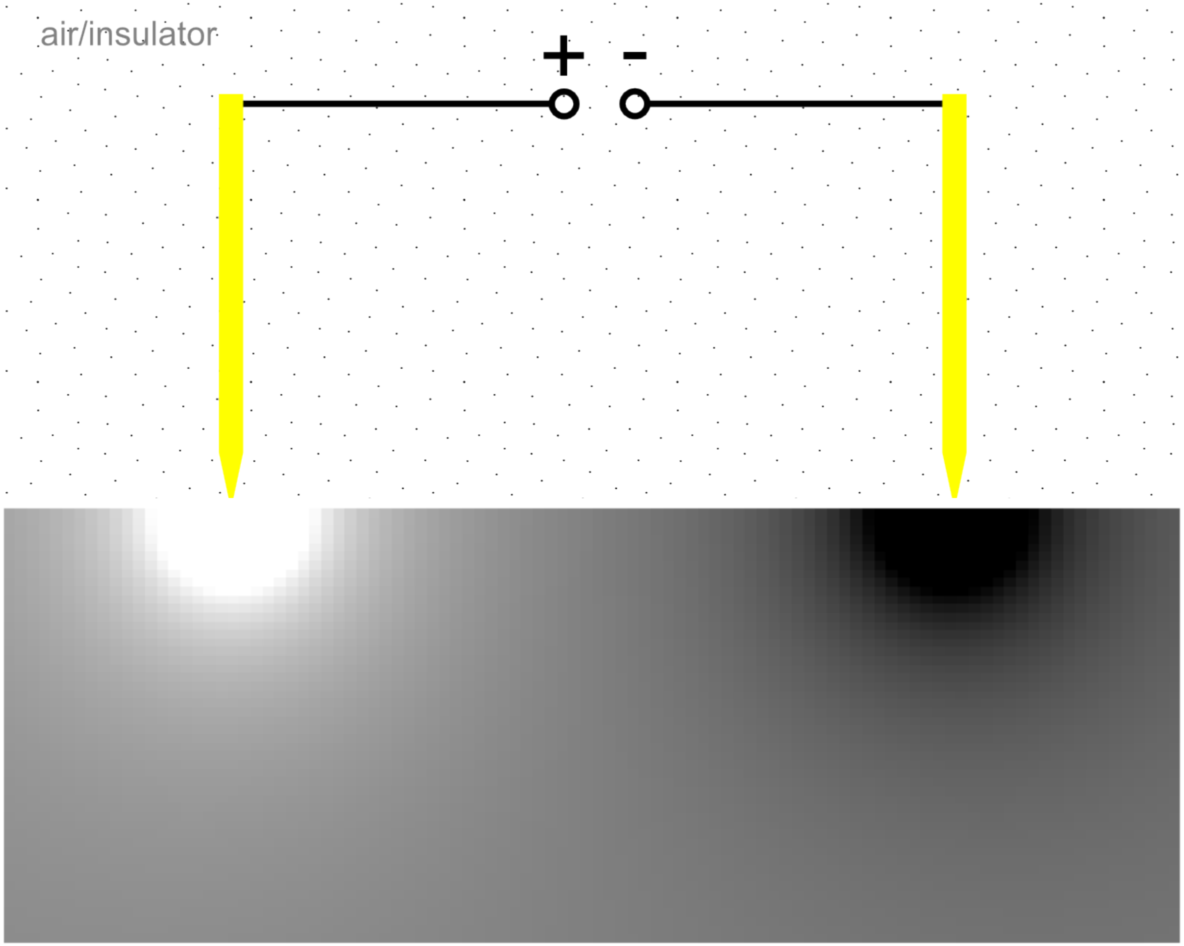
What is the potential due to the electrodes at arbitrary point P within the conductor?

3*a*/2?
to measure resistance one uses four, equally spaced (by *a*), electrodes

*point-like contacts between the electrode and the conductor

Basics of resistance measurements

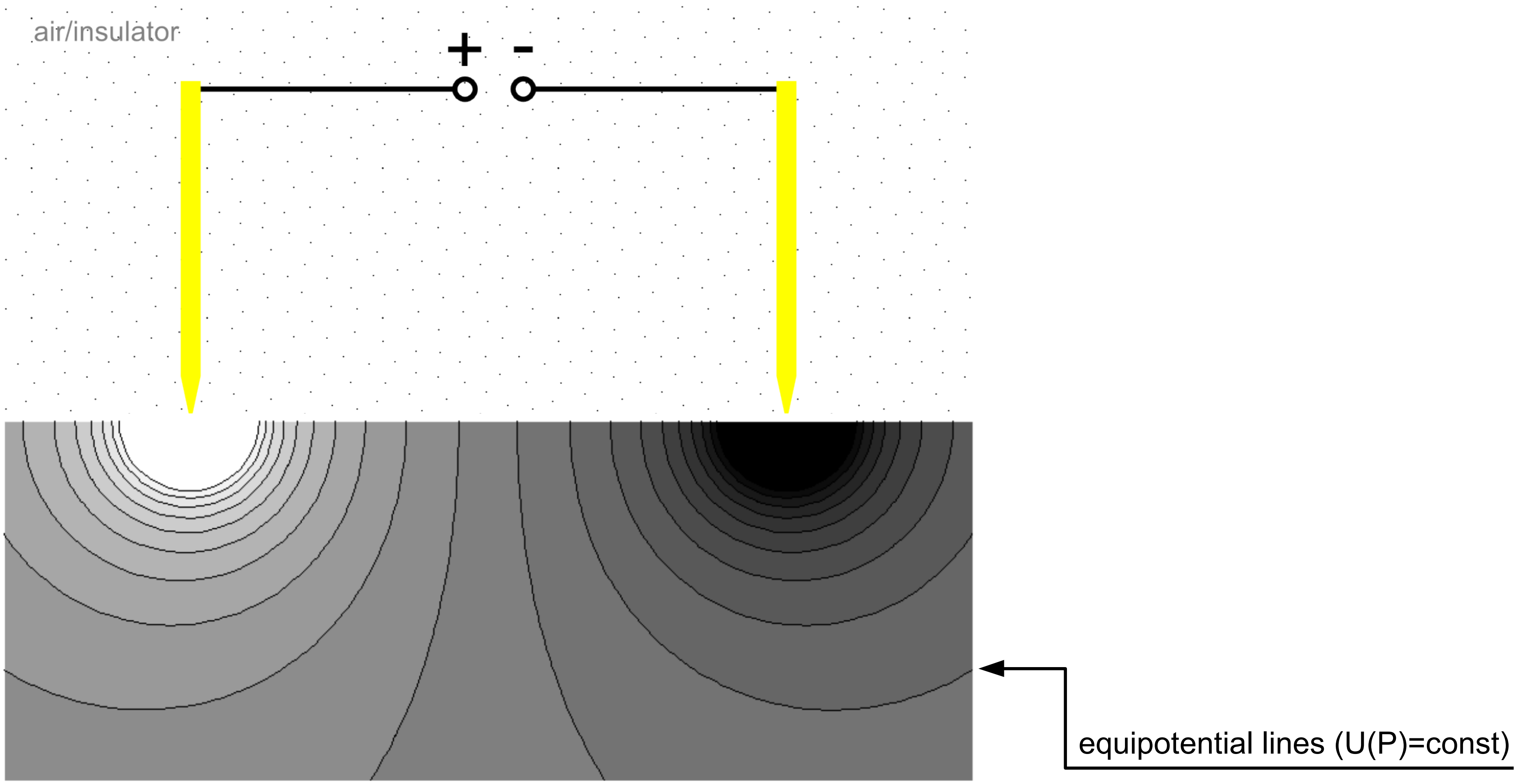
- The potential at point P is the sum of the potentials due to both electrodes. Using Eq. 6 we get:



$$U(P)=\frac{\rho}{2\pi R_1}I-\frac{\rho}{2\pi R_2}I=\frac{I\rho}{2\pi}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{I\rho}{2\pi}\left[\frac{1}{\left(\left(x+\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}-\frac{1}{\left(\left(x-\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}\right] \quad (7)$$

Basics of resistance measurements

- The potential at point P is the sum of the potentials due to both electrodes. Using Eq. 6 we get:

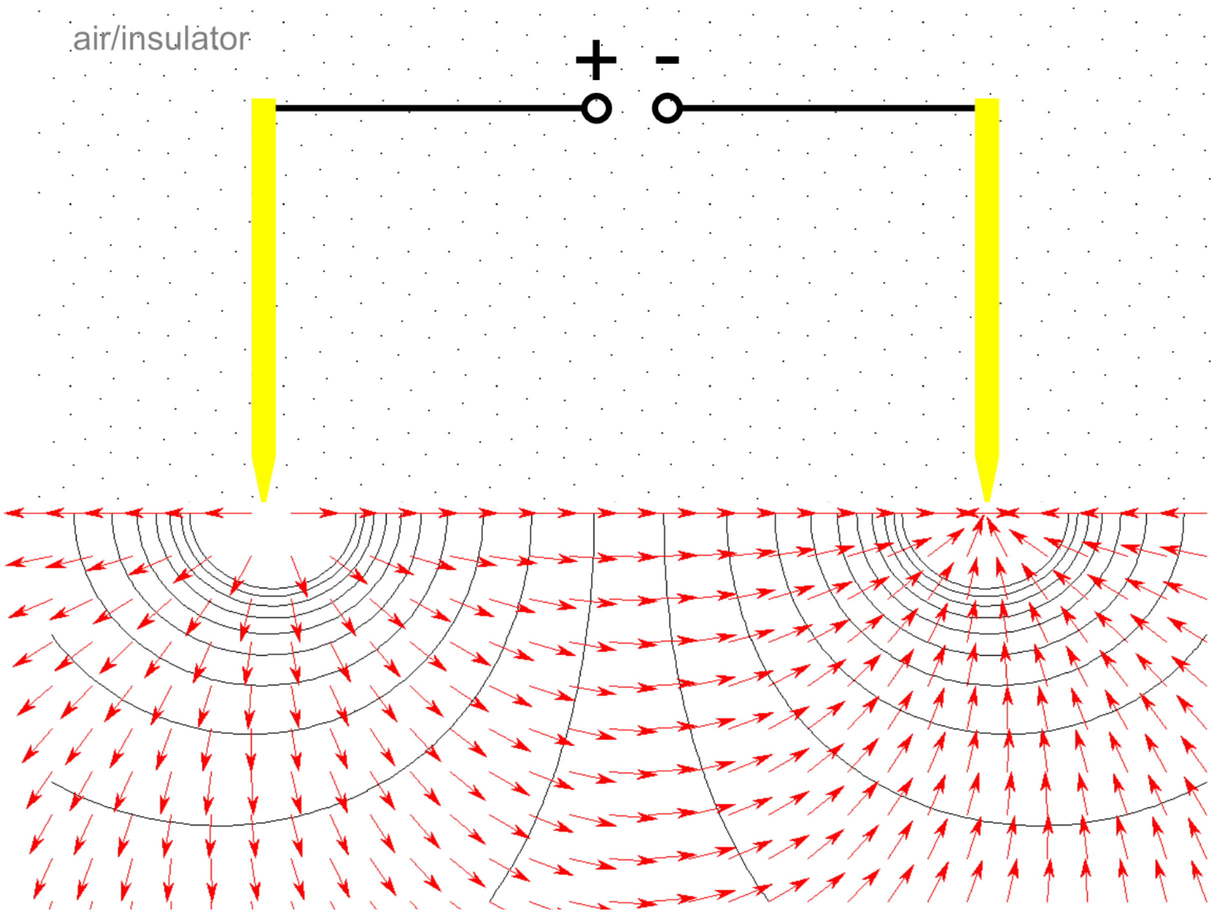


equipotential lines (U(P)=const)

$$U(P)=\frac{\rho}{2\pi R_1}I-\frac{\rho}{2\pi R_2}I=\frac{I\rho}{2\pi}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{I\rho}{2\pi}\left[\frac{1}{\left(\left(x+\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}-\frac{1}{\left(\left(x-\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}\right]$$

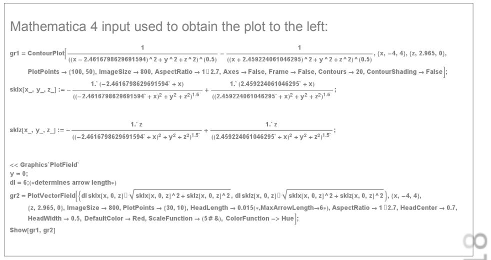
Basics of resistance measurements

- The potential at point P is the sum of the potentials due to both electrodes. Using Eq. 6 we get:



Current flows from the plus electrode to the minus one in the direction determined by the gradient of the potential

Current vector is everywhere perpendicular to equipotential lines*



Mathematica 4 input used to obtain the plot to the left:

```
gr1 = ContourPlot[1/((x - 2.4616798629691594)^2 + y^2 + z^2)^(0.5) - 1/((x + 2.459224061046295)^2 + y^2 + z^2)^(0.5), {x, -4, 4}, {z, 2.965, 0},
    PlotPoints → {100, 50}, ImageSize → 800, AspectRatio → 1/2.7, Axes → False, Frame → False, Contours → 20, ContourShading → False];
sklx[x_, y_, z_] := -(1.` (-2.4616798629691594` + x))/((-2.4616798629691594` + x)^2 + y^2 + z^2)^1.5` + (1.` (2.459224061046295` + x))/((2.459224061046295` + x)^2 + y^2 + z^2)^1.5`;


sklz[x_, y_, z_] := -(1.` z)/((-2.4616798629691594` + x)^2 + y^2 + z^2)^1.5` + (1.` z)/((2.459224061046295` + x)^2 + y^2 + z^2)^1.5`;


<< Graphics`PlotField`
y = 0;
dl = 6;(*determines arrow length*)
gr2 = PlotVectorField[{dl sklx[x, 0, z]/Sqrt[sklx[x, 0, z]^2 + sklz[x, 0, z]^2], dl sklz[x, 0, z]/Sqrt[sklx[x, 0, z]^2 + sklz[x, 0, z]^2]}, {x, -4, 4},
    {z, 2.965, 0}, ImageSize → 800, PlotPoints → {30, 10}, HeadLength → 0.015(*,MaxArrowLength→6*), AspectRatio → 1/2.7, HeadCenter → 0.7,
    HeadWidth → 0.5, DefaultColor → Red, ScaleFunction → (5 # &), ColorFunction -> Hue];
Show[gr1, gr2]
```

