5

MAGNETIC MATERIALS AND HYSTERESIS

Magnetic hysteresis

Poznań 2019

Maciej Urbaniak

5
Magnetic hysteresis

- General properties of magnetic hysteresis
- Rate-dependent hysteresis
- Preisach model

Essential parameters of commercial magnetic materials

ARNOLD
MAGNETIC TECHNOLOGIES

RECOMA®

Summary of main grades of $SmCo_5$ and Sm_2Co_{17}

Product	Designator[(1)]	(BH)max				Br				Hcb				Intrinsic Coercivity Hcj				Density	Magnetizing Field [(2)]	Temperature Coefficient of Br (20-150˚C)	Maximum Operating Temperature [(3)]
		kJ/m³		MGOe		T		kG		kA/m		kOe		kA/m		kOe		g/cm³	kA/m	%/K	˚C
		typ	min	typ	min	typ	min	typ	min	typ	min	typ	min	typ	min	typ	min	typ	min	typ	
Recoma 18	(135/200) A	143	135	18.0	17.0	0.87	0.83	8.7	8.3	650	600	8.2	7.5	2400	2000	30	25	8.4	>2000	-0.045	250
Recoma 20	(140/200) A	160	140	20.1	17.6	0.90	0.85	9.0	8.5	700	640	8.8	8.0	2400	2000	30	25	8.4	>2000	-0.045	250
Recoma 22	(155/200) T	175	155	22.0	19.5	0.94	0.90	9.4	9.0	730	680	9.2	8.6	2400	2000	30	25	8.4	>2000	-0.045	250

source: www.arnoldmagnetics.com, 2019.05.25

Depending on the use (e.g. permanent magnets or transformers applications) different parameters of the hysteresis are most important. In hard magnets applications the (BM)max* and coercivity are most relevant. In electrical steel for transformers hysteresis losses and attainable polarization ($\mu_0 M$) are crucial.

thyssenkrupp Electrical Steel GmbH

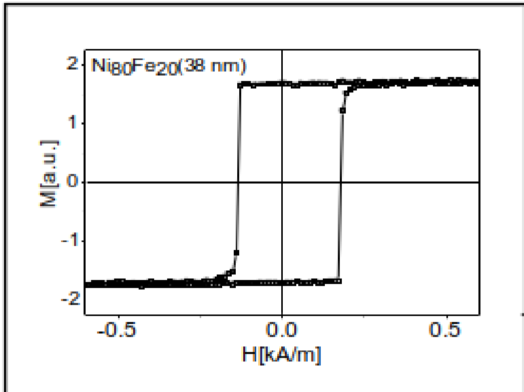
powercore® C: Guaranteed magnetic properties

Grade	Thickness	Thickness	Typical core loss at				Guaranteed core loss at		Typical polarization at	Guaranteed polarization at
	[mm]	[inch]	1.5 T	1.7 T	1.5 T	1.7 T	1.7 T	1.7 T		
			50 Hz	50 Hz	60 Hz	60 Hz	50 Hz	60 Hz	800 A/m	800 A/m
			W/kg	W/kg	W/lb	W/lb	W/kg	W/lb	typ. T	min. T
C 120-23	0.23	0.009	0.77	1.18	0.46	0.71	1.20	0.72	1.83	1.78
C 120-27	0.27	0.011	0.80	1.18	0.48	0.71	1.20	0.72	1.83	1.80
C 130-27	0.27	0.011	0.83	1.23	0.50	0.74	1.30	0.78	1.83	1.78

source: www.thyssenkrupp-steel.com/en/electricalsteel, 2019.05.29

* see my lecture from 2012 (www.ifmpan.poznan.pl/~urbaniak/Wyklady2012/urbifmpan2012lect3_02.pdf)

Exemplary M(H) hystereses of thin films



J. Magn. Magn. Mater. **190**, 187 (1998)

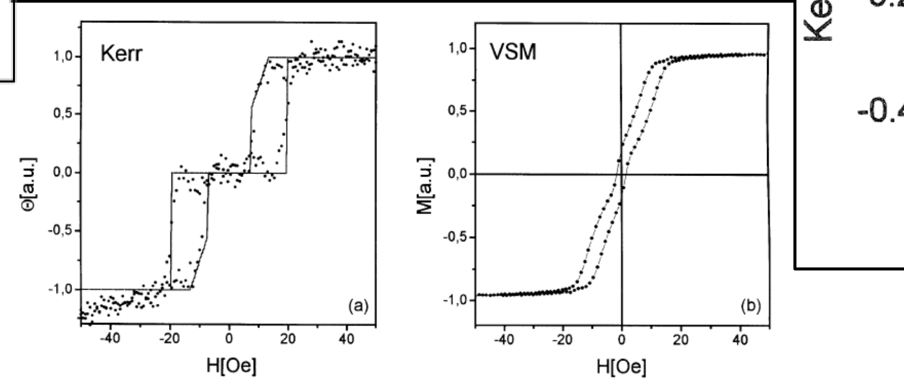
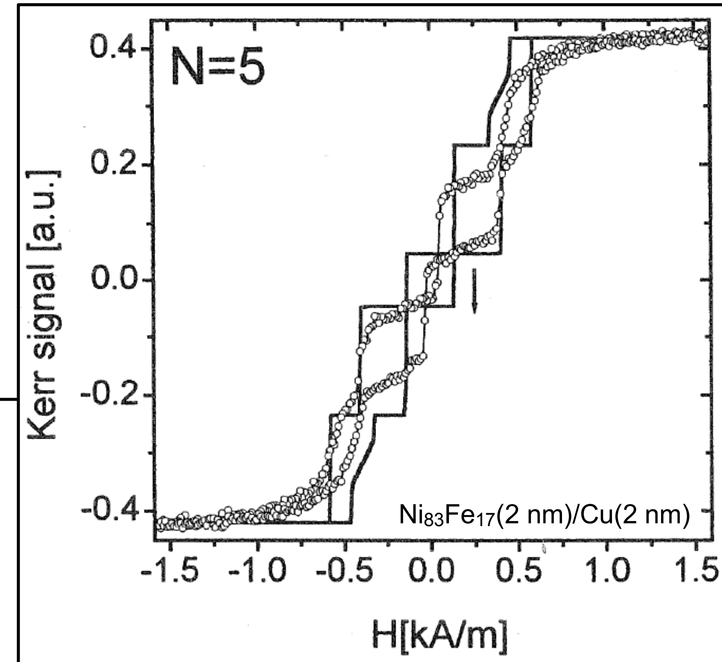
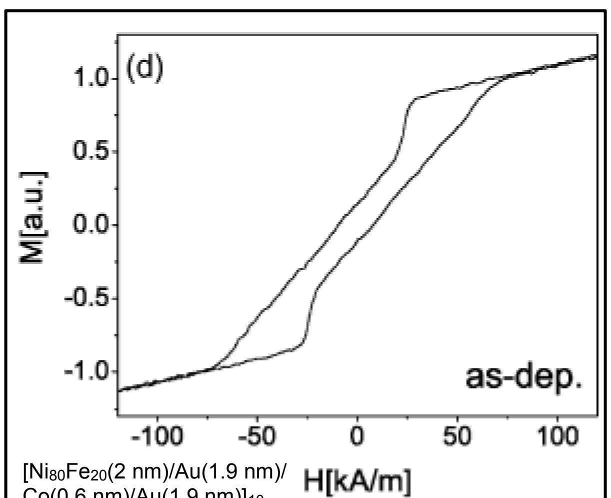


Fig. 2. The field dependence of (a) the Kerr rotation $\Theta_K(H)$ (line shows a fit according to the model of Dieny (see text)) and (b) magnetisation $M(H)$ of Py(2 nm)/Cu(2 nm) multilayer with 101 magnetic sublayers.

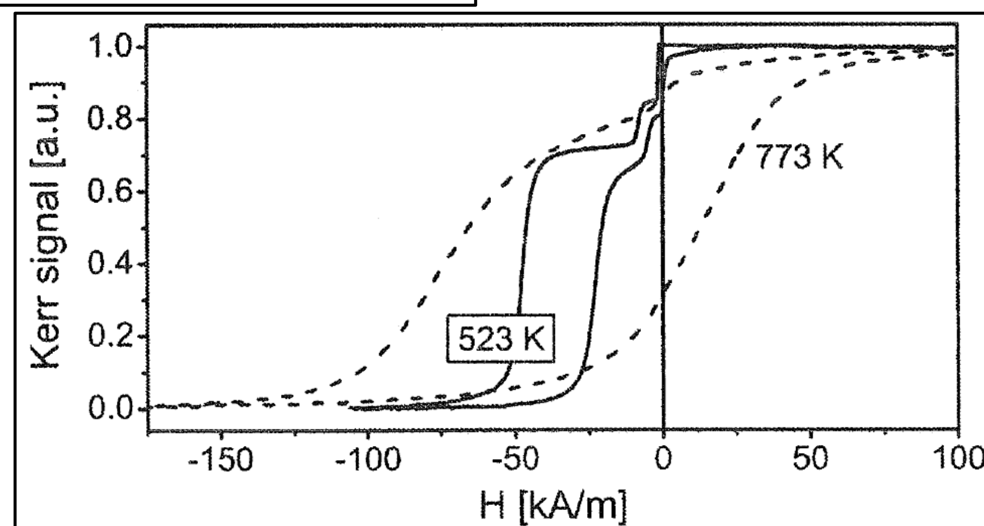
Phys. Stat. Sol. (a) **186**, 423 (2001)