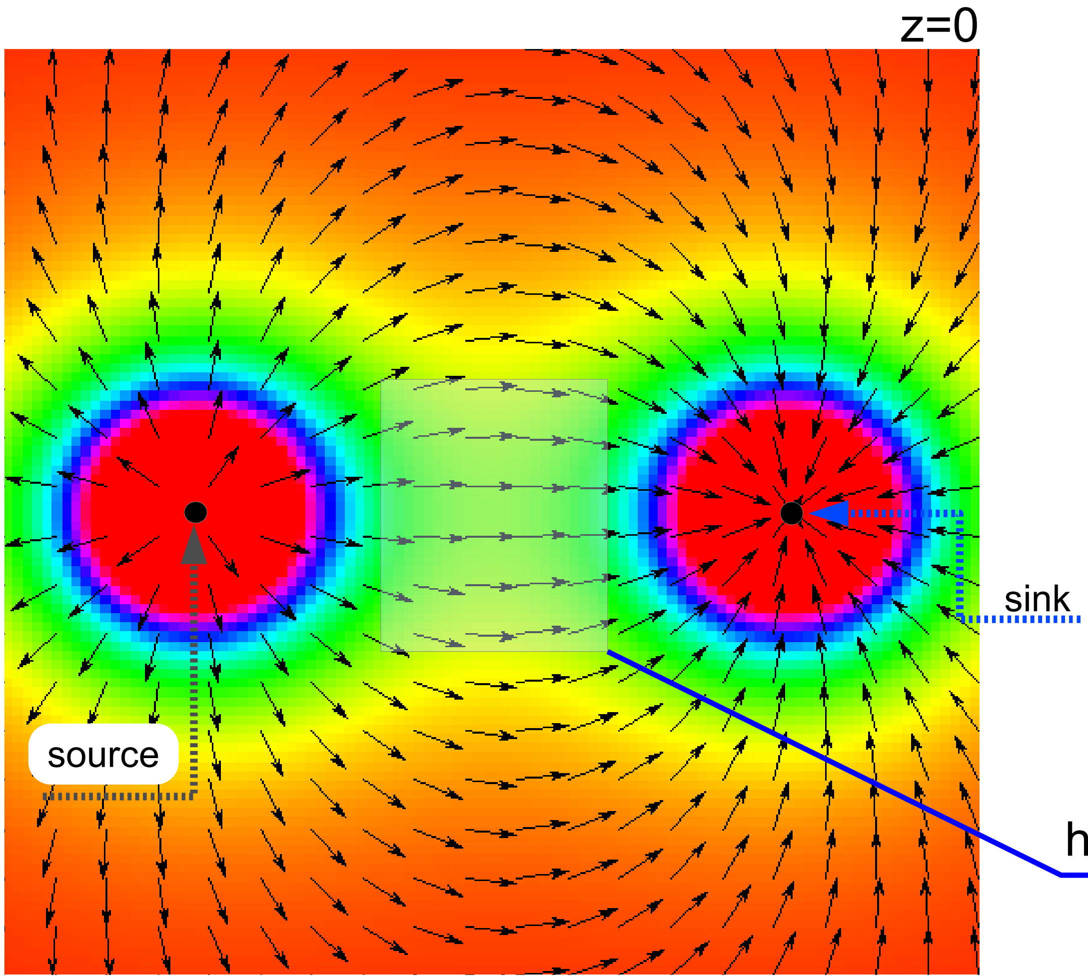
equipotential lines (U(P)=const)

$$U(P)=\frac{\rho}{2\pi R_1}I-\frac{\rho}{2\pi R_2}I=\frac{I\rho}{2\pi}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{I\rho}{2\pi}\left[\frac{1}{\left(\left(x+\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}-\frac{1}{\left(\left(x-\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}\right]$$

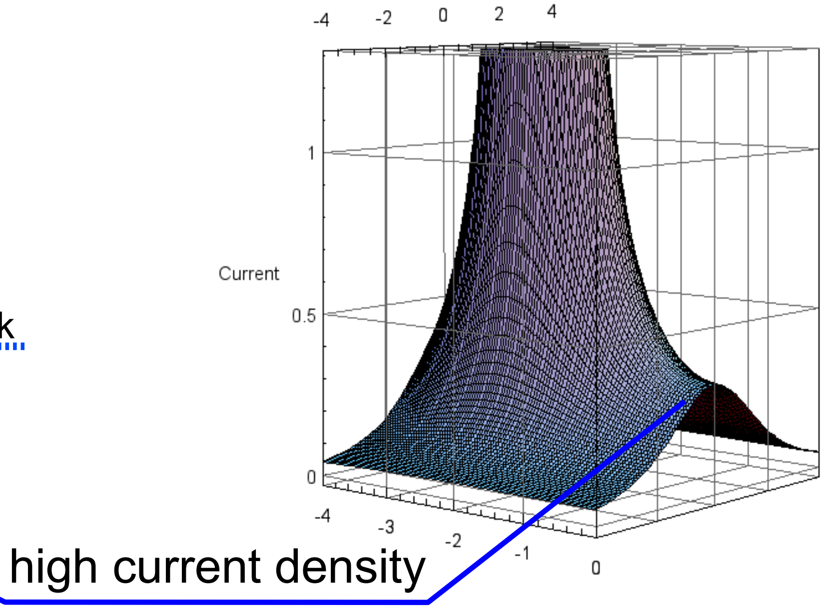
*lengths of arrows do not show current amplitude; the arrows show only current direction!

Basics of resistance measurements

- Top view of the two electrodes*:



Current flows from the plus electrode to the minus one in the direction determined by the gradient of the potential

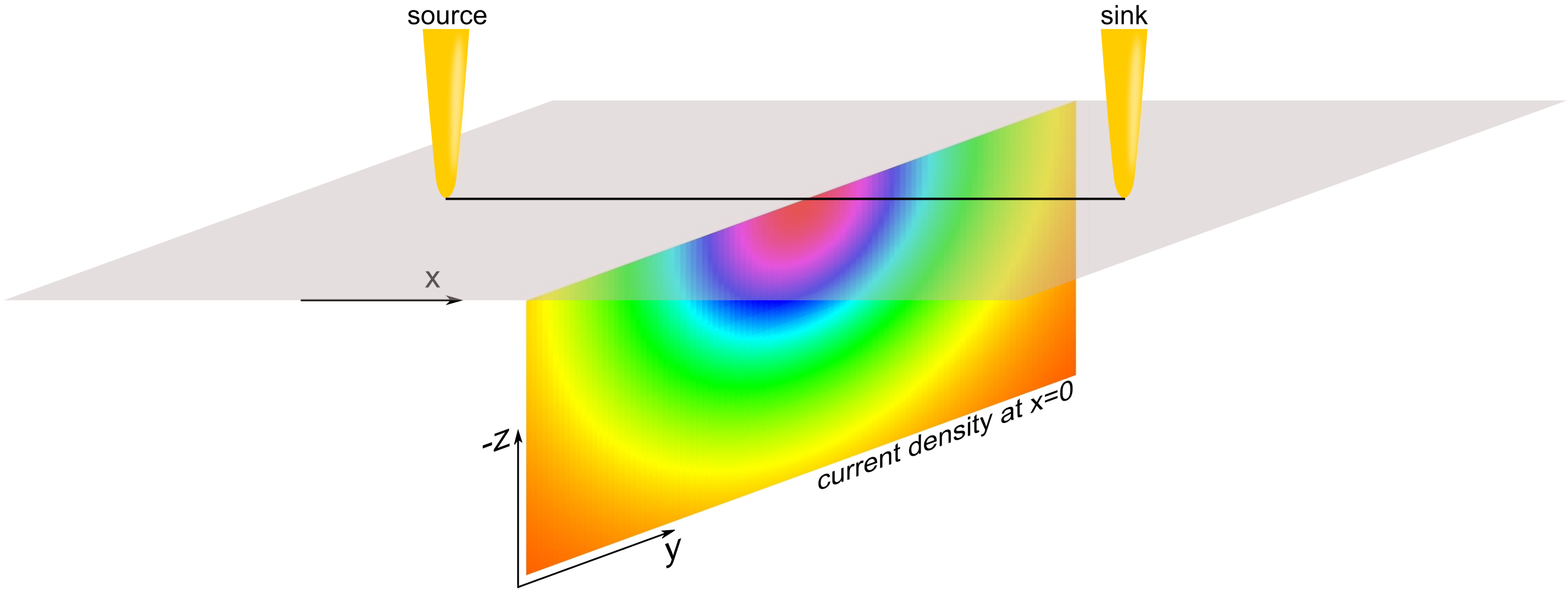


$$U(P)=\frac{\rho}{2\pi R_1}I-\frac{\rho}{2\pi R_2}I=\frac{I\rho}{2\pi}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{I\rho}{2\pi}\left[\frac{1}{\left(\left(x+\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}-\frac{1}{\left(\left(x-\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}\right]$$

*lengths of arrows do not show current amplitude; the arrows show only current direction!

Basics of resistance measurements

- Depth of current penetration [8]:



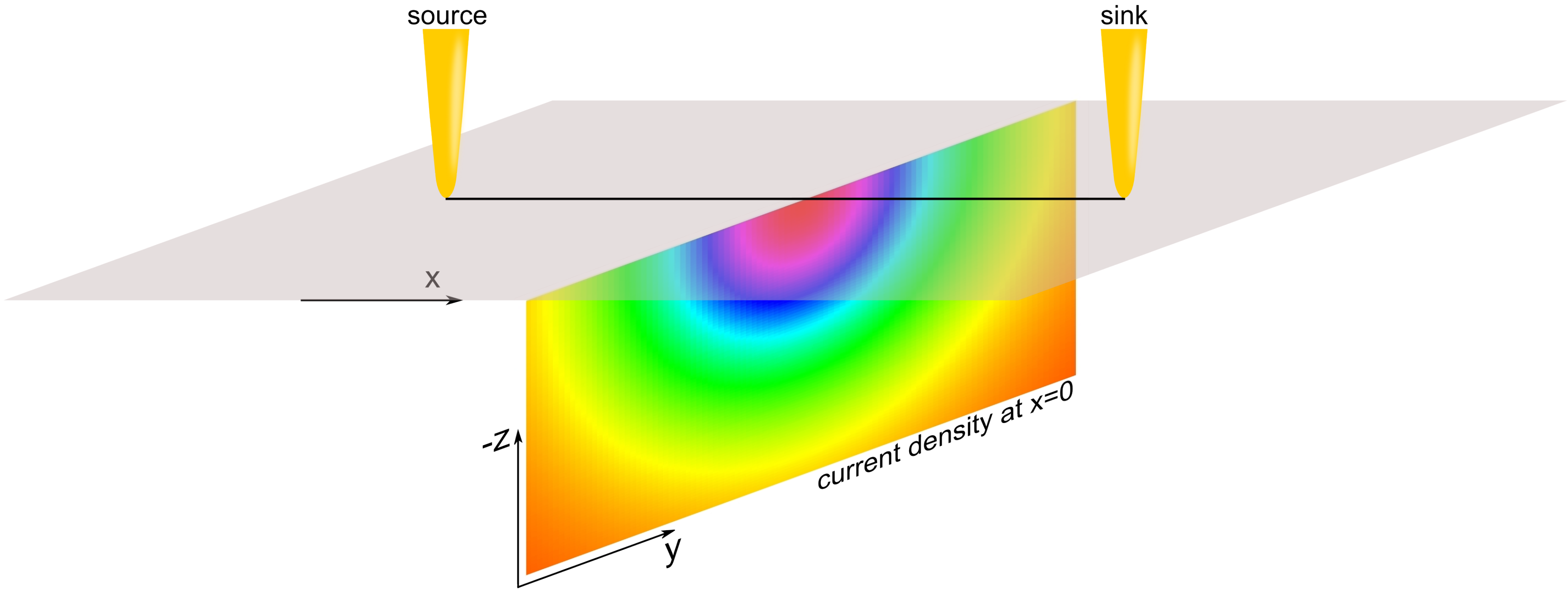
- Following van Nostrand we investigate the current density at the mid-plane between the two electrodes (x=0)
- The current density at any given point is given by:

$$\vec{j}=\frac{\vec{E}}{\rho}=-\frac{1}{\rho}\nabla U$$

- Using Eq.(7) we get: $$\vec{j}=-\frac{I}{2\pi}\nabla\left[\frac{1}{\left(\left(x+\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}-\frac{1}{\left(\left(x-\frac{3a}{2}\right)^2+y^2+z^2\right)^{\frac{1}{2}}}\right]$$

Basics of resistance measurements

- Depth of current penetration [8]:



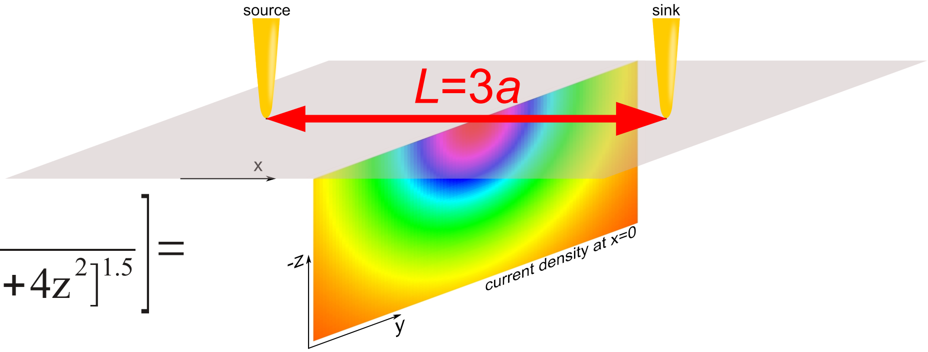
- and for the components of current density respectively:

$$j_x = -\frac{I}{2\pi}\left[\frac{x-(3/2)a}{\left((x-(3/2)a)^2+y^2+z^2\right)^{\frac{3}{2}}} - \frac{x+(3/2)a}{\left((x+(3/2)a)^2+y^2+z^2\right)^{\frac{3}{2}}}\right]$$

$$j_y = -\frac{I}{2\pi}\left[\frac{y}{\left((x-(3/2)a)^2+y^2+z^2\right)^{\frac{3}{2}}} - \frac{y}{\left((x+(3/2)a)^2+y^2+z^2\right)^{\frac{3}{2}}}\right]$$

$$j_z = -\frac{I}{2\pi}\left[\frac{z}{\left((x-(3/2)a)^2+y^2+z^2\right)^{\frac{3}{2}}} - \frac{z}{\left((x+(3/2)a)^2+y^2+z^2\right)^{\frac{3}{2}}}\right]$$