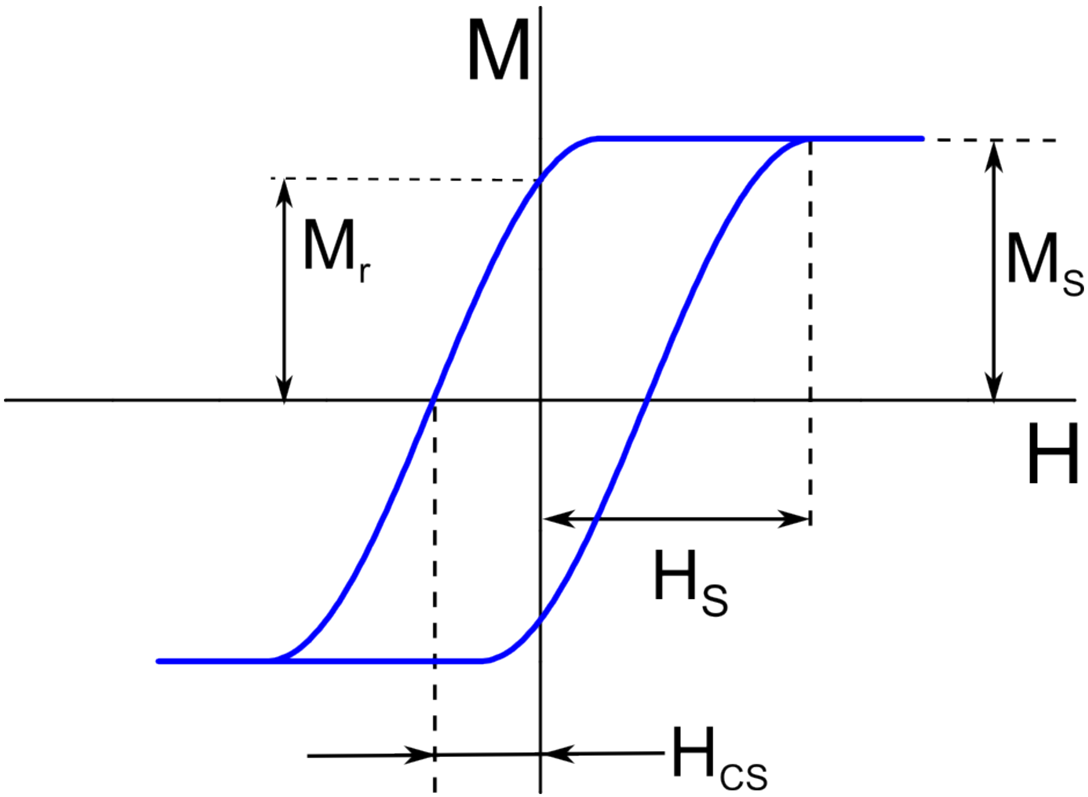
J. Alloys Compd. **454**, 57 (2008)



Phys. Stat. Sol. (a) **199**, 284 (2003)



$Ni_{80}Fe_{20}$(4 nm)/$Mn_{83}Ir_{17}$(15 nm)/$Co_{70}Fe_{30}$(3 nm)/Al(1.4nm)+Ox/$Ni_{80}Fe_{20}$(4 nm)/Ta(3 nm)

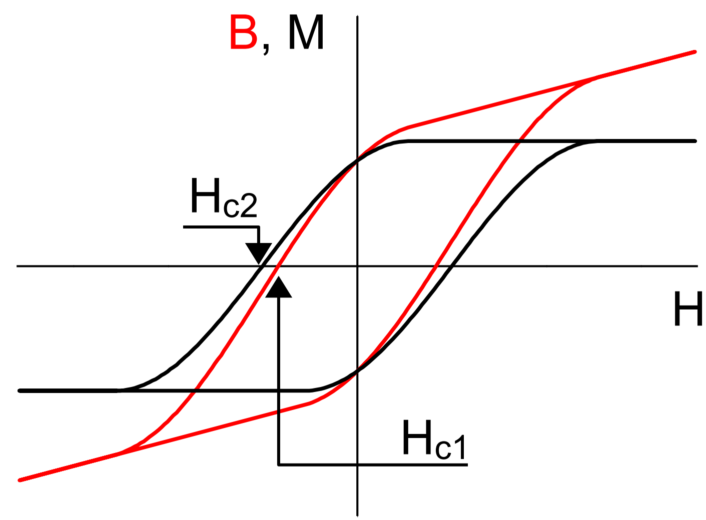


•A hysteresis loop can be expressed in terms of ***B***(***H***) or ***M***(***H***) curves.

•In soft magnetic materials (small H_s) both descriptions differ negligibly [1].

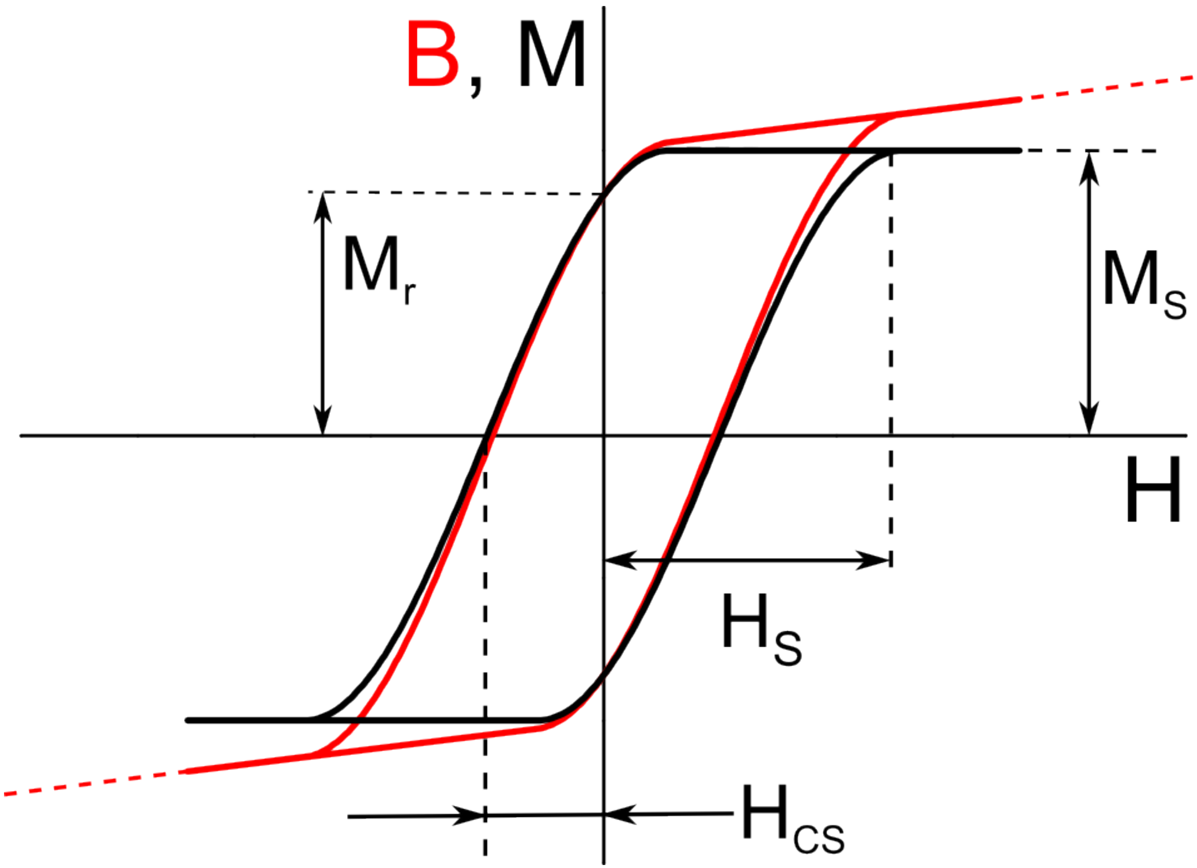
•In hard magnetic materials both descriptions differ significantly leading to two possible definitions of coercive field (and coercivity).

•***M***(***H***) curve better reflects the intrinsic properties of magnetic materials.



Hysteresis nomenclature

The magnetic hysteresis can be presented both as ***B***(***H***) and ***M***(***H***)* dependencies.



intrinsic induction:

$$\vec{B}_i = \vec{B} - \mu_0 \vec{H} = \mu_0 \vec{M}$$

coercive field strength – field required to reduce the **magnetic induction** to zero after the material has been symmetrically cyclically magnetized.