Basics of resistance measurements



- We need only the current density component that is perpendicular to yz plane (i.e. j_x)
- at x=0 j_x is given by:

$$j_x=-\frac{I}{2\pi}\left[\frac{3a}{[(9/4)a^2+y^2+z^2]^{1.5}}\right]=-\frac{I}{2\pi}\left[\frac{3a}{(1/8)[9a^2+4y^2+4z^2]^{1.5}}\right]=$$

$$4^{(3/2)}=8$$

$$-\frac{I}{\pi}\left[\frac{12a}{[9a^2+4y^2+4z^2]^{1.5}}\right]$$ or introducing $$L:=3a \qquad j_x=\frac{I}{\pi}\left[\frac{4L}{[L^2+4y^2+4z^2]^{1.5}}\right]$$

- To calculate the total current I_1 flowing above the given depth z_i we integrate the current density j_x (see the drawing to the right) [8]:

$$I_1=\frac{4LI}{\pi}\int_0^{z_1}\int_{-\infty}^{\infty}\left[\frac{1}{[L^2+4y^2+4z^2]^{1.5}}\right]dydz$$

The problem is symmetric about y=0 plane so we can replace integration from -∞ to ∞ by 2 integrals from 0 to ∞

$$I_1=\frac{4LI}{\pi}\int_0^{z_1}\int_{-\infty}^{\infty}\left[\frac{1}{[L^2+4y^2+4z^2]^{1.5}}\right]dydz=\frac{4LI}{\pi}\int_0^{z_1}\left[2\int_0^{\infty}\left[\frac{1}{[L^2+4y^2+4z^2]^{1.5}}\right]dy\right]dz=\int_0^{z_1}\left[2\left[\frac{4ILy}{[\pi(L^2+4z^2)L^2+4y^2+4z^2]^{0.5}}\right]_0^{\infty}\right]dz=$$

$$\int_0^{z_1}\left[\frac{4IL}{\pi L^2+4\pi z^2}\right]dz=I\ \frac{2}{\pi}\arctan\left(\frac{2z_1}{L}\right)$$

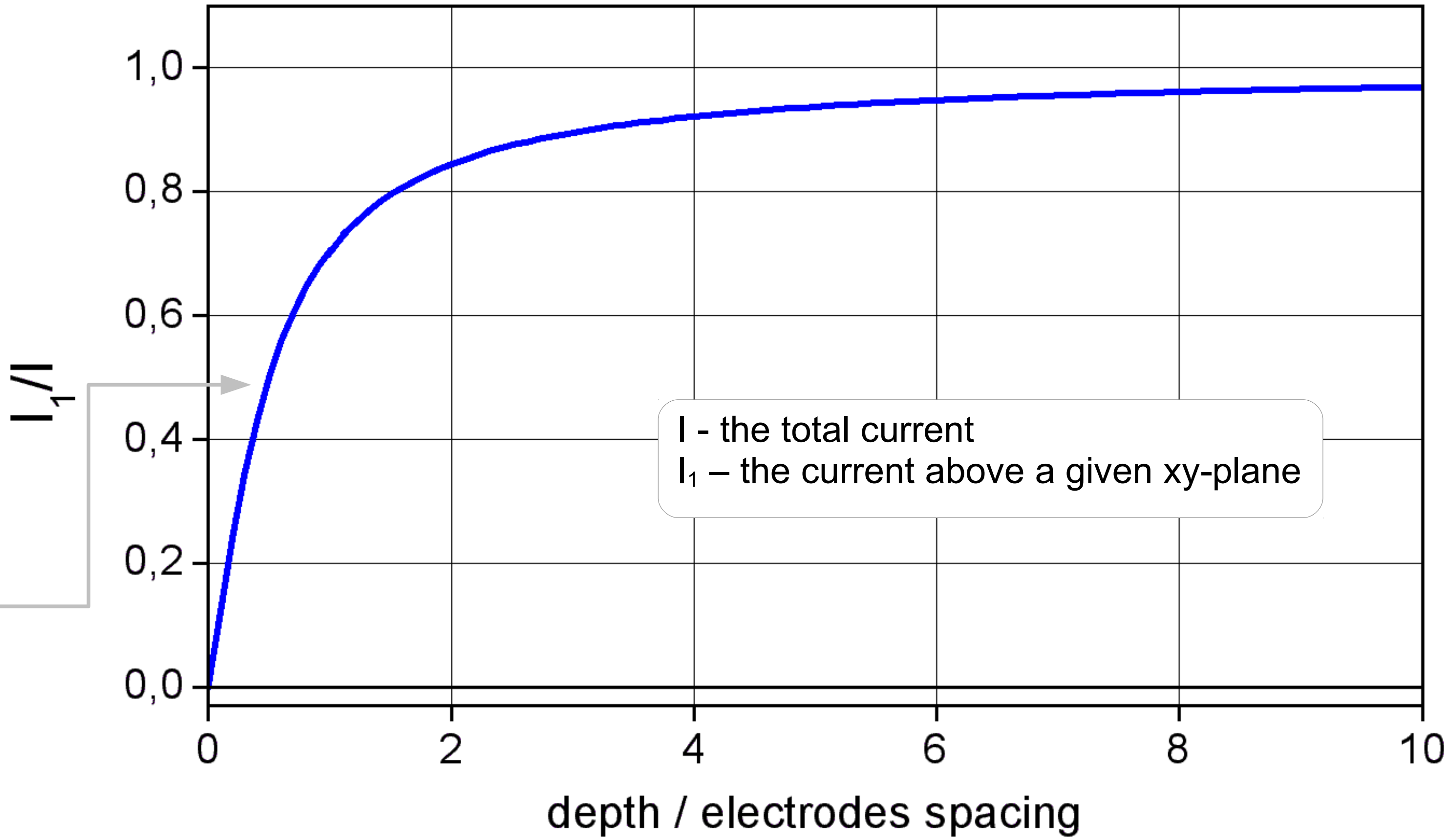
$$\boxed{I_1=I\ \frac{2}{\pi}\arctan\left(\frac{2z_1}{L}\right)}$$

the total current above z_1 depth crossing x=0 plane

I - the total current
I_1 – the current above a given xy-plane

Basics of resistance measurements

$$\frac{I_1}{I}=\frac{2}{\pi}\arctan\left(\frac{2z_1}{L}\right)$$



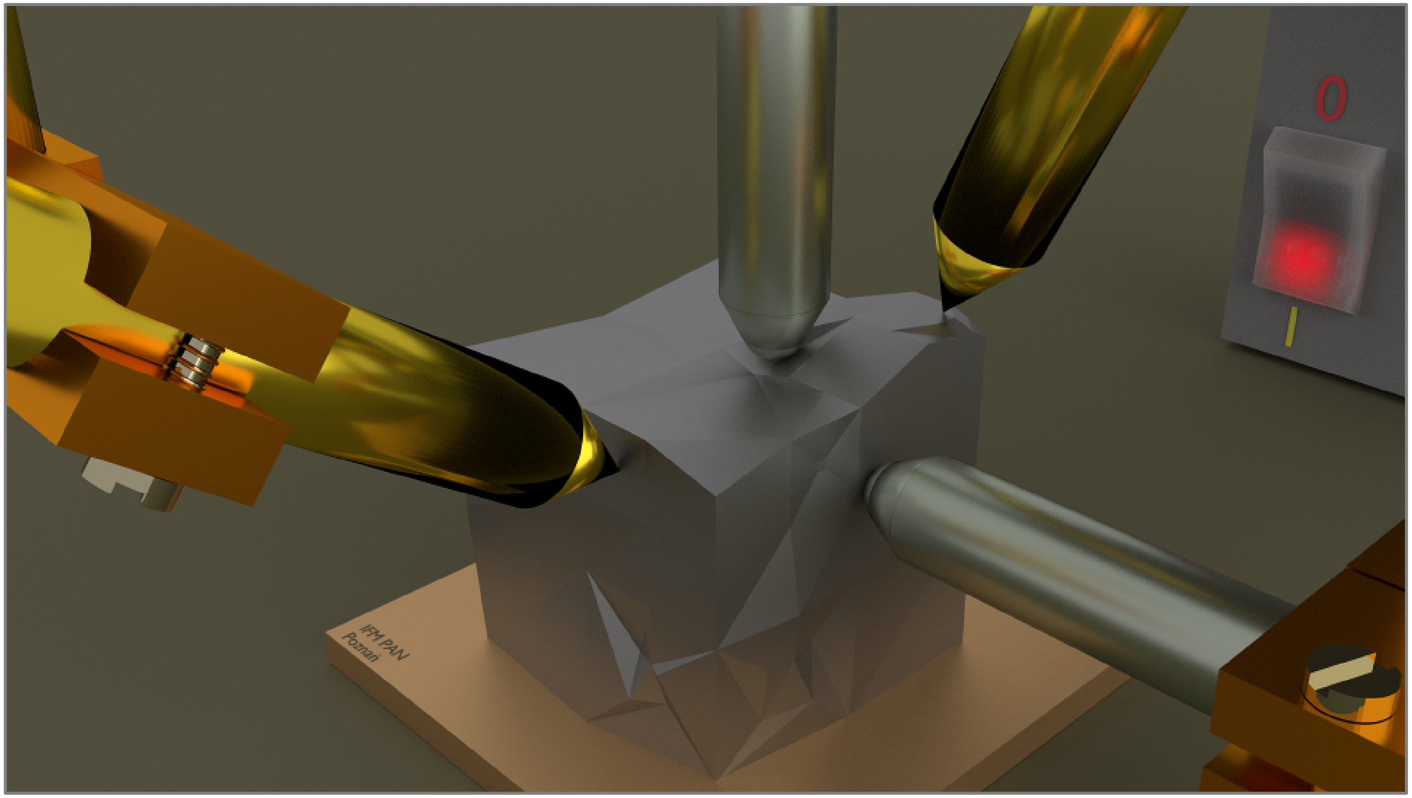
$$\frac{2}{\pi}\arctan(2\times0.5)=0.5$$

- “only half of the current penetrates to a depth greater than half of the distance between **the current electrodes***” [8]
- 70.5% of the total current passes above depth equal to the distance between the current electrodes

*note that till now we were talking only about the current electrodes

Basics of resistance measurements

- **the current electrodes** establish current flow within the investigated conductor
- they can be arbitrarily placed and the sample can have any shape



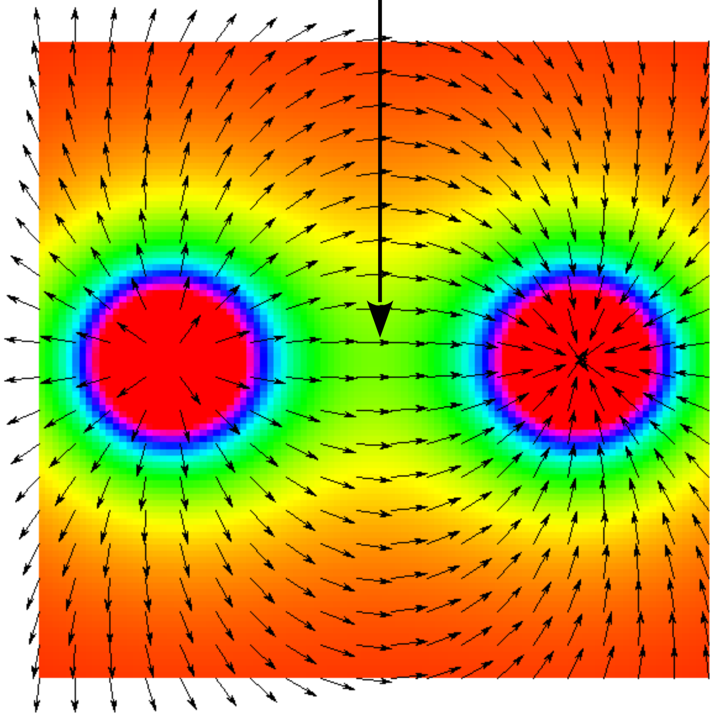
- in practice, when the resistivity is a quantity of interest, both the placement of electrodes relative to the sample as well as the shape of the sample are standardized; this allows the use of analytical expressions relating the current and voltage applied to the current electrodes
- usually **a second pair of electrodes is used to measure the voltage** between defined points of the sample (in solid state physics usually on its surface; in electric soundings of geophysics [5] the electrodes may be placed within the sample)

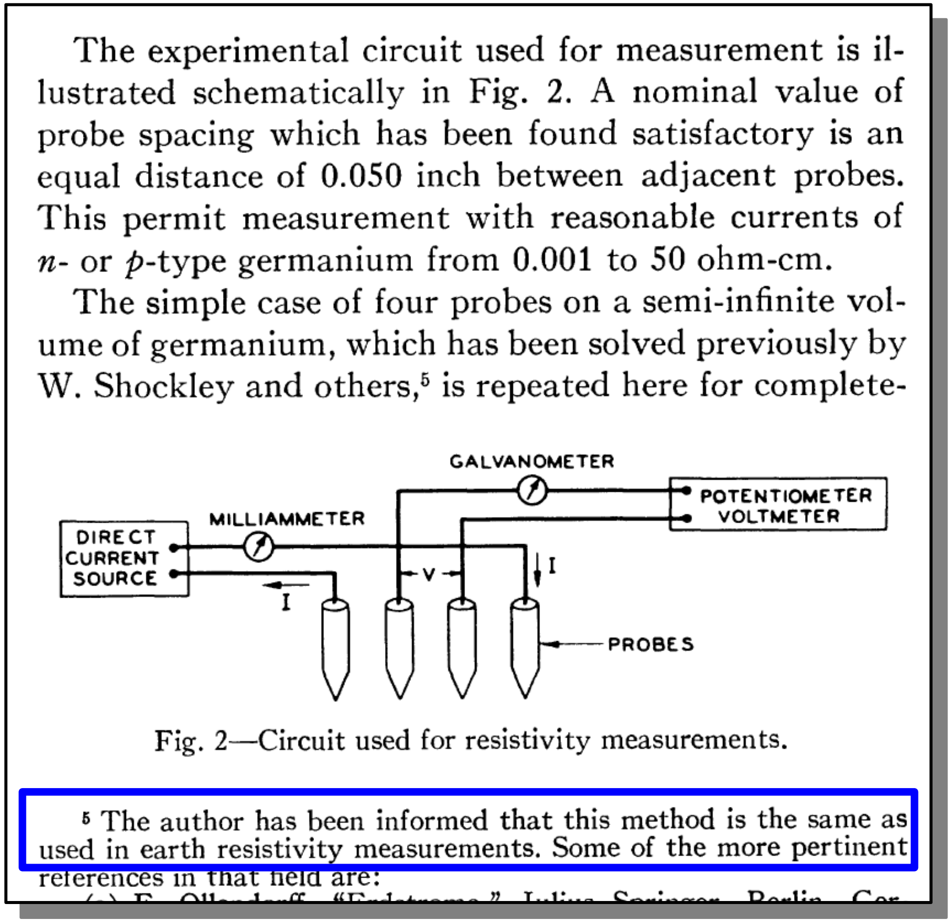
Basics of resistance measurements

The use of the voltage electrodes in addition to the current electrodes has two principal reasons:

- high input resistance of voltmeter makes voltage measurements almost independent of a resistance of connecting wires
- the typical 4-point configuration (with all electrodes on a line) is more sensitive to the resistance of material between voltage electrodes, where the current flows almost parallelly to the probe axis;

this property is important when measuring isotropic samples





The experimental circuit used for measurement is illustrated schematically in Fig. 2. A nominal value of probe spacing which has been found satisfactory is an equal distance of 0.050 inch between adjacent probes. This permit measurement with reasonable currents of *n*- or *p*-type germanium from 0.001 to 50 ohm-cm.

The simple case of four probes on a semi-infinite volume of germanium, which has been solved previously by W. Shockley and others,[5] is repeated here for complete-

Fig. 2—Circuit used for resistivity measurements.

[5] The author has been informed that this method is the same as used in earth resistivity measurements. Some of the more pertinent references in that field are:

graphics taken from L.B. Valdes, Proceedings of the I R E **42**, 420 (1954)

Basics of resistance measurements

There is a multitude of popular electrode configurations

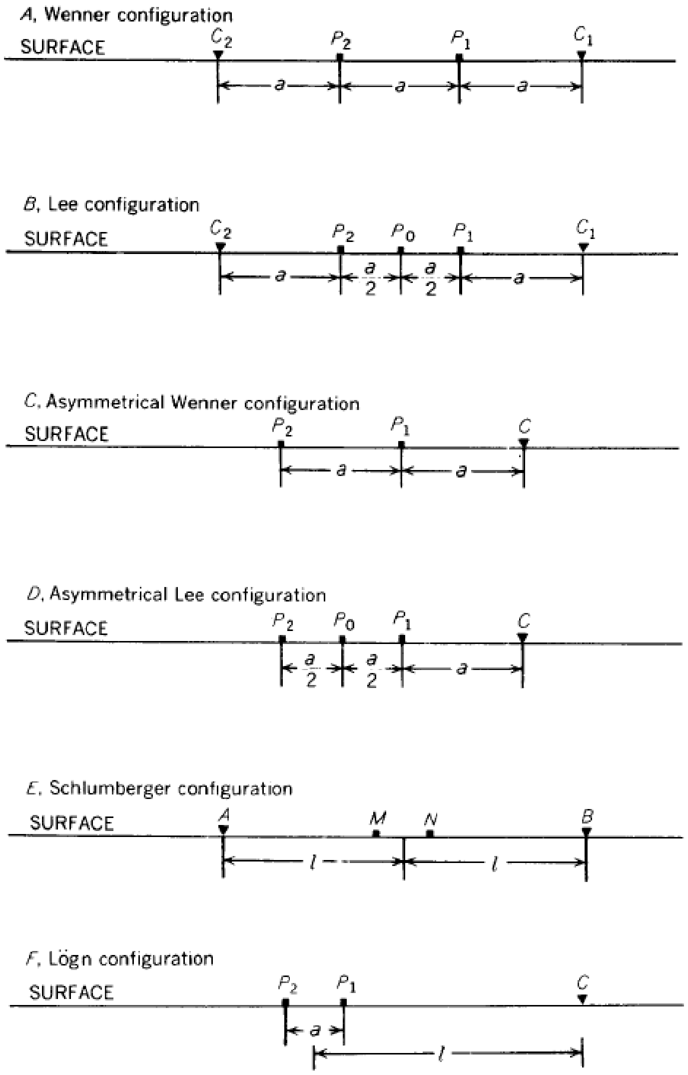


FIGURE 10.—Various electrode configurations to be studied in this work.

The one most relevant to solid state physics is the **Wenner configuration***:

- the four electrodes are equidistant and collinear
- inner electrodes are used to measure voltage

In solid-state physics this arrangement is usually called simply **four-point probe** or four-electrode probe

graphics from [8] R.G. Van Nostrand, K.L. Cook,
Interpretation of resistivity data,
US Geological Survey Professional Paper 499, 1966

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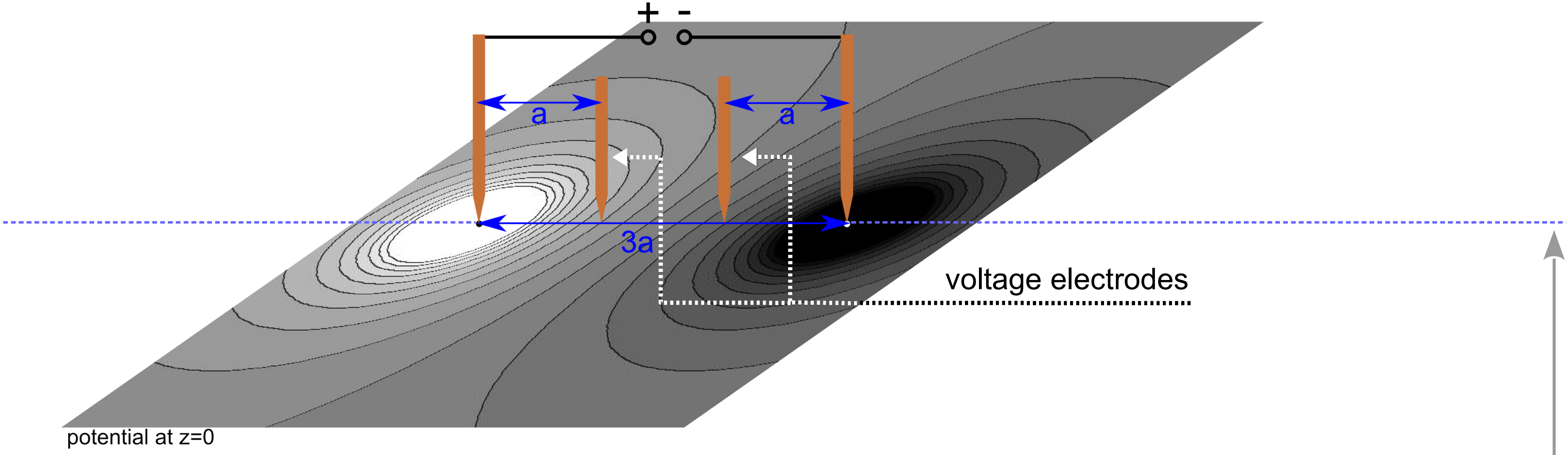
*Wenner alpha configuration [9], in solid state physics the use of this array is called **Kelvin method** [16]

Basics of resistance measurements

Let us consider the voltage (potential difference) between two voltage electrodes placed collinearly with the current electrodes **on the surface of the semi-infinite sample**



We have derived previously the expression for the potential due to the two current electrodes (Eq. 7) placed at *x*=-3a/2 and *x*=+3a/2; for *z*=0 (the surface of the sample) and *y*=0 (all electrodes lie on the x-axis) it transforms to:

$$U(x,y=0,z=0)=\frac{I\rho}{2\pi}\left[\frac{1}{\left(\left(x+\frac{3a}{2}\right)^{2}\right)^{\frac{1}{2}}}-\frac{1}{\left(\left(x-\frac{3a}{2}\right)^{2}\right)^{\frac{1}{2}}}\right]$$

this expression gives the potential along that line

Basics of resistance measurements

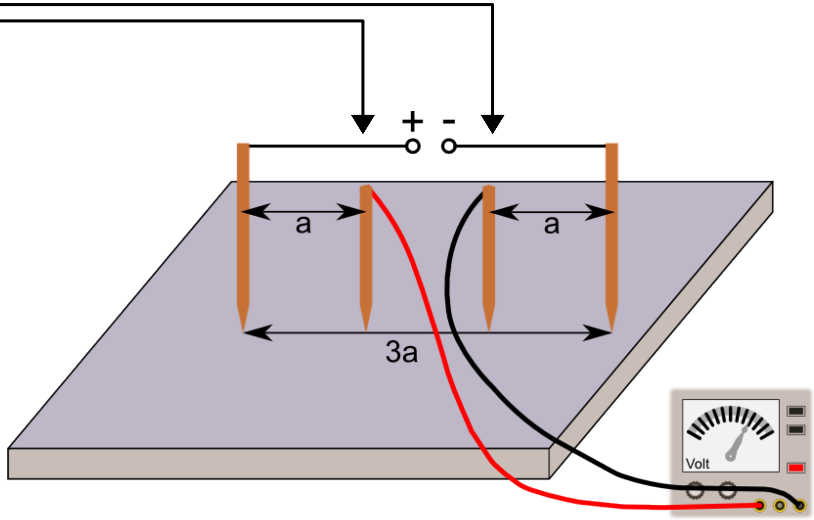
The potential difference is given by:

$$\Delta U = U(x=-a/2, y=0, z=0) - U(x=a/2, y=0, z=0) = \frac{1}{2\pi a} I \rho$$

The so called apparent resistivity is given by:

$$\boxed{\rho_a = 2\pi a \frac{V}{I}}$$

this relation characterizes Wenner alpha array placed on a homogeneous half-space sample



Apparent resistivity is the resistivity of a homogeneous sample that for a given array of electrodes would produce the same voltage drop for a given current flowing between current electrodes [9].