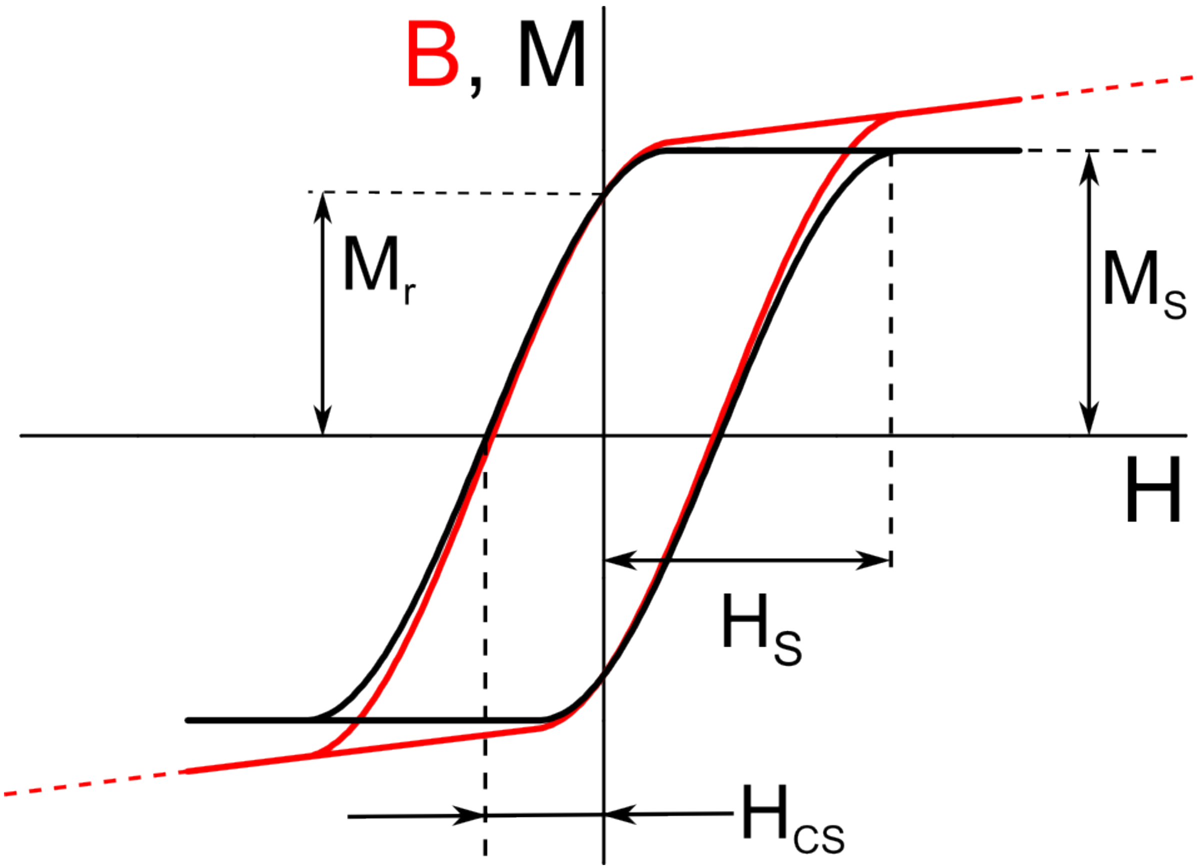
intrinsic coercive field strength – field required to reduce the **intrinsic induction** to zero after...

coercivity, *H*cs—the maximum value of coercive field strength that can be attained when the magnetic material is symmetrically cyclically magnetized to *saturation induction*, B_S.

*μ_0 ***M***(H) dependence is called a intrinsic hysteretis loop [5]

Hysteresis nomenclature

The magnetic hysteresis can be presented both as ***B***(***H***) and ***M***(***H***)* dependencies.



saturation induction, *B*s—the maximum intrinsic induction possible in a material

saturation magnetization, **M**s:

$$\vec{M}_s = \vec{B}_s / \mu_0$$

demagnetization curve—the portion of a dc hysteresis loop that lies in the second (or fourth quadrant). Points on this curve are designated by the coordinates, B_d and H_d.

remanence, B_{dm}—the maximum value of the remanent induction for a given geometry of the magnetic circuit.

paraprocess, forced magnetization—after domain walls disappear at technical saturation further increase of the external field leads to the asymptotic increase of magnetization to that corresponding to the absolute zero of temperature [7]

*for the glossary of magnetic measurements terms see ASTM, A 340 – 03a, 2003 [5]

Hysteresis losses

We use the following formula [8] for the energy density of the magnetic field:

$$\rho_E = \frac{1}{2}\vec{B}\cdot\vec{H}$$

underline: complex quantity

Introducing complex permeability, $\underline{\mu} = \mu_0(\mu'_R - i\mu''_R)$, we get [6]:

$\vec{B} = \mu\vec{H} = \mu_0(\vec{M} + \vec{H})$

$$\underline{\tilde{\vec{B}}} = \underline{\mu}\,\underline{\tilde{\vec{H}}} = \mu_0(\mu'_R - i\mu''_R)\,\underline{\vec{H}}\,e^{i(\omega t + \alpha)}$$

$\underline{\vec{H}} = (H'_x + iH''_x, H'_y + iH''_y, H'_y + iH''_y)$

The mean power dissipated in a periodic system is given by [6]:

$$\overline{P} = \overline{\Re(\tilde{\vec{H}})\Re\left(\frac{\partial\underline{\tilde{\vec{B}}}}{\partial t}\right)} = \frac{1}{2}\Re\left[(\mu_0 * (\mu' - i*\mu'')) * \omega * (H'_x + iH''_x) * (H'_x - iH''_x)\right]$$

$\leftarrow \frac{\partial\rho_E}{\partial t} = \vec{H}\frac{\partial\vec{B}}{\partial t}$

$$= \frac{1}{2}\mu_0\omega\mu''(H'^2_x + H''^2_x) = \boxed{\frac{1}{2}\mu_0\omega\mu'' H^2}$$

dissipative term in complex magnetic permeability

$\overline{\Re(\tilde{\vec{H}}_1)\Re(\tilde{\vec{H}}_2)} = \frac{1}{2}\Re(\underline{H}_1\cdot\underline{H}_2{}^*)$

[6], page 240

conjugate

*"Magnetic Anisotropy" → pole avoidance principle

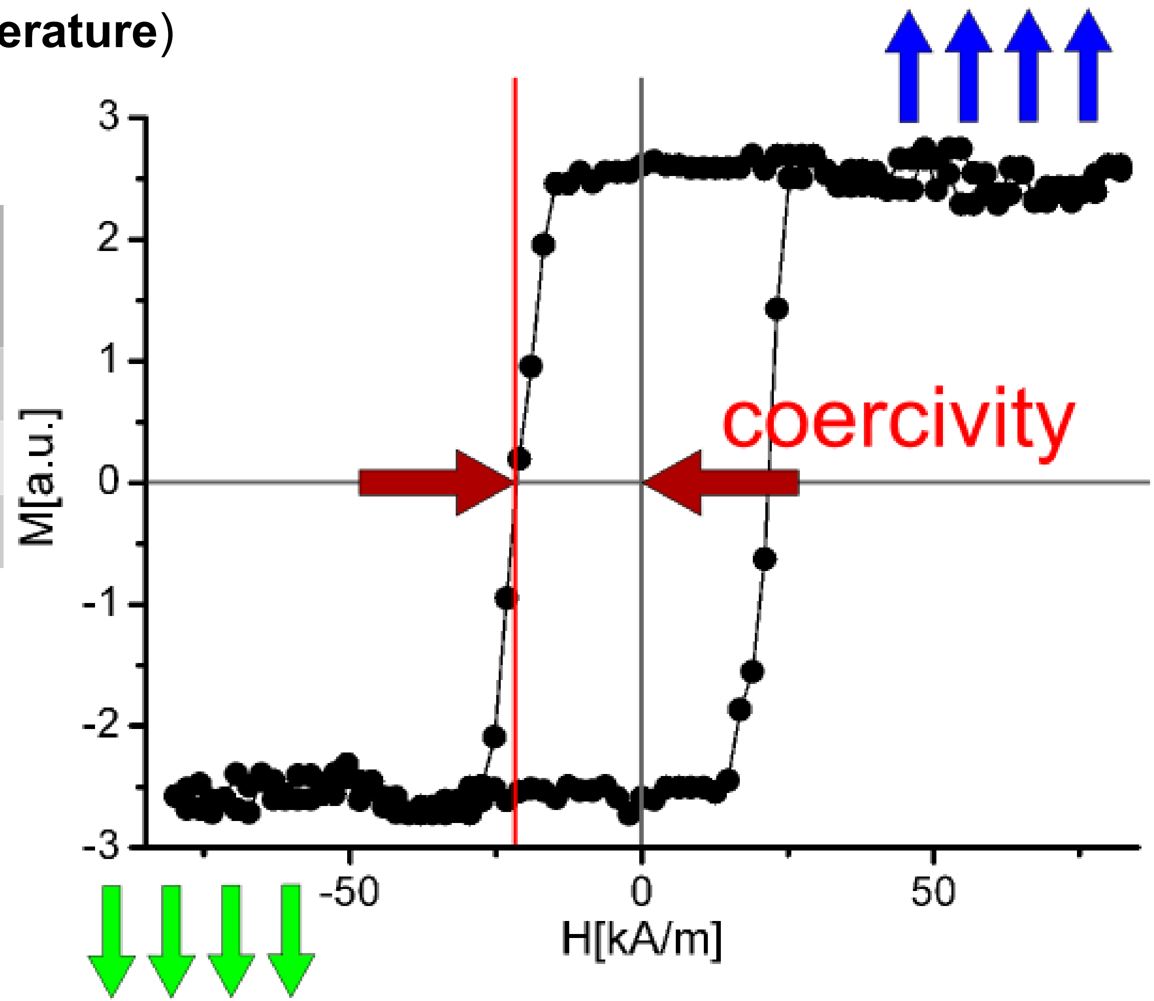
Ferromagnetic materials

Most notable features of ferromagnetic materials:
- high initial susceptibility/permeability
- they usually retain magnetization after the removal of the external field – **remanence**
- the magnetization curve (B-H or M-H) is nonlinear and hysteretic
- they lose ferromagnetic properties at elevated temperatures (**Curie temperature**)

The notable examples:

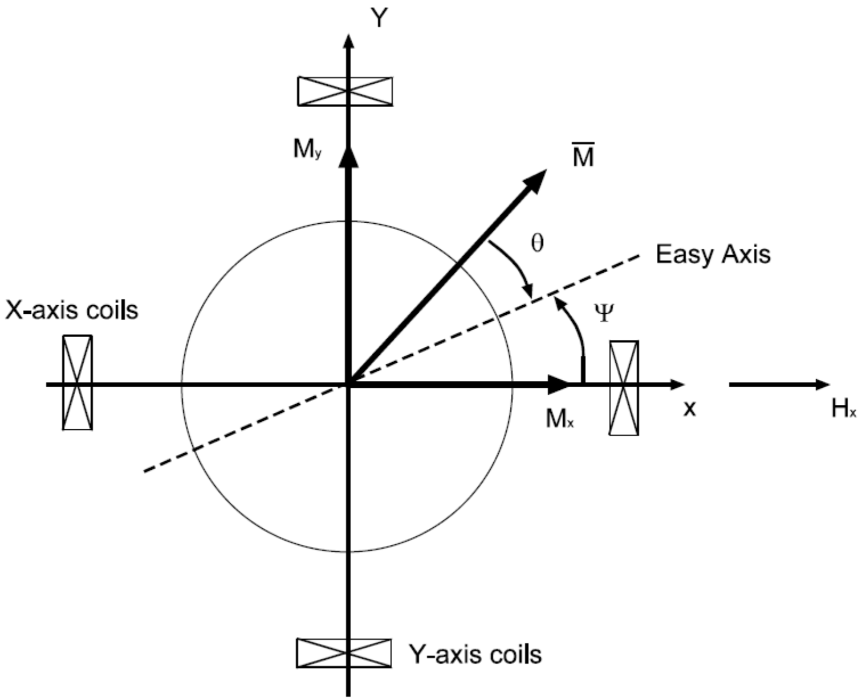
	M_S[kA/m]@RT	T_C[K]
Fe	1714	1043
Co	1433	1403
Ni	485	630

M_S[Am^2/kg]@RT
217.75
161
54.39



M(H) hysteresis – vector picture

Because field ***H*** and magnetization ***M*** are vector quantities the full description of hysteresis should include information about the magnetization component perpendicular to the applied field – it gives more information than the scalar measurement.
Vector Vibrating Sample Magnetometer (VVSM):



Rev. Sci. Instrum., Vol. 69, No. 9, September 1998
E.O. Samwel, T. Bolhuis, J.C. Lodder

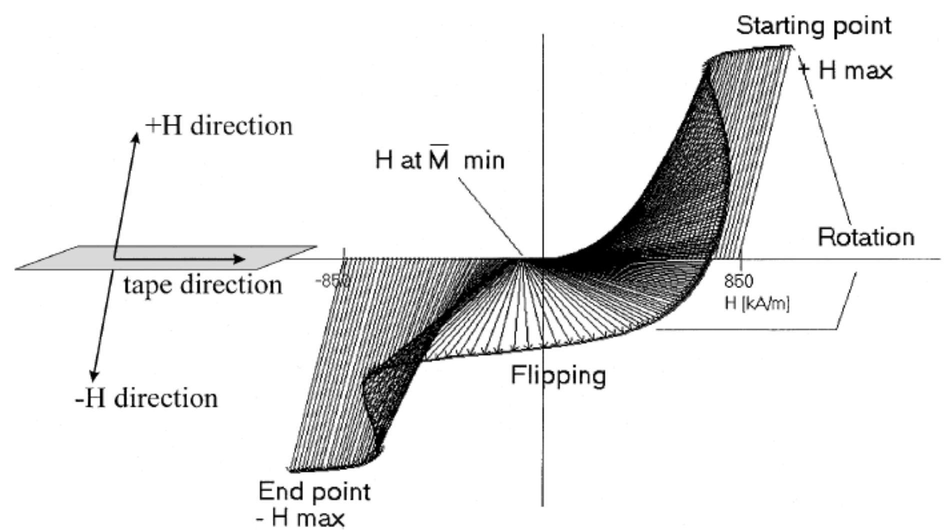


FIG. 1. Reversal process in a γ-Fe_2O_3 audio tape at 80°. The arrows indicate the magnetization vector. The downward half of a hysteresis loop is shown. The horizontal direction represents the sample plane, the vertical direction the normal to the tape.

Arrows showing magnetization direction start at point on field axis corresponding to the appropriate external field value

image from Magnetic Anisotropy: Measurements with a Vector Vibrating Sample Magnetometer, B. C. Dodrill, J. R. Lindemuth, and J. K. Krause
Lake Shore Cryotronics; www.lakeshore.com/Documents/Magnetic Anisotropy.pdf

M(H) hysteresis – vector picture

Vector Vibrating Sample Magnetometer (VVSM):

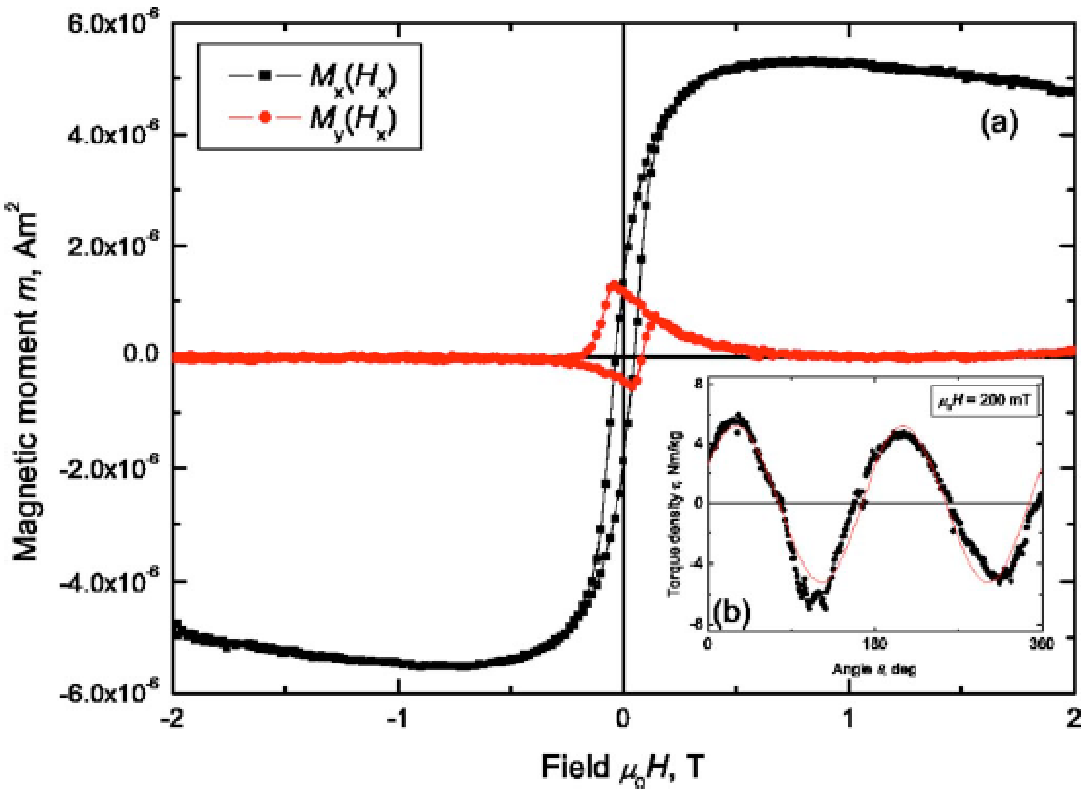


FIG. 2. (Color online) Magnetization curve of CrO_2 magnetic recording tape with the field applied perpendicular to the plane (a), 1 s per point. Torque curve for the same sample recorded at 200 mT, 100 ms per data point.

- VVSM magnetometer with two compact Halbach cylinders
- The vector measurement principle can be used with MOKE magnetometers too:

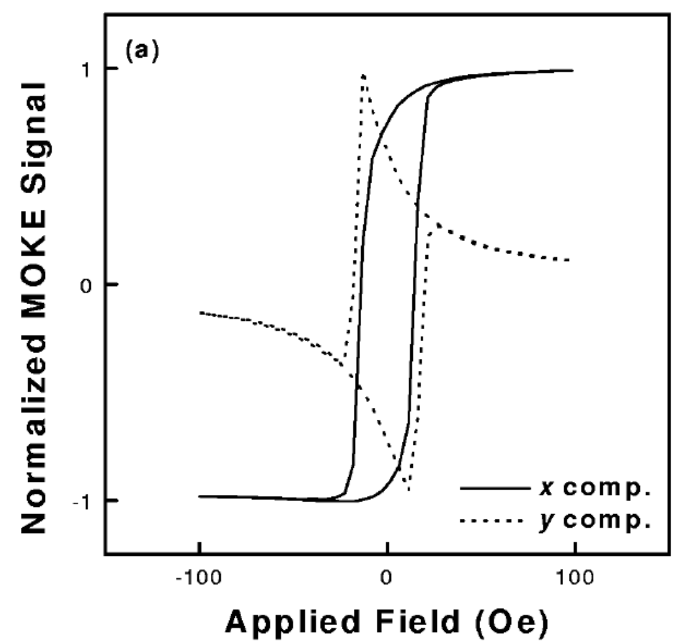


Fig. 3. Hysteresis loops of the x and the y components measured by using the MOKE for α=45°.