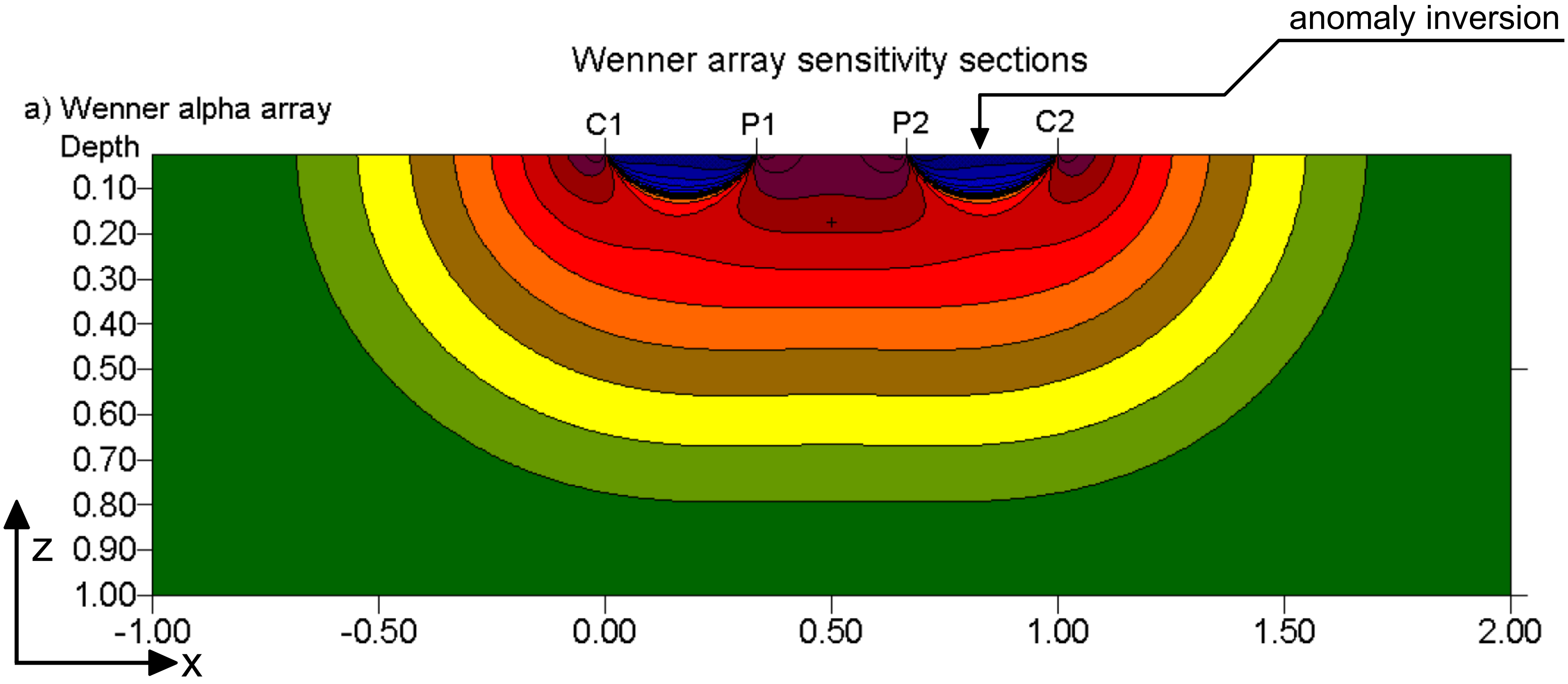
Basics of resistance measurements

- Sensitivity of the Wenner array

- contributions for all *y*-values are added giving S(*x*,*z*) map

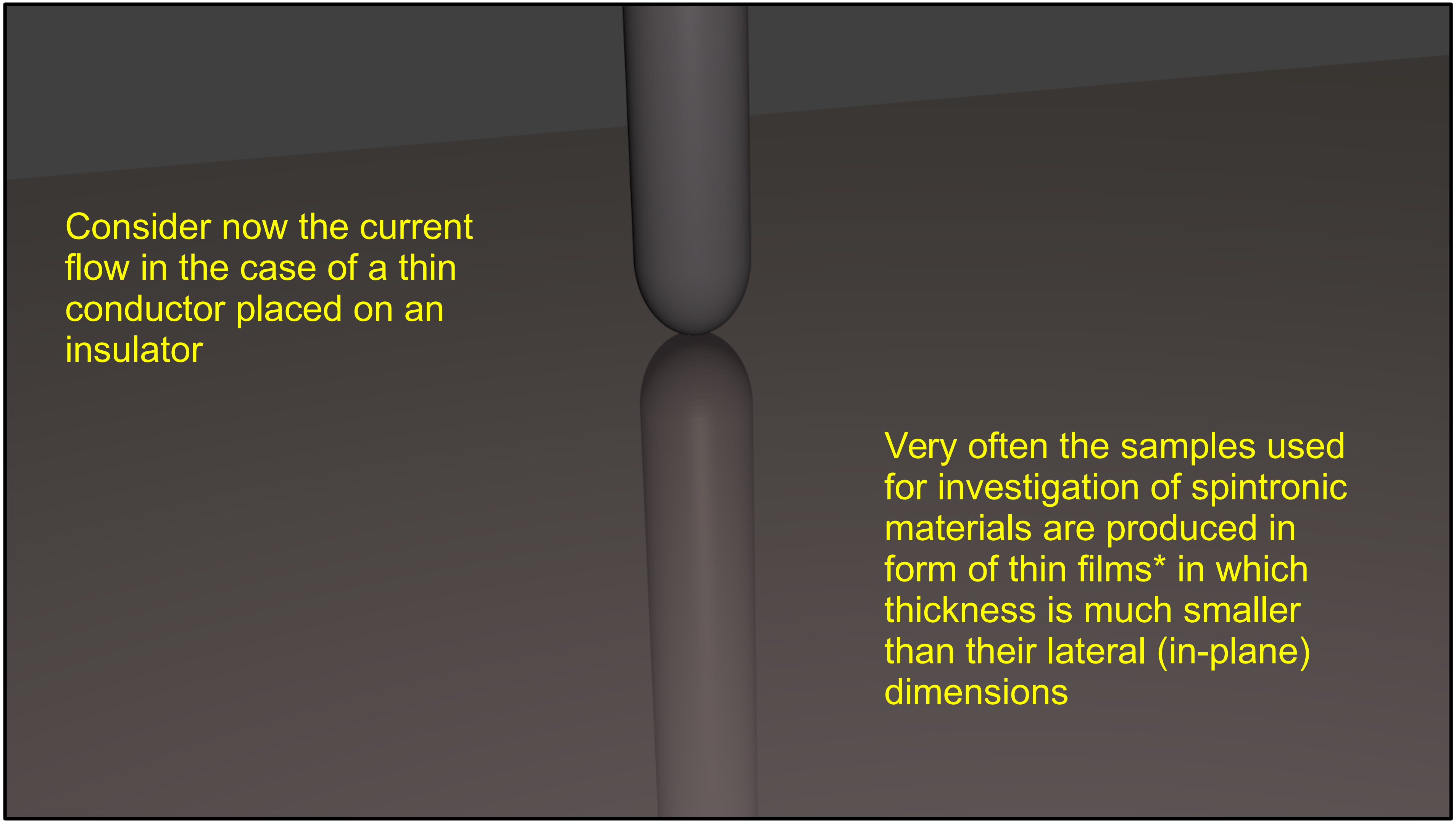
> ,,*The sensitivity function basically tells us the degree to which a change in the resistivity of a section of the subsurface will influence the potential measured by the array*" - M.H. Loke [9]

- Sensitivity is high close to electrodes
- Sensitivity is high between voltage electrodes
- Large negative values of sensitivity show between current and voltage electrodes (anomaly inversion [9])



graphics from
[9] M.H. Loke,
Tutorial: 2_D and 3-D electrical imaging surveys,
2012

Basics of resistance measurements



Consider now the current flow in the case of a thin conductor placed on an insulator

Very often the samples used for investigation of spintronic materials are produced in form of thin films* in which thickness is much smaller than their lateral (in-plane) dimensions

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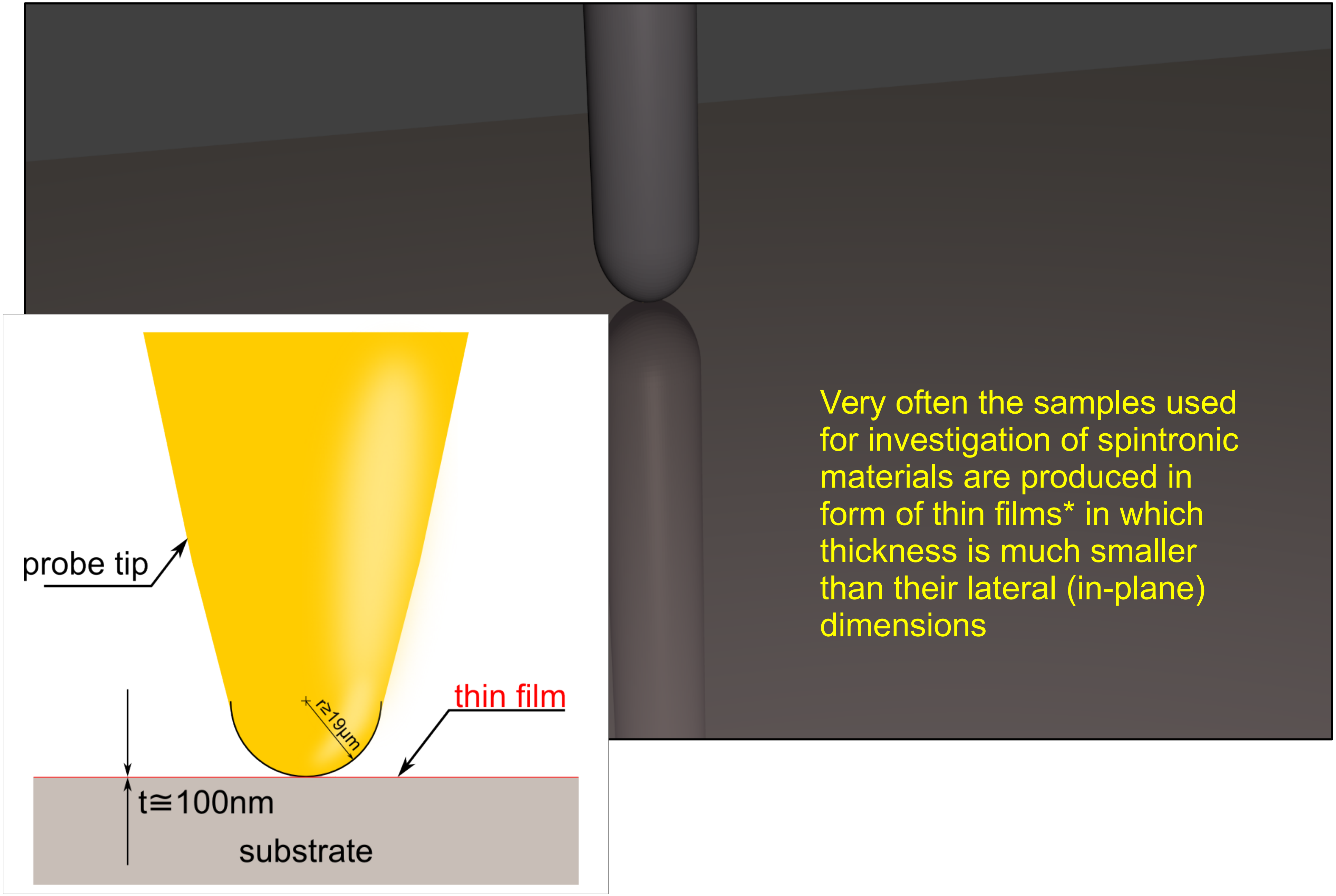
*to be structured when used in functioning devices

Basics of resistance measurements



Very often the samples used for investigation of spintronic materials are produced in form of thin films* in which thickness is much smaller than their lateral (in-plane) dimensions

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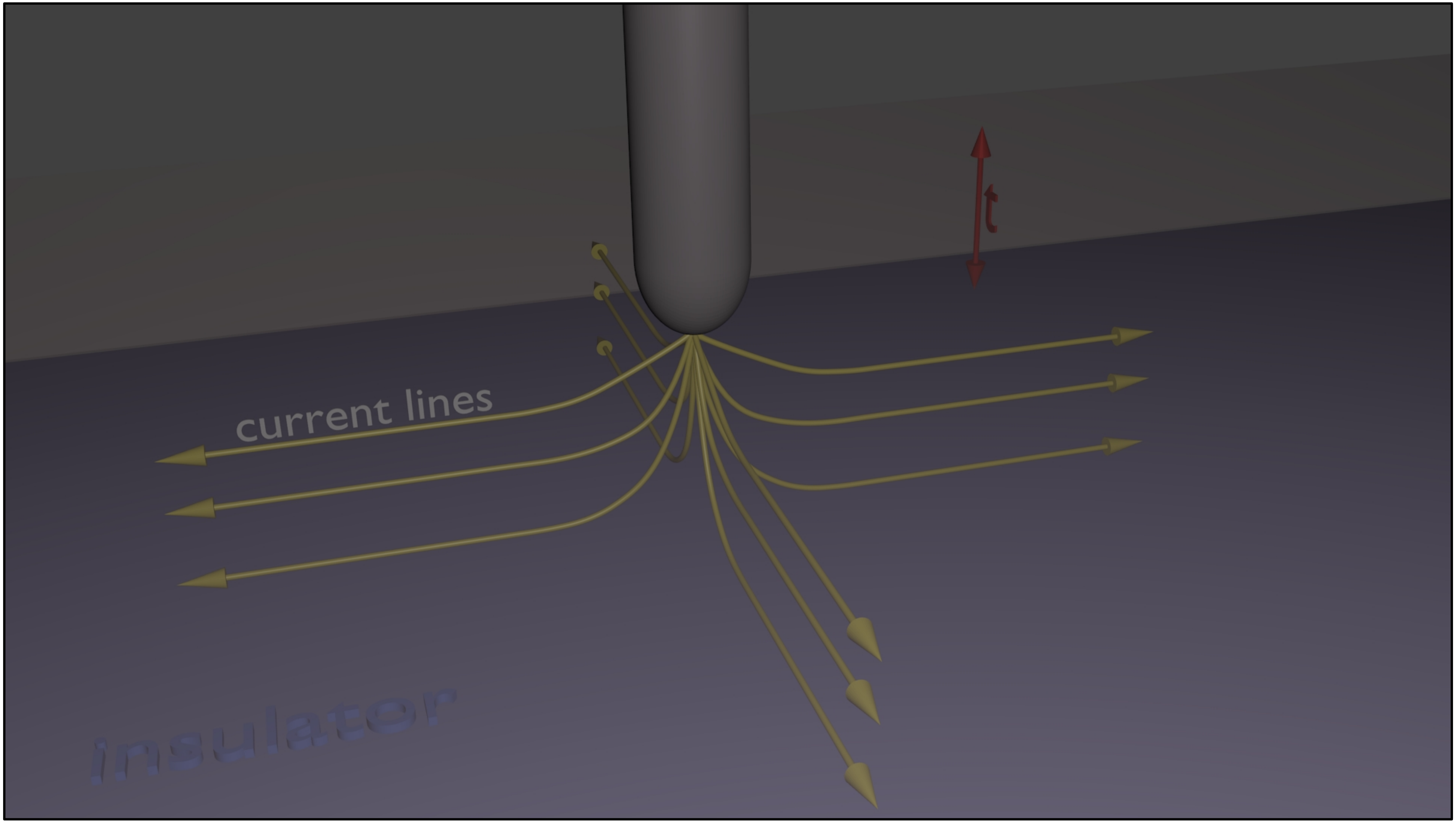
*to be structured when used in functioning devices

Basics of resistance measurements



- from previous analysis it follows that direct in the vicinity of a contact point between the electrode and the conductor/sample the current flows radially from the point
- symmetry of the problem suggests that far from the contact point the current flows radially from the parallel to the surface* of the sample at the contact point

*the current cannot flow into the insulator (see slide 16)

Basics of resistance measurements

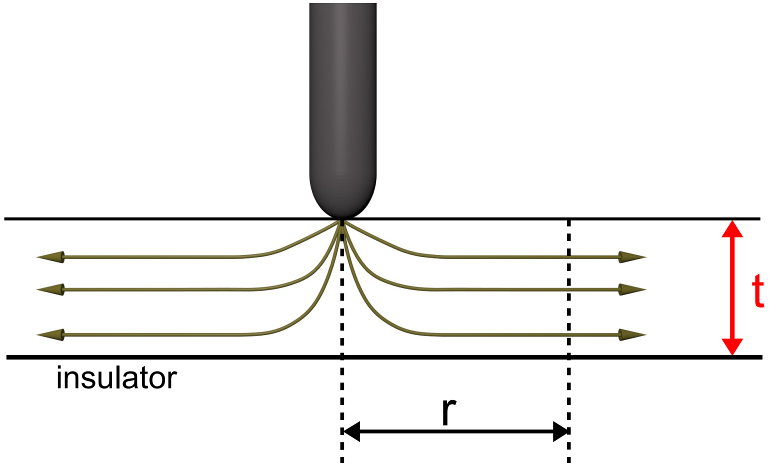
Indeed, it can be shown [5, p.246] that radial component of electric field E_r, on the surface of the conductor, is inversely proportional to in-plane distance from the contact point:

$$E_r = \frac{I\rho}{2\pi r t} \qquad if \qquad \frac{r}{t} \gg 1 \quad (8)$$

$$r = \sqrt{x^2 + y^2}$$

$$E_r = -\frac{\partial U}{\partial r}$$

t-thickness of the sample



- With an increase of *r* the current density vector becomes horizontal and independent of *z* coordinate.

- In a system composed of a number of layers with different conductivities the layers can be treated as conductors connected in-parallel if the distance from the current electrode *r* is much greater than the total thickness of the system [5, p.248] – this statement may not apply in case of electrically thin films, i.e. such in which mean free path of electrons is comparable to the thickness of the individual layers.

Basics of resistance measurements

Four-point probe in case of thin conducting film on an insulator

Like in the case of the array placed on the sample occupying a half-space we have for the potential difference **due the source electrode** (from Eq. 8):

$$\Delta U(r_1,r_2)=\int_{r_1}^{r_2}\frac{I\rho}{2\pi r t}dr=\left[\frac{I\rho\log(r)}{2\pi t}\right]_{r_1}^{r_2}$$

$$E_r=\frac{I\rho}{2\pi r t}\quad if\quad \frac{r}{t}\gg 1\quad (8)$$

In case of Wenner alpha array we have for the potential difference at the voltage electrodes due to both source and sink electrodes (spacing 3a):

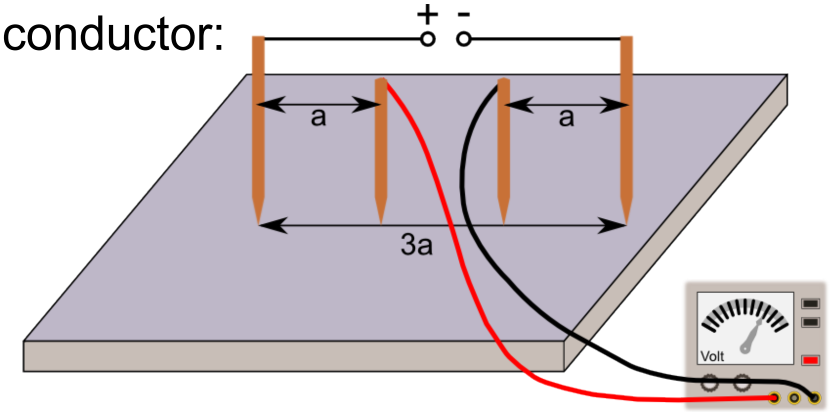
$$V=2\left[\frac{I\rho\log(r)}{2\pi t}\right]_{a}^{2a}=\frac{I\rho\log(2)}{\pi t}$$

2 comes from the fact that both current electrodes produce the same potential difference between the voltage electrodes

We have then for a four-point probe array on a thin* conductor:

$$\boxed{\rho=\frac{\pi t}{\ln(2)}\frac{V}{I}}$$

note that this expression is independent of electrode spacing



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*probe spacing much greater than the thickness of the conductor (only then the approximation of radial current flow holds)

Basics of resistance measurements

Four-point probe in case of thin conducting film on an insulator

In thin film physics one often speaks about the so called sheet resistance:

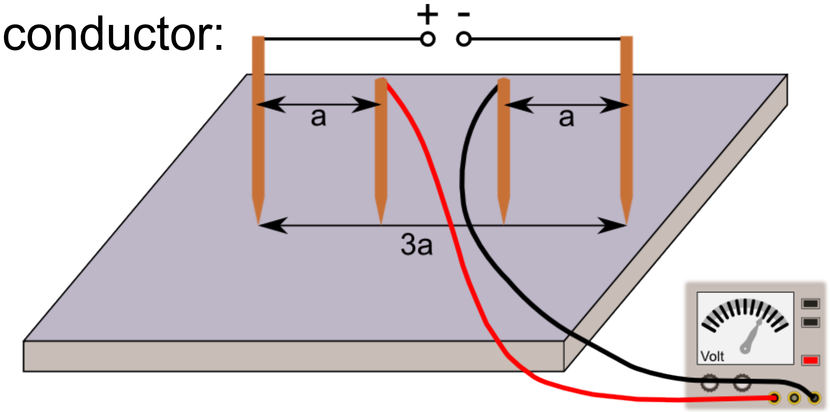
$$R_s = \frac{\rho}{t} \quad [Ohm] \quad \rightarrow \quad R_s = \frac{\pi}{\ln(2)} \frac{V}{I}$$

R_s characterizes the sample from the point of view of external circuit giving its apparent resistance.

We have then for a four-point probe array on a thin* conductor:

$$\boxed{\rho = \frac{\pi t}{\ln(2)} \frac{V}{I}}$$

note that this expression is independent of electrode spacing



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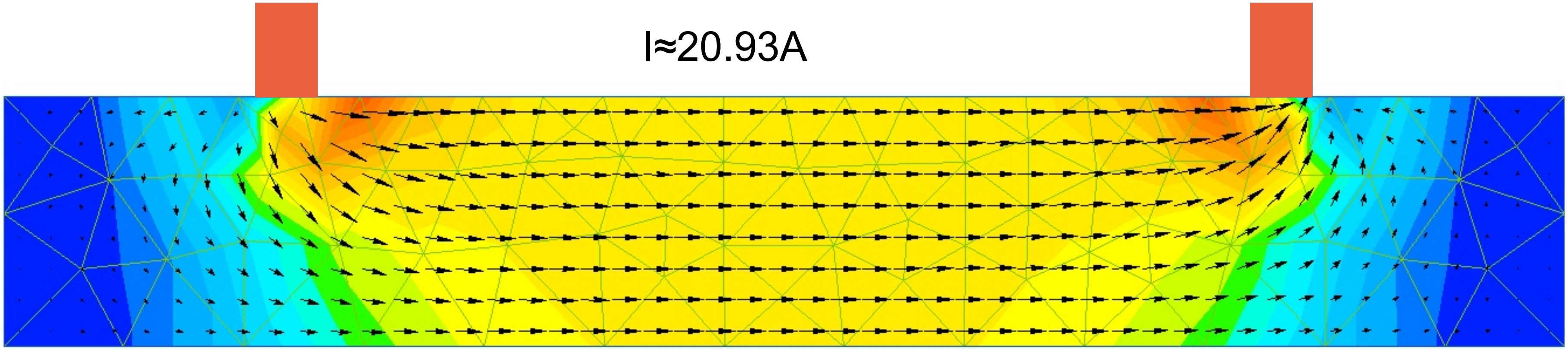
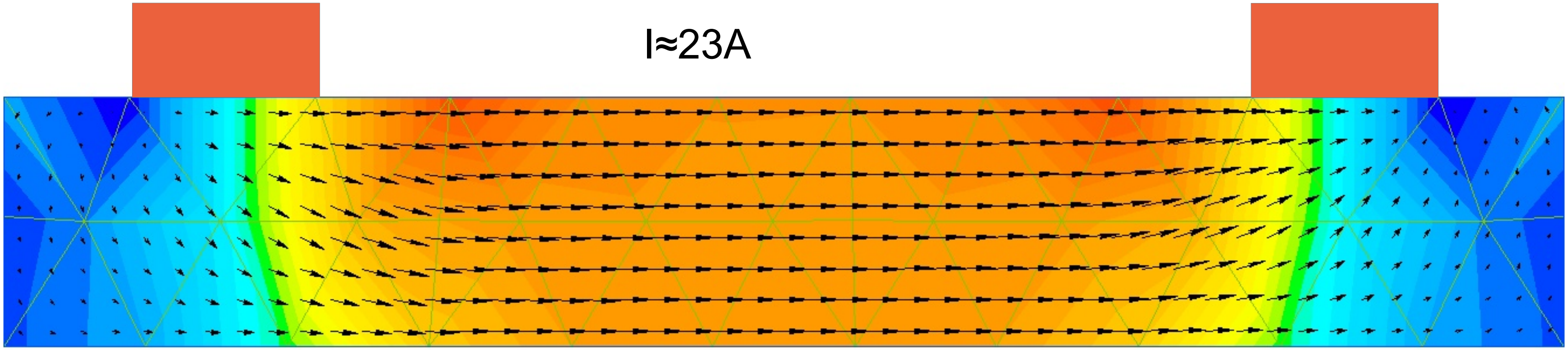
*probe spacing much greater than the thickness of the conductor (only then the approximation of radial current flow holds)

Basics of resistance measurements

Influence of an electrode size on current flow





the system is infinitely extended in the direction perpendicular to the plane of the image – the electrodes are in fact strips

the current flow was calculated with a QuickField™ Student Edition software (version 5.10.1.1141) from Tera Analysis Ltd., www.quickfield.com; not that the mesh nodes are limited to 255 so the spatial resolution of the graph is not sufficient to show the details of the flow, particularly in the vicinities of the electrodes and sample corners.

Basics of resistance measurements

Sensitivity of the four-point array

Wang *et al.*: Sensitivity study of micro four-point probe measurements on small samples

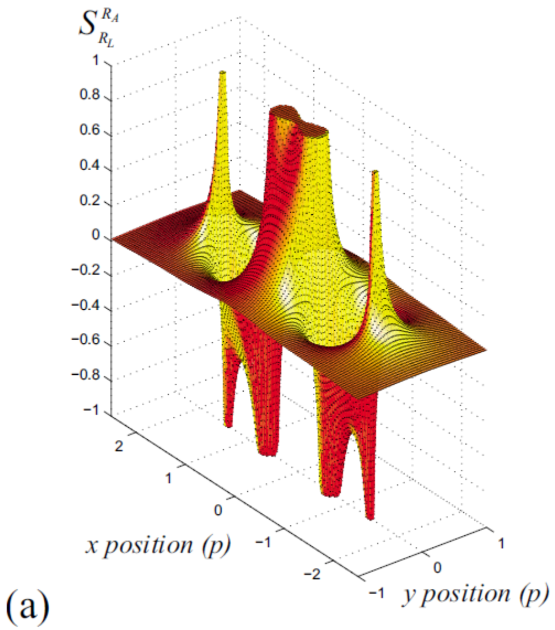


FIG. 2. (Color online) Sensitivity of the measured four-point probe resistance to local sheet resistance variations for single configuration measurements of (a) R_A, (b) R_B, and (c) R_C, respectively, on an infinite sample. The singular peaks have been truncated to show the smaller values around the probes clearly.

source of graphics: [10] Fei Wang, Dirch H. Petersen, Torben M. Hansen, Toke R. Henriksen, Peter Bøggild, and Ole Hansen, J. Vac. Sci. Technol. B 28, C1C34 (2010)

> ,,*The sensitivity function basically tells us the degree to which a change in the resistivity of a section of the subsurface will influence the potential measured by the array*” - M.H. Loke [9]

- like in the case of the array placed on a conductor occupying a half-space sensitivity is highest on the array-line

- anomaly inversion [9] is present too

- note that the sensitivity is higher when smaller samples are used; maximum sensitivity for the Wenner array placed on a longer symmetry axis of 2a x 5a sample is about twice that of the prob placed on an infinite sample [10]

Basics of resistance measurements

Geometrical correction factors

- Our derivations of the basic equations characterizing four-point probe assumed infinite samples
- In practice the sample dimensions are very often comparable with the electrodes spacing ***a*** and correction factors ***F*** are needed to account for that [11,12]
- For the case of thin films:

$$\rho=\frac{\pi t}{\ln(2)}\frac{V}{I} \longrightarrow \rho=F\frac{V}{I}$$

infinite thin film

thin film with dimensions comparable to electrodes spacing

- Calculation of factors *F* is very demanding in most cases but they are already calculated/ tabulated for all practically important cases

- In practice, however, it is enough to use a sample in which all relevant dimensions (width and height in case of thin films) are 5 or more times greater than the electrode spacing [12] – the corrections become negligible and *often unnecessary* if other sources of measurement errors dominate.

Basics of resistance measurements

$$\rho=\frac{\pi t}{\ln(2)}\,\frac{V}{I}$$

infinite thin film

$$\rho=F\,\frac{V}{I}$$

sample with dimensions comparable to electrodes spacing

1954 Valdes: Resistivity Measurements on Germanium for Transistors 423

expression for the Wenner configuration on a half-space (slide 48)

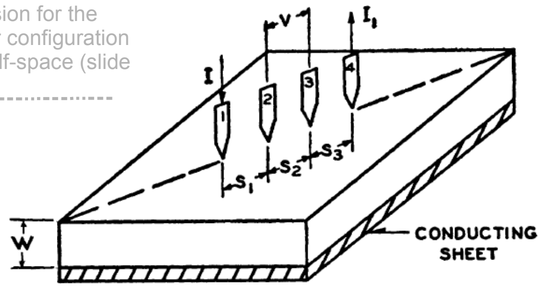


Fig. 9—Resistivity measurements on a thin slice.