S. J. Kang et al.,Journal of the Korean Physical Society, 37 414 (2000)

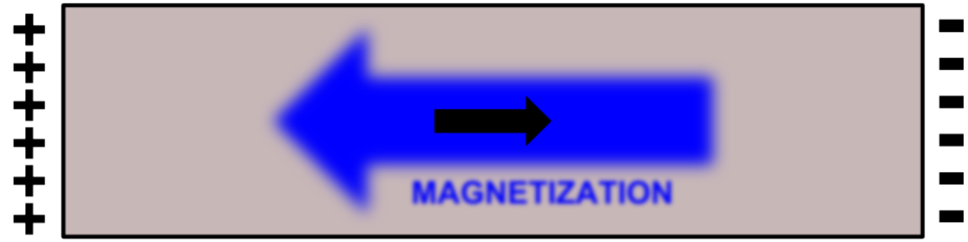
P. Stamenov and J. M. D. Coey, JOURNAL OF APPLIED PHYSICS **99**, 08D912 (2006)

M(H) hysteresis – shape of the specimen

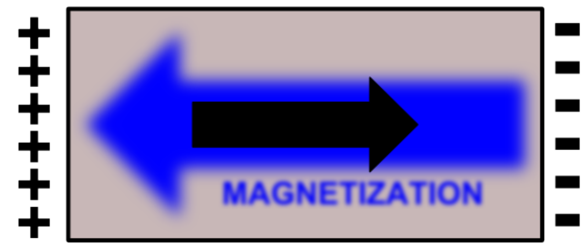
The demagnetizing field* changes the inner field within the sample which **can** lead to the change of characteristic fields of the hysteresis loop (switching fields, coercive field)

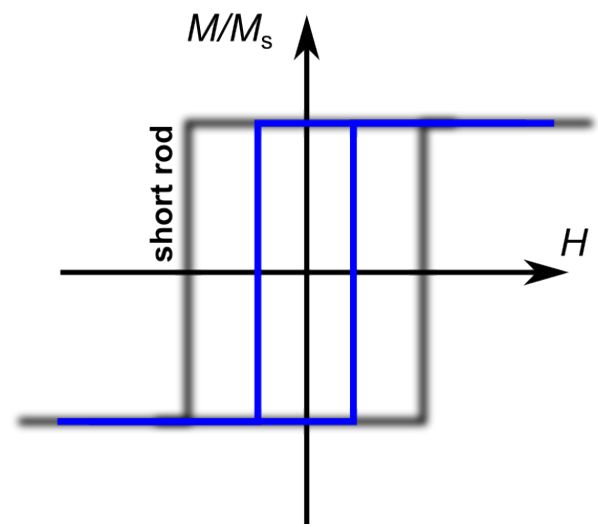
$$\vec{B}=\mu_0\left(-N\cdot\vec{M}+\vec{M}\right)$$

LONG ROD - weak demagnetization field



short rod -STRONG DEMAGNETIZATION FIELD





* see lecture 2 and 3

M(H) hysteresis – shape of the specimen

Shape of the sample influences the measured hysteresis not only through the scaling of ***H***-axis. The demagnetizing field can influence the character of the ***M***(***H***) dependence:

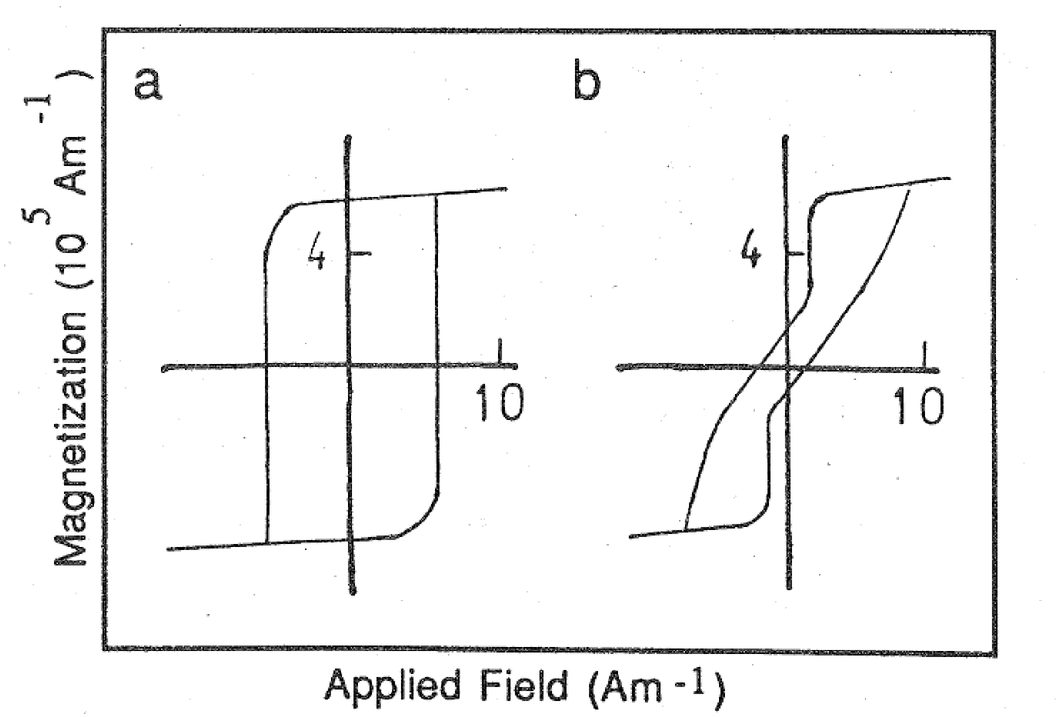


Fig. 1.- Hysteresis loops of two Fe-rich wires ($Fe_{77.5}Si_{7.5}B_{15}$) with different length: 13.1 cm(a) and 5.2 cm (b).

- The long wire made of a magnetically soft amorphous material shows a typical bistable behavior.
- Closure domains extend up to about 3cm from ends into the wire.
- In short wires (less than 7cm) the collapse of closure domains suppresses bistable behavior.

'There exists nothing that we can straightforwardly call the "hysteresis loop of iron".' - Bertotti [1]

To associate an experimental curve with a given material one has to state [1]:

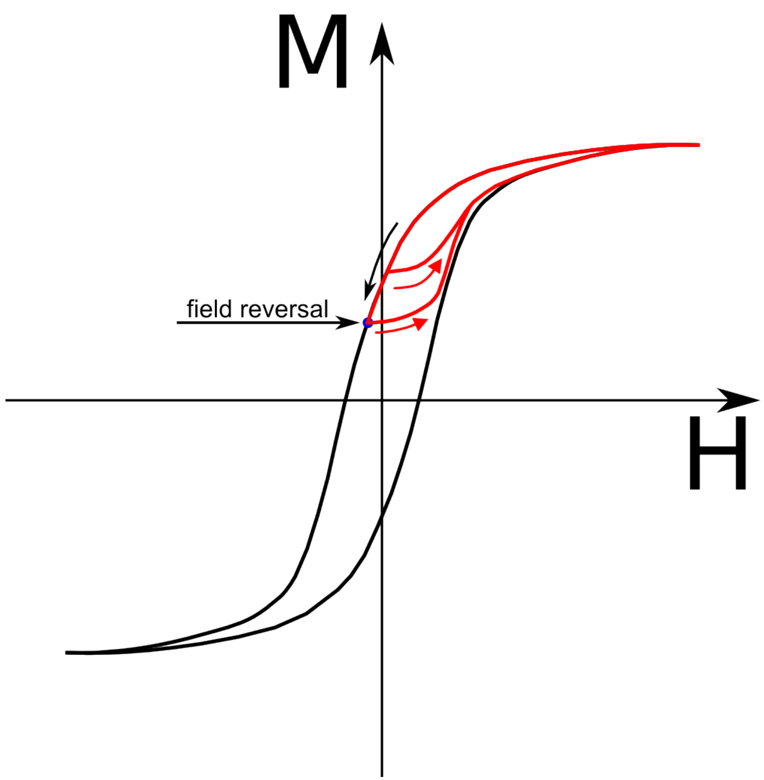
- the experimental conditions
- geometry of the experiment
- spatial scale of the experiment (*i.e., what is the size of the volume/area we get signal from*)

M. Vázquez, C. Gómez-Polo, D.-X. Chen, A. Hernando, IEEE Trans. Magn. **30,** 907 (1994)

M(H) hysteresis – cont'd

Technical saturation - state reached under field such that further increase of field does not change hysteresis properties

- Each point at the interior of the saturation loop can be reached in a infinite number of ways
- *First order return branch* – start at saturation and reversal at some point of saturation loop
- Second, third... order return branch- two, three... reversal fields
- *Ac-demagnetization* – sequence of large number of finely spaced reversals with decreasing values of reversal fields leads to **demagnetized state** (zero remanent magnetization)
- *Anhysteretic curve* – ac-demagnetizing field superimposed on constant field



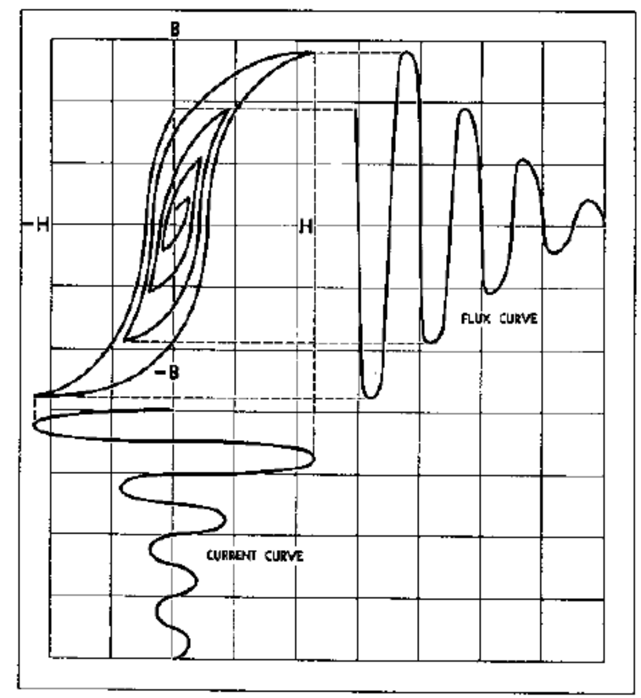
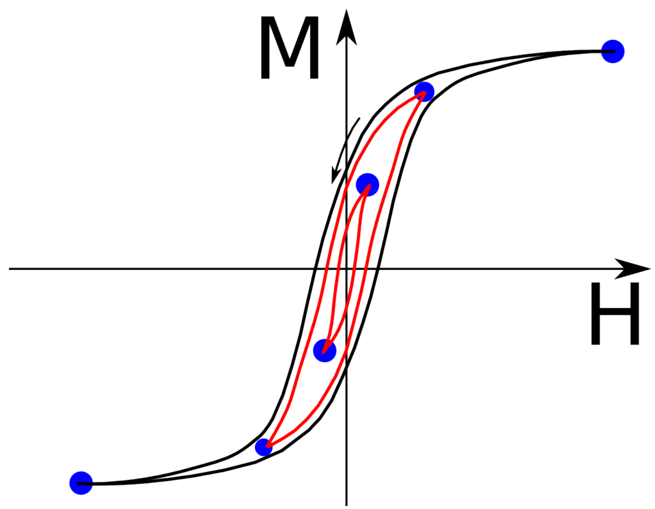


Figure 7.27: Demagnetization flux curve projected from hysteresis curve.

image from "Liquid Penetrant and Magnetic Particle Testing at Level 2"
INTERNATIONAL ATOMIC ENERGY AGENCY, 2000

- *Normal magnetization curve* – start from demagnetized state and cycle field with increasing amplitude; the line connecting the tips of the curve (places where the field sweep changes direction) [1]*.



Schematic drawing of normal curve – line connecting blue dots. *Note*: in real measurement all points are on the same **M**(***H***) curve obtained with field increasing somewhat like that:

$$H(t) = Const\ t\ \cos(\omega t),$$

where $\omega \gg Const$

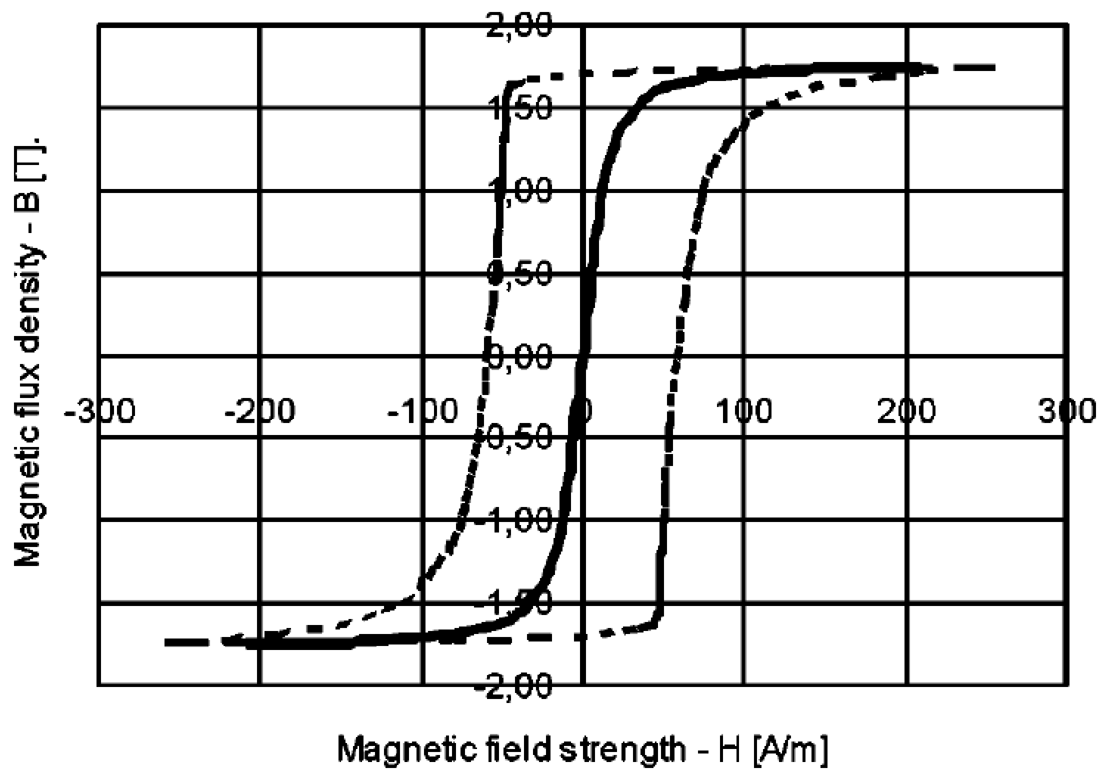


FIG. 1. Anhysteretic magnetization curve (continuous line) and hysteresis loop (dot line) of the magnetic materials.

Normal curve is similar but not equal to initial magnetization curve

*The normal curve can denote the ***B***(***H***) dependence too.

J. Kwiczala and B. Kasperczyk , J. Appl. Phys. 97, 10E504 (2005)

Barkhausen effect

- The magnetic moments of the magnetized body have different local surroundings (grains in polycrystals, dislocations, deformations, inclusions, interface/surface roughness etc.)
- These sources of disorder are coupled to the magnetization through exchange, anisotropy, magnetoelastic and magnetostatic interactions [1]
- As a result energy landscape of the system exhibits numerous local minima.
- At normal temperatures the heights of the energy barriers separating local minima is enough to keep the system in initial state.
- Applied field destroys that equilibrium; if it changes with time the system jumps abruptly to consecutive minima – this leads to a noncontinuous change of magnetic moment/magnetization with time