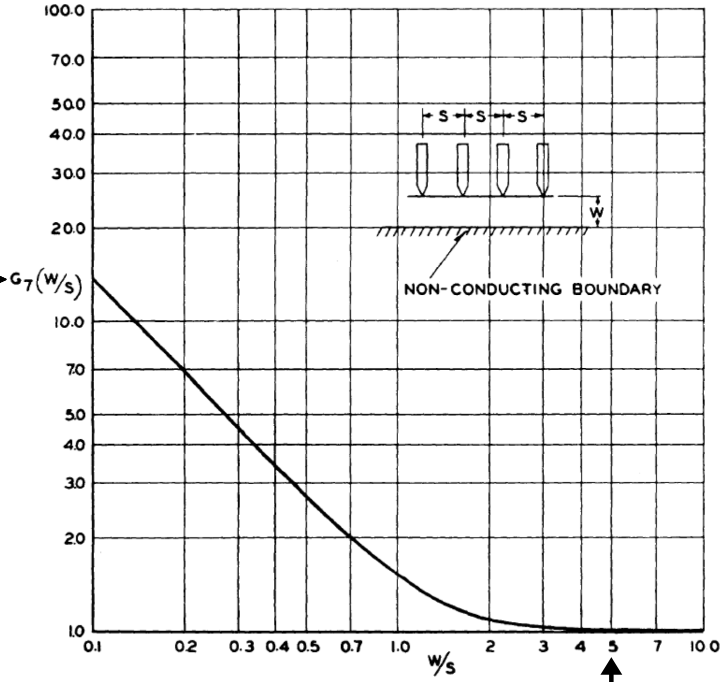


Fig. 11—Correction divisor for probes on a thin slice with a nonconducting bottom surface.

$$\rho=2\pi a\frac{V}{I}\times\frac{1}{G_7}$$

If the thickness of a conductor is 5 or more times greater than probe spacing (other dimensions infinite) the expression for Wenner alpha array on a half-space conductor can be used*

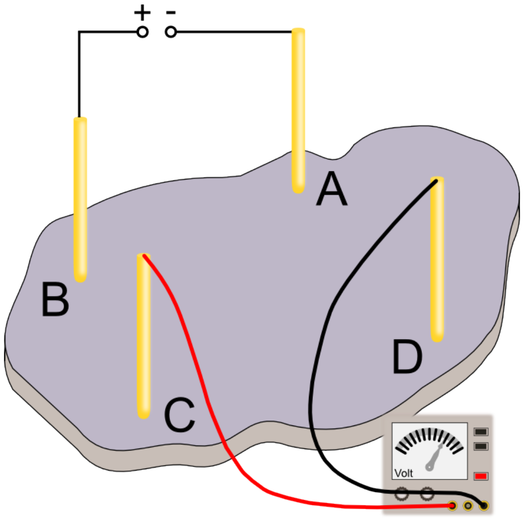
*it introduces only minor deviations from the exact value

Basics of resistance measurements

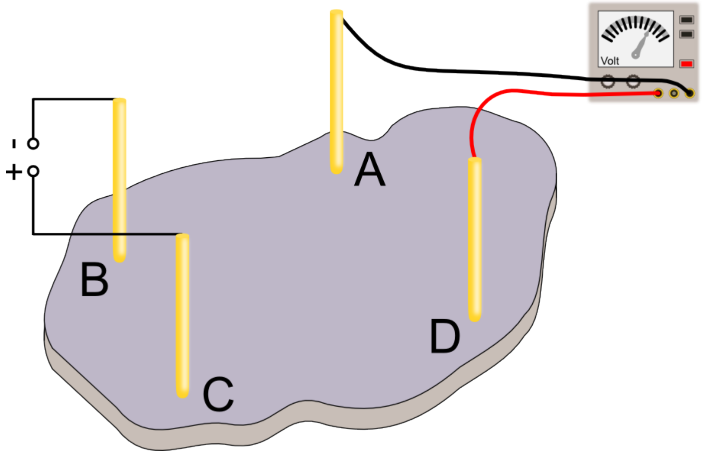
Van der Pauw's method [13] – used to measure thin samples of arbitrary shape:

- simply connected geometry (no areas of a different conductivity- holes, inclusions)
- point-like contacts
- homogeneous and isotropic electric conductivity

Two consecutive measurements are performed with each pair of electrodes used once as the current electrodes and once as the voltage electrodes – two resistances $R_{AB,CD}$ and $R_{BC,DA}$ are obtained



$R_{AB,CD}$



$R_{BC,DA}$

Basics of resistance measurements

It can be shown than in van der Pauw's method [13,16] the resistances $R_{AB,CD}$ and $R_{BC,DA}$ are related by the following formula:

$$\exp\left(-\pi\frac{R_{AB,CD}}{R_s}\right)+\exp\left(-\pi\frac{R_{AB,CD}}{R_s}\right)=1$$

$$R_s=\frac{\rho}{t} \qquad [Ohm]$$

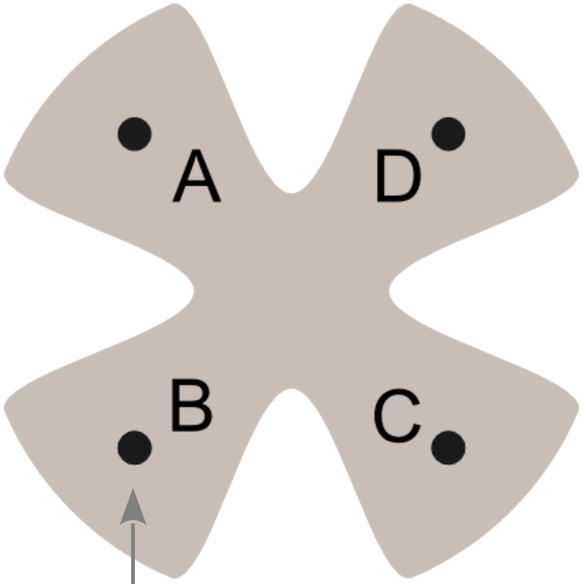
If the device and the electrode placement is symmetric the expression simplifies to:

$$\exp\left(\pi\frac{R_{AB,CD}}{R_s}\right)=2$$

Which gives for the sheet resistance:

$$\boxed{R_s=\pi\frac{R_{AB,CD}}{\ln(2)}}$$

example



Note that van der Pauw assumes point-like contacts between the electrodes. In real systems the contacts are extended but the proper modification of the method allows use of **arbitrary symmetric contacts** on a symmetric device [13,16]

Basics of resistance measurements

- When measuring resistances in a normal range (>10 Ω [14]) the 2-wire method can be used to obtain relative changes of resistance (e.g., in magnetoresistance measurements common in spintronics)
- If however the resistance of a device under test (DUT) is smaller the typical lead resistance (1mΩ to 10mΩ* [14]) makes the use of a 4-point method necessary
- If the measurements involves low voltages it may be necessary to cancel out **thermoelectric voltage** by making two voltage measurements, with reversed current polarization, and taking the average (in ferromagnetic metals room temperature thermoelectric coefficients are in 20×10^{-6} V/K range)
- Since it is usually not possible to use large area contacts in laboratory measurements special care must be taken to avoid spurious contact resistances

image source: http://www.bmius.com/c-128-standards.aspx

The standard is equipped with large contacts to minimize effects of the contact resistance

Honeywell four-terminal
1 Ohm resistance standard

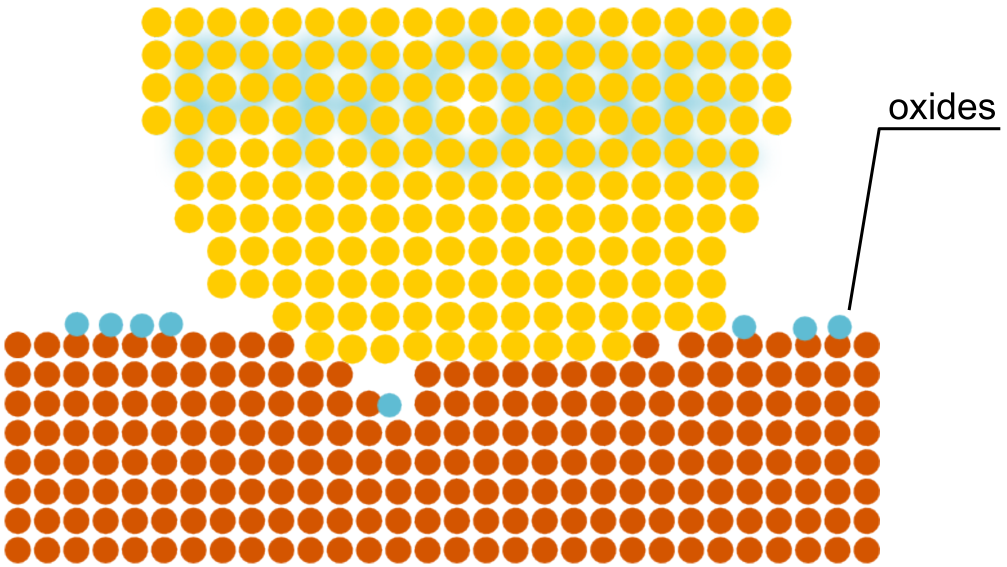
*it may be overly optimistic: my DMM (not the cheapest one) has about 30mΩ

Basics of resistance measurements

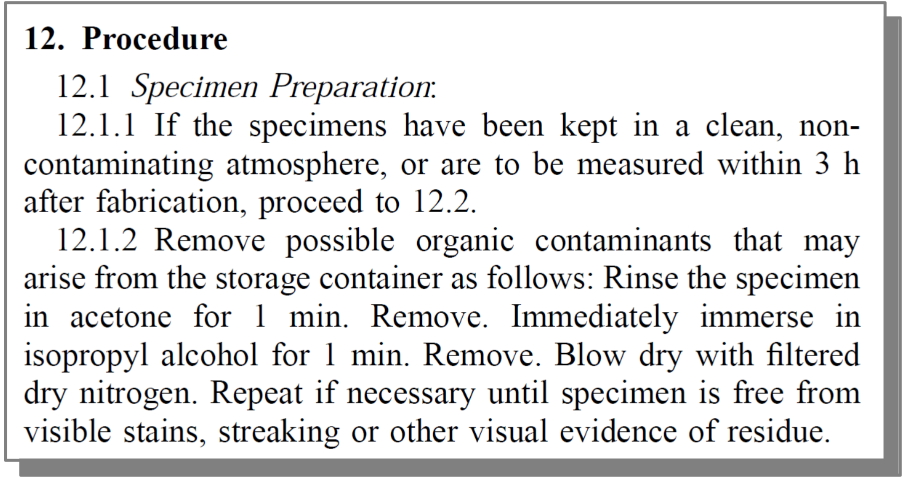
- **Contact resistance** – additional resistance due to a contact between two elements (influenced by contamination, oxide layers etc.)
- In devices the specific contact resistance $R_{c,spec}$ should be below $10^{-5}\ \Omega\text{cm}^2$ [16]*; it should be noted that the effective $R_{c,spec}$ depends on the contact geometry
- To assure the repeatability of measurements the normal force of the probe on the sample should be controlled [17]

Digression:
always use standardized procedures of surface cleaning prior to the measurement (not necessary with fresh samples)

Schematic of a contact geometry:



excerpt from Standard Test Method for Sheet Resistance Uniformity Evaluation by In-Line Four-Point Probe with the Dual-Configuration Procedure, American Society for Testing and Materials 1997 [17]:



12. Procedure

12.1 *Specimen Preparation*:

12.1.1 If the specimens have been kept in a clean, non-contaminating atmosphere, or are to be measured within 3 h after fabrication, proceed to 12.2.

12.1.2 Remove possible organic contaminants that may arise from the storage container as follows: Rinse the specimen in acetone for 1 min. Remove. Immediately immerse in isopropyl alcohol for 1 min. Remove. Blow dry with filtered dry nitrogen. Repeat if necessary until specimen is free from visible stains, streaking or other visual evidence of residue.

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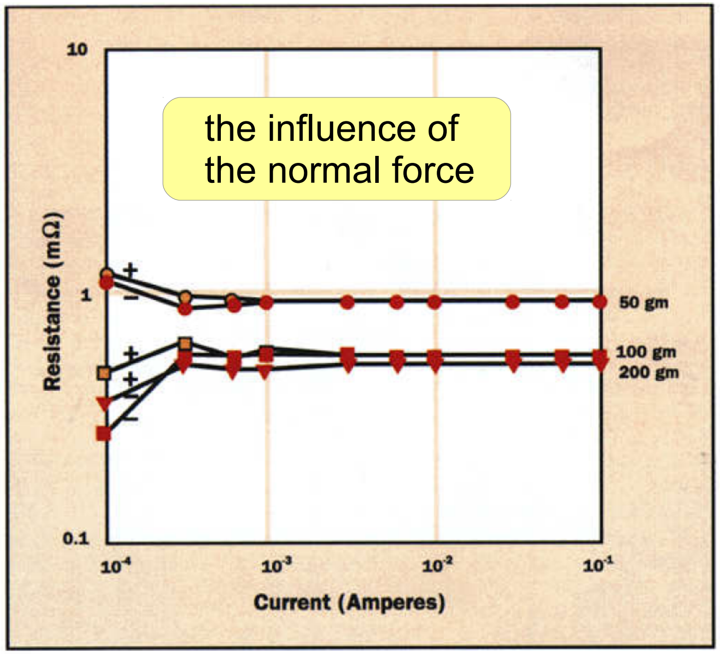
* to get the contact resistance one divides $R_{c,spec}$ by contact area

Basics of resistance measurements

- Contact resistance – additional resistance due to a contact between two elements (influenced by contamination, oxide layers etc.)



the influence of the normal force

Figure 3. Contact resistance vs. current for silver-plated contacts. Data are shown at 50-, 100- and 200-gram weight normal forces with both forward and reverse current.

Graphics from:
E.M. Bock, Jr.
AMP Journal of Technology **3**, 64 (1993) [15]

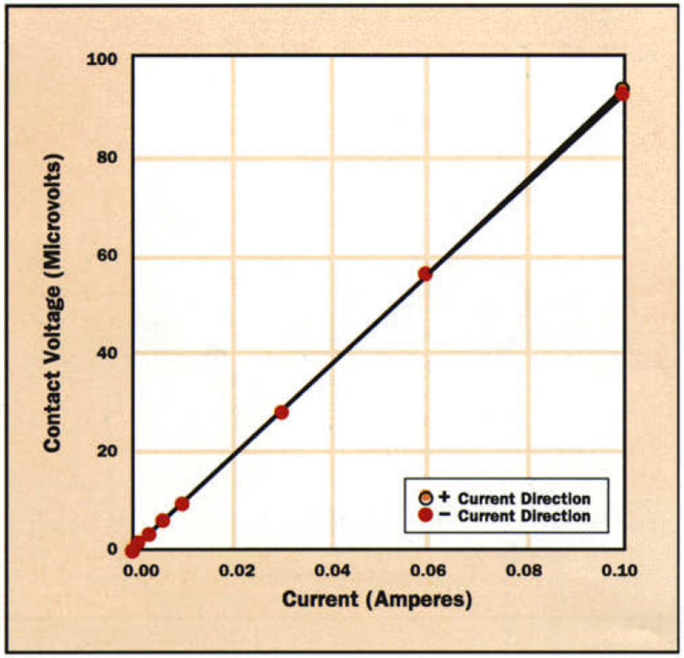


Figure 4. Contact voltage plotted vs. current for silver-plated contacts loaded to 50-gram weight. Data are shown for both forward and reverse current.