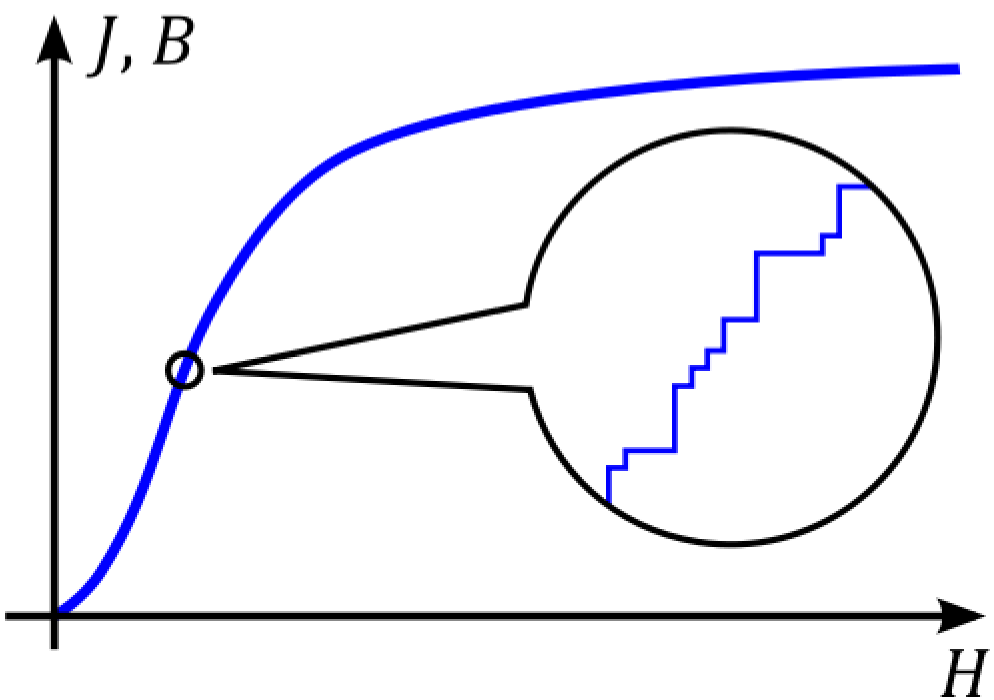


Image source: Wikimedia Commons; author: User:Stannered

- Barkhausen effect is closely related to the presence of magnetic domains

Barkhausen effect

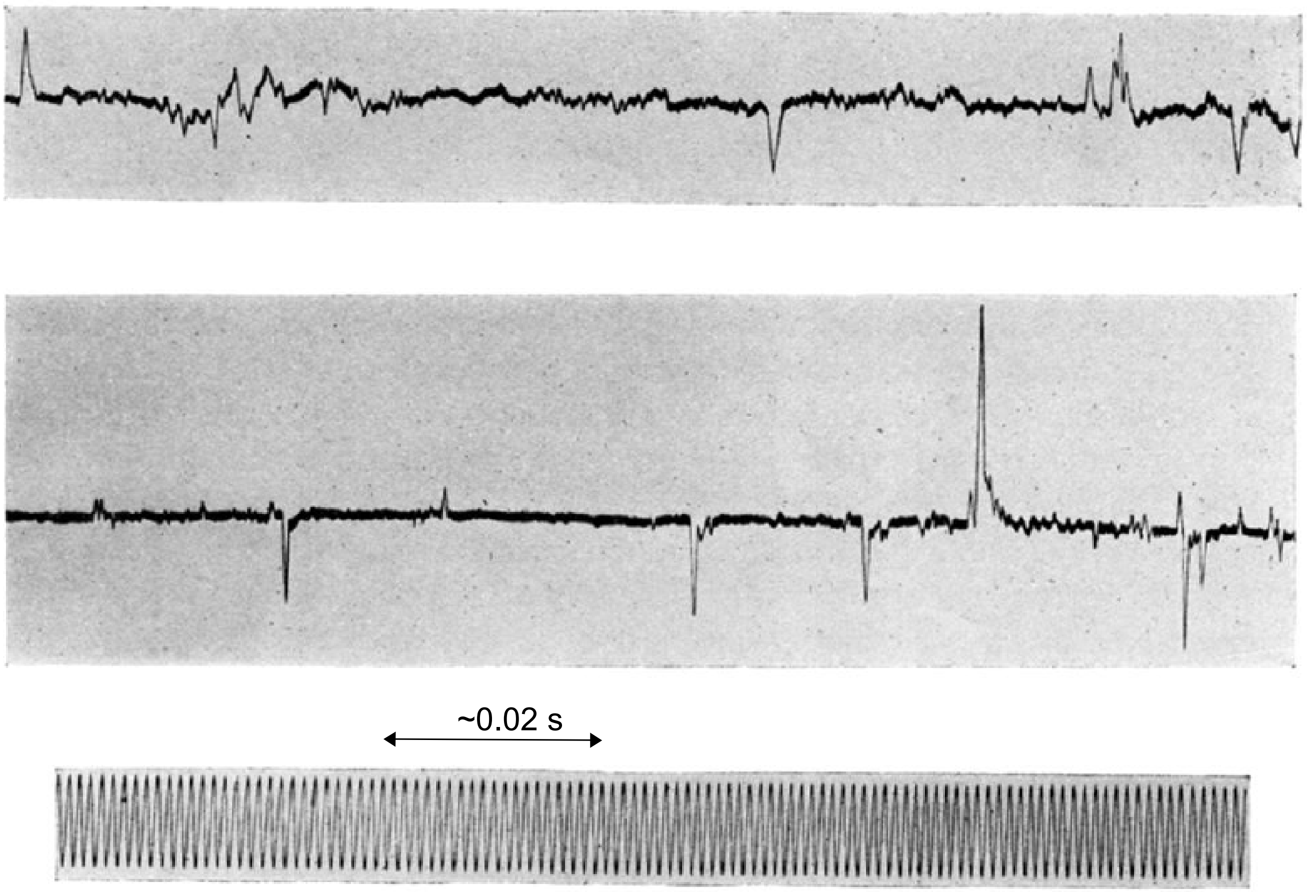


Fig. 16. Retouched oscillographic records of Barkhausen effect. Upper trace, record for sample 2 mm in diameter containing 78 percent Ni and 22 percent Fe. Middle trace, record for sample 1 mm in diameter containing 81 percent Ni and 19 percent Fe. Lower trace, timing line of frequency 1000 cycles per second.

R. M. Bozorth, Phys. Rev. **34**, 792 (1929)

Dynamic effects – eddy current losses

- The shape of the M(H) loop depends on field sweep rate:

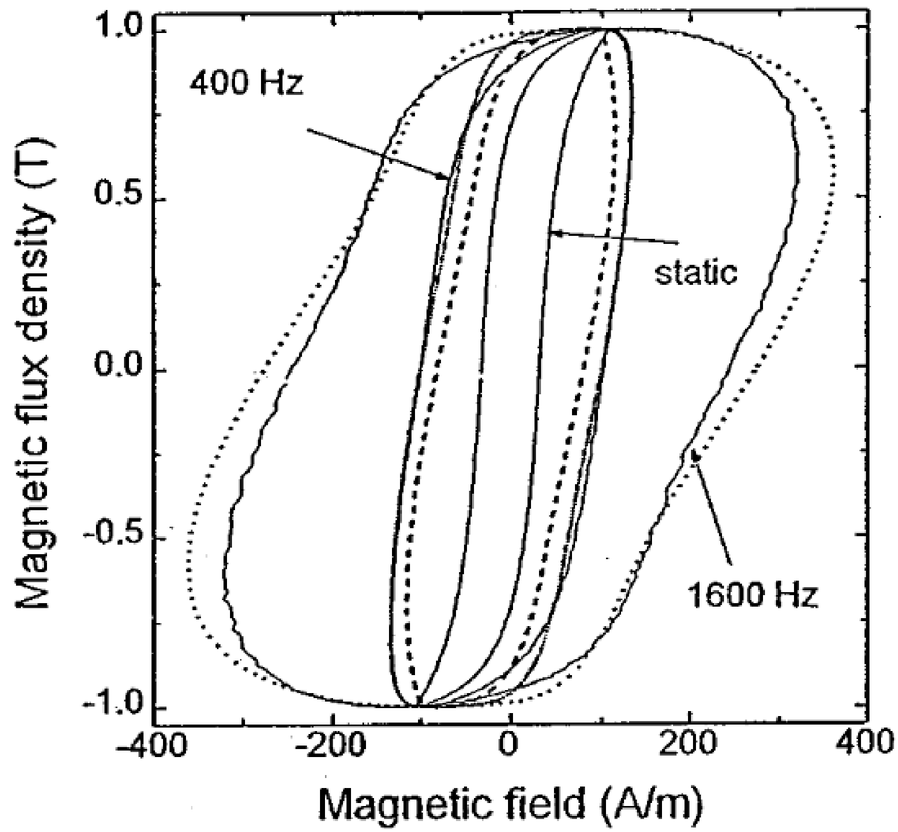


FIG. 3. Dynamic hysteresis loops at 1 T, for 0, 400, 1600 Hz. Continuous lines measurements. Dotted lines FEM prediction based on dynamic PM. Broken line. FEM prediction based on conventional PM.

- Measurement on commercial (AST 27/ 35) non-oriented Fe–Si alloys (2.9 wt. % Si, 0.4 wt. % Al), with average grain size of 75 μm. The lamination thickness was 0.334 mm.

- With increasing frequency *f* the are of the saturation loop increases – this means that the amount of energy irreversibly turned into heat in every cycle increases with *f*.

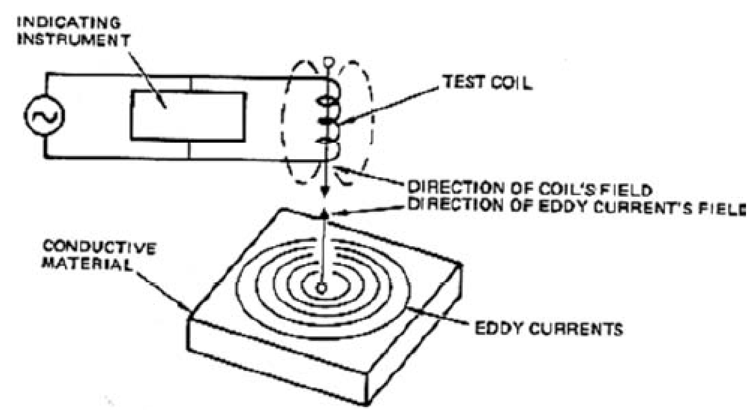


FIG. 2.17. Basic eddy current testing equipment.

image from "Eddy Current Testing at Level 2..." INTERNATIONAL ATOMIC ENERGY AGENCY, Vienna 2011

V. Basso, G. Bertotti, O. Bottauscio, F. Fiorillo, M. Pasquale, M. Chiampi and M. Repetto, J. Appl. Phys. 81, 5606 (1997)

Dynamic effects – eddy current losses

- The energy transformed into heat in one cycle is [1]:

$$\frac{P}{f}=\oint_{loop} H_{applied}\,dB\,,$$ where P is a power loss and $\frac{P}{f}$ is a loss per cycle

- The losses can be formally expressed as (Joule's law):

$$\frac{P}{f}=\frac{1}{V}\int_V d^3r\int_0^{1/f}|j(\vec{r},t)|^2\rho(\vec{r})$$

- In real systems the eddy current distribution is not known. It can be though expressed phenomenologically as [1]:

$$\boxed{\frac{P}{f}=C_0+C_1 f+C_2 f^{-1/2}}$$

The coefficients may be functions of magnetization

- The above equation can be applied to the broad variety of magnetic materials with different domain structure.

Dynamic effects – loss separation

- It turns out that the constants in the $P/f = C_0 + C_1 f + C_2 f^{-1/2}$ equation can be attributed to different processes influencing the hysteresis [1]:
- -*hysteresis loss* – scale of Barkhausen effect – eddy currents induced by the small jumps of domain walls fragments - C_0

  -*classical loss* – related to specimen geometry – calculated from Maxwell's equations assuming homogeneous, conducting material with no domain structure - C_1

  -*excess loss* – caused by eddy currents accompanying steady motion of domain walls under the influence of external field

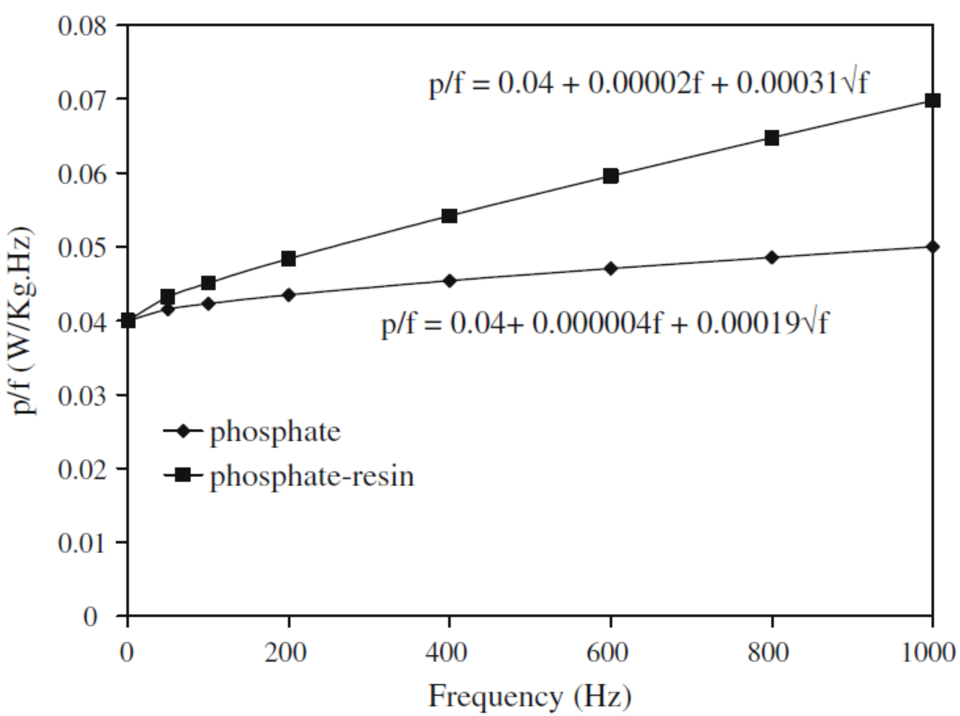


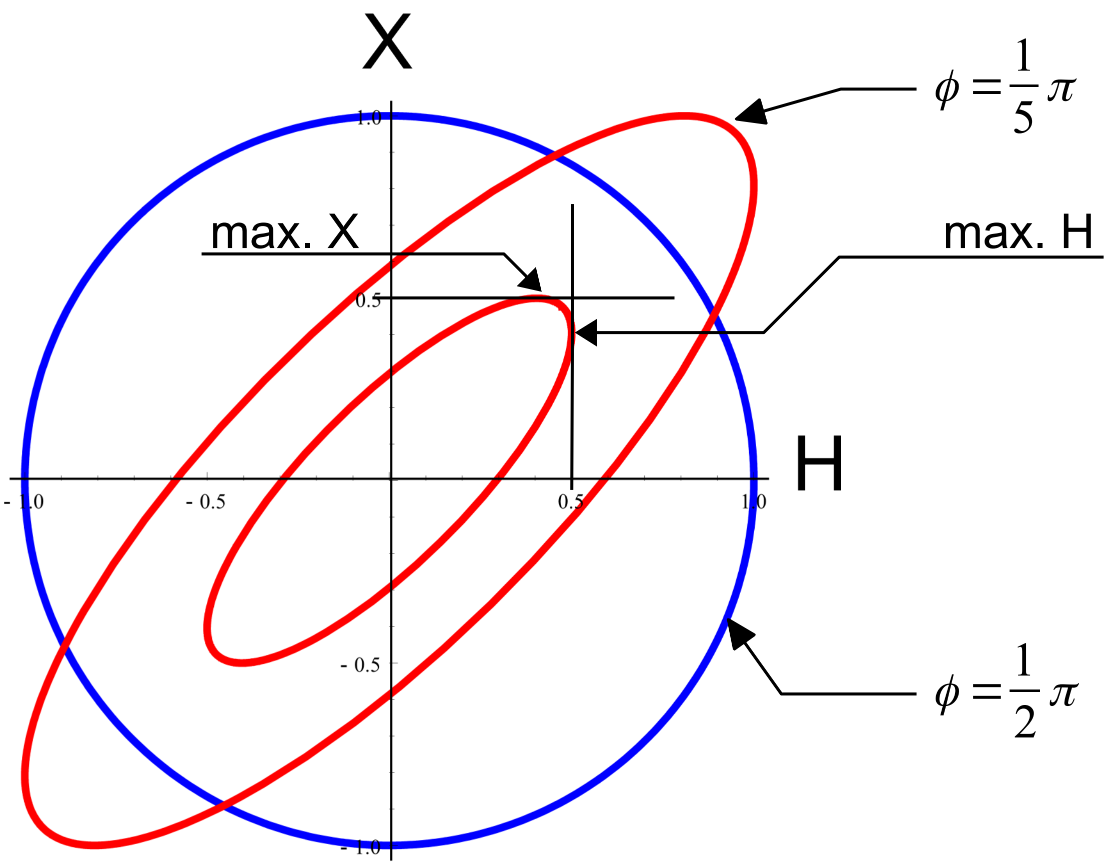
Fig. 10. Behavior of the loss per cycle for phosphated and phosphate–resin insulated samples.

[1] A.H. Taghvaei, H. Shokrollahi, K. Janghorban, H. Abiri, Materials and Design 30 (2009) 3989–3995

Hysteresis lag

- The word hysteresis, coined by Scotsman James Alfred Ewing from ancient Greek ὑστέρησις (hysterēsis, "shortcoming"), denotes in general that the output is lagging the input [1].
- As an example consider a system:

$$H(t)=H_0\cos(\omega t) \qquad X(t)=X_0\cos(\omega t-\varphi)$$



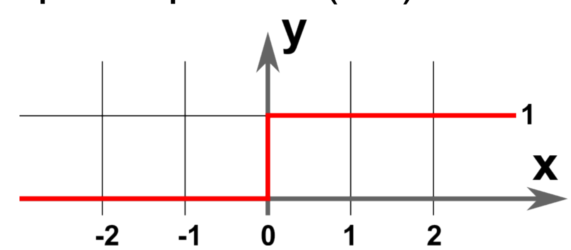
- In general the response to sinusoidal excitation is not sinusoidal.
- Undistorted response is characteristic for *linear systems* where the *superposition principle* holds.

Hysteresis lag

- We have time-invariant system which responses at time t to an input impulse $\delta(t-t_0)$ that occurred at t_0. The response can be often written as [1]:

$$\Phi(t)=\chi_i\delta(t)+\Phi_d(t)\Theta(t)$$

Heaviside step function - response follows excitation



- The terms represent the instantaneous and delayed response. It is assumed that:

$\Phi_d(t)\rightarrow 0 \quad as \quad t\rightarrow\infty$ which means that after long enough time after excitation the system is at X=0.

- Using a superposition principle the response to a arbitrary input is:

$$X(t)=\chi_i H(t)+\int_{-\infty}^{t}\Phi_d(t-t')H(t')dt' \qquad (1)$$

- In frequency domain it can be rewritten to give [1]:

$$X_\omega=\chi(\omega)H_\omega \qquad with \qquad \chi(\omega)=\chi_i+\int_0^{\infty}\Phi_d(t)e^{-i\omega t}dt$$

- The generalized susceptibility is a complex number: $\chi(\omega)=\chi'(\omega)+\chi''(\omega)$

- From Euler's formula* and the fact that the response Φ is a real function it follows that**:

$$\chi'(-\omega)=\chi'(\omega) \qquad \chi''(-\omega)=-\chi''(\omega)$$

**because $\cos(-\omega)=\cos(\omega)$ and $\sin(-\omega)=-\sin(\omega)$

*$e^{ix}=\cos(x)+i\sin(x)$

Hysteresis lag

for very slow excitation ω=0

- Further, from $\chi''(-\omega)=-\chi''(\omega)$ we have: $\boxed{\chi''(0)=0}$

out of phase relative to excitation

- The response of the system is:

$$X=R.e(\chi H)=R.e[(\chi'+i\chi'')H_0 e^{-i\omega t}]=H_0(\chi'\cos(\omega t)+\chi''\sin(\omega t))$$

$$X=\chi' H_0\cos(\omega t)$$

In quasi-static limit where the excitation varies arbitrarily slow there is no lag of response relative to input and there is **no hysteresis**

- Thermal fluctuations in the limit of very slowly varying excitation lead the system to the absolute energy minimum

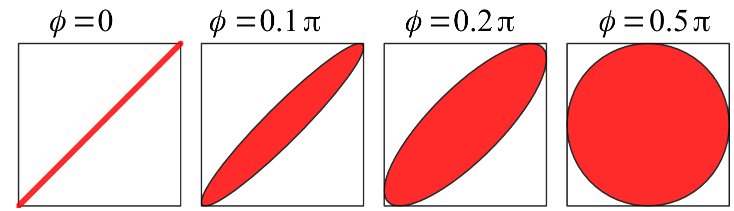
Hysteresis dissipation

- We consider the case when H and X are conjugate variables; HdX represents the work done on the system by external forces [1].
- The first law of thermodynamics may be written as:

$$dU = H\ dX + \delta Q$$

- If the system undergoes cyclic changes its internal energy is not changed (T=const) and the work done on the system is dissipated as heat:

$$\oint_{cycle} H\ dX = -\oint_{cycle} \delta Q$$



- In any hysteretic system the work dissipated in each cycle is given by the area of the loop described by ***X***(***H***). For the case of linear system (see 4 slides back) we get [1]:

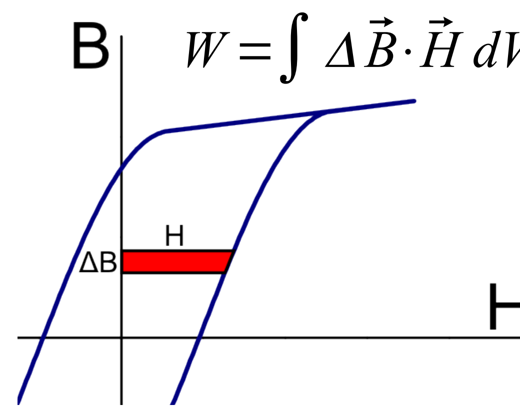
$$W = \oint