- Special finish/plating is usually applied to measurement probe tip to optimize its properties; the finish is usually made out of noble metal (electroplating, vacuum deposition)
- Contact resistance is dependent on the probe force
- Down to the 1 nA* measuring current the resistance of the contact is constant – completely ohmic contacts (the same was true for gold coated probe tips) [15]

* which is much less than the currents you will usually use in your measurements

Basics of resistance measurements

Tip wear – the probes should be regularly checked for any damage/contamination as they may change contact resistance, the flow of current in the sample [18] and damage the sample itself [17]

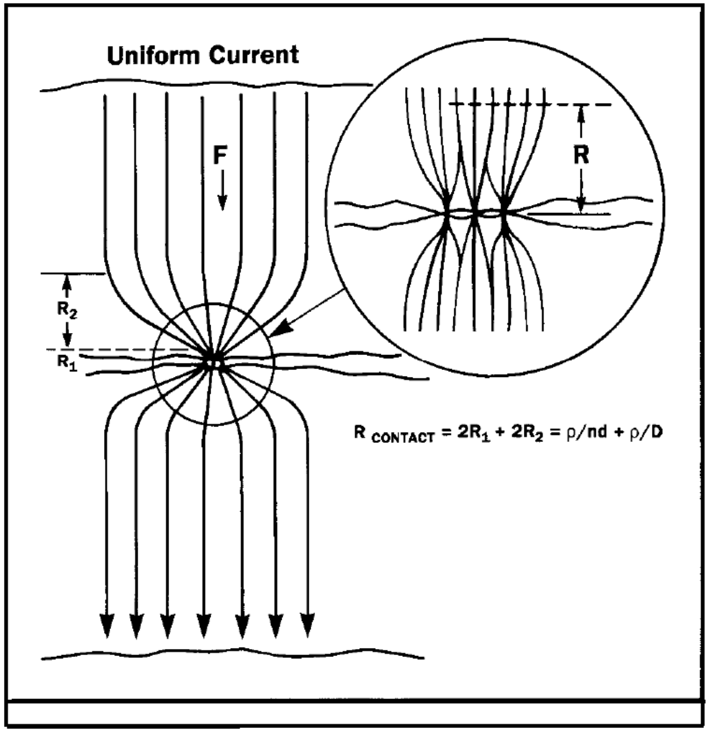


Figure 2. Schematic illustration of constriction resistance. Constriction resistance is a geometric effect.

graphics from:
Connector Design - Materials and Connector Reliability
R. S. Mroczkowski, AMP Incorporated 1993

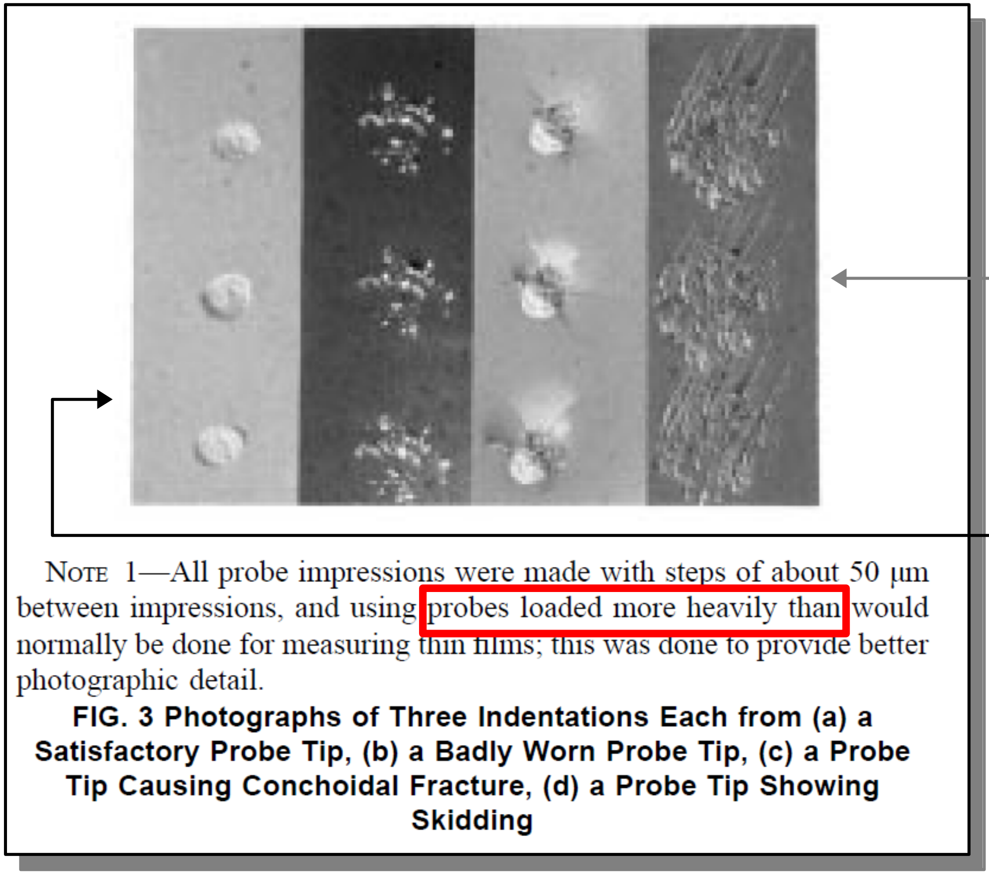
Joule heating in the vicinity of the constriction is another factor that may influence the measurement

$$P = R I^2$$

Basics of resistance measurements

Tip wear – if one has an access to a microscope it is advisable to check the probe quality by making *Visual Inspection of Probe Impressions* [17]:

- this test should be performed for new or refurbished probes
- the test should be performed if the standard deviation of the measurements performed at different locations on the homogeneous sample exceeds some previously set limit*



NOTE 1—All probe impressions were made with steps of about 50 µm between impressions, and using probes loaded more heavily than would normally be done for measuring thin films; this was done to provide better photographic detail.

FIG. 3 Photographs of Three Indentations Each from (a) a Satisfactory Probe Tip, (b) a Badly Worn Probe Tip, (c) a Probe Tip Causing Conchoidal Fracture, (d) a Probe Tip Showing Skidding

always attempt to make a good impression

probe tip not driven parallelly to a normal to the surface

graphics from:
Standard Test Method for Sheet Resistance Uniformity Evaluation by In-Line Four-Point Probe with the Dual-Configuration Procedure
American Society for Testing and Materials 1997 [17]

*American Society for Testing and Materials suggests s.dev of **less than 0.1%** for sets of 10 measurement at one point [17]

Measurement errors etc.

Accuracy, precision etc. (definitions from International vocabulary of metrology(VIM) [20])

- **measurement accuracy** - closeness of agreement between a measured quantity value and a true quantity value of a measurand„ (note: <<'Measurement accuracy' is sometimes understood as closeness of agreement between measured quantity values that are being attributed to the measurand. >>)
- „the concept 'measurement accuracy' is not a quantity and is not given a numerical quantity value*. A measurement is said to be more accurate when it offers a smaller *measurement error*.*"
- **measurement error** - measured quantity value minus a *reference quantity value*
- **reference quantity** value can be a true quantity value of a measurand, in which case it is unknown, or a conventional quantity value, in which case it is known.
- **measurement precision** - closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions.
- „*Measurement precision* is usually expressed numerically by measures of imprecision, such as standard deviation, variance, or coefficient of variation under the specified conditions of measurement."
- **repeatability** condition of measurement - „condition of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time"
- **reproducibility** condition of measurement - „condition of measurement, out of a set of conditions that includes different locations, operators, measuring systems, and replicate measurements on the same or similar objects" [20] and/or different reference standard [21]

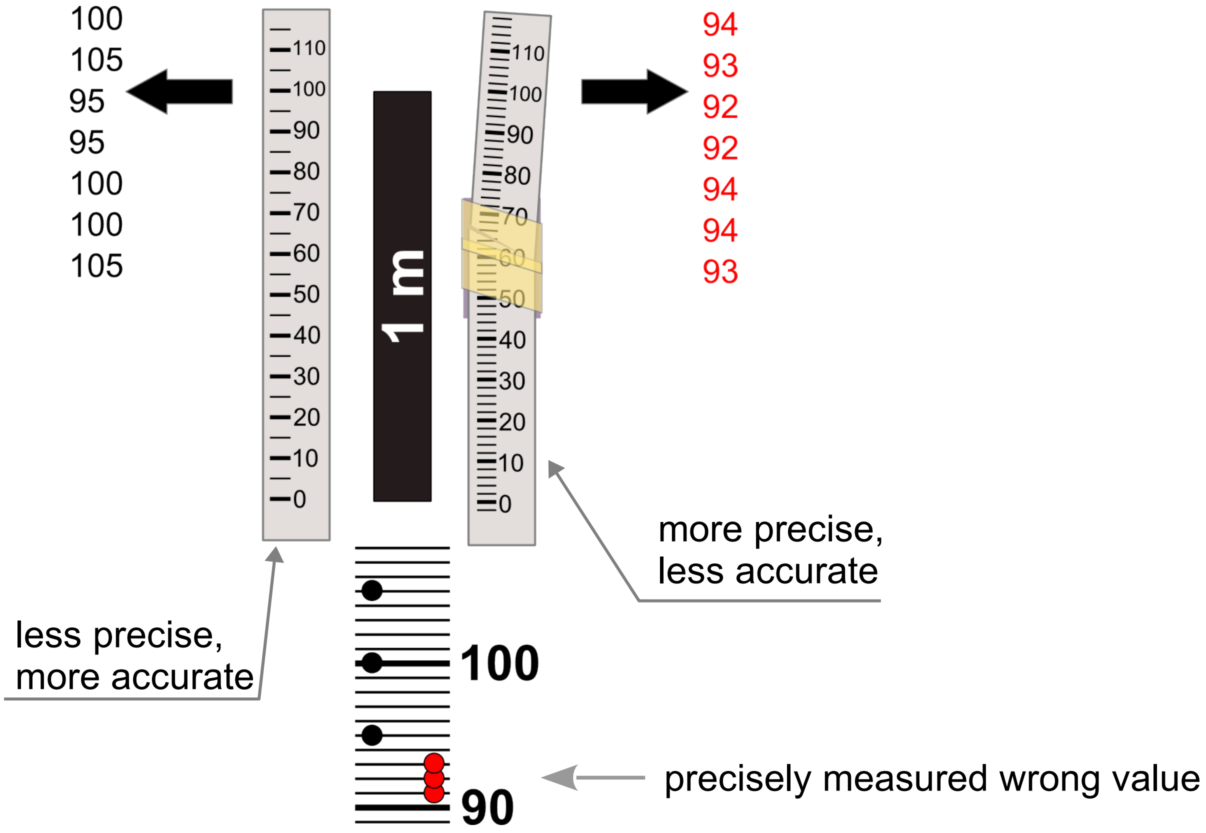
*note however that many manufacturers give a numerical value to it ([14], Yokogawa DS200 etc.):
% of reading+% of range etc.

Measurement errors etc.

Accuracy, precision etc. (definitions from International vocabulary of metrology(VIM) [20])

- **measurement uncertainty** - non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used

The parameter may be, for example, a standard deviation called **standard measurement uncertainty** (or a specified multiple of it)



less precise, more accurate

more precise, less accurate

precisely measured wrong value

Measurement errors etc.

Johnson noise – thermal noise (phonons, magnons etc.) results in random fluctuations of a voltage at the terminals of every resistor

The mean-square noise voltage of that noise is given by [22]*:

$$V_{rms}^{2}=4kTR\Delta f$$

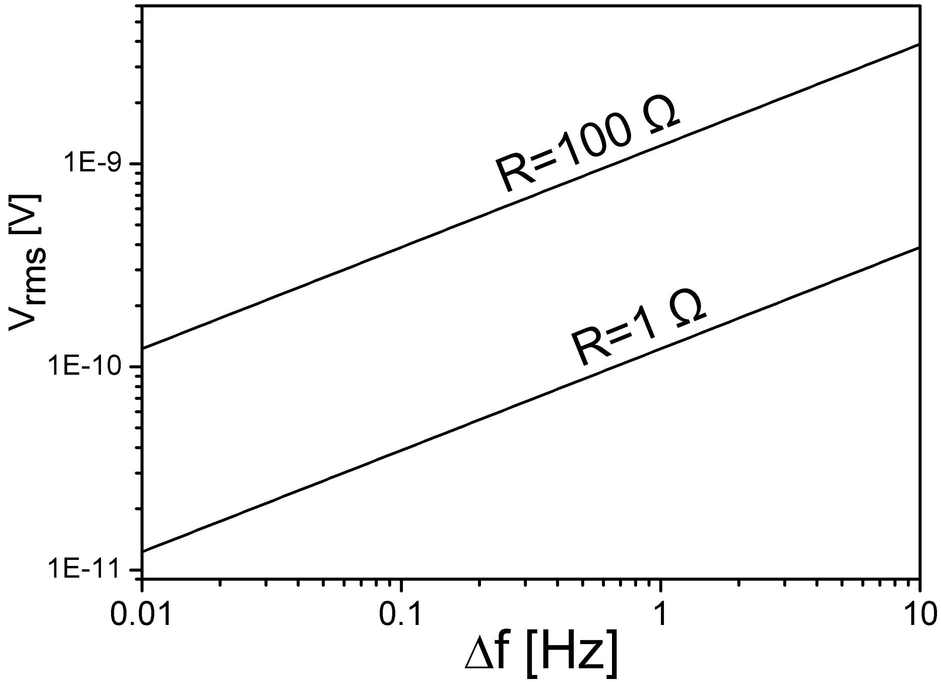
k – Boltzmann's constant
T – temperature
R – resistance
Δf – bandwidth of the measurement

To get a peak-to-peak noise one should triple the V_{rms} (for normal distribution some 99.7% of values lies within that range)

- bandwidth -"*The range of frequencies that can be conducted or amplified within certain limits. Bandwidth is usually specified by the –3dB (half-power) points.*" [14]

$$10\log_{10}0.5=-3.0103...$$

-3dB corresponds roughly to 30% decrease of a voltage



in "static" (direct current) measurements with typical voltmeters (low bandwidth) the Johnson noise can usually be neglected provided that sample resistance is not too high (say less than 10 kΩ)

---

*at room temperature this approximation (constant noise for every frequency) holds up to tens of gigahertz [23]

Conclusions

- resistivity measurements involve knowledge of sample and probes geometry
- contact resistance must be taken into account only in exceptional cases
- in many laboratory applications the relative change of resistivity and not the resistivity itself is the quantity of interest
- standard equipment is usually sufficient to conduct measurements (currents and voltages are relatively high)

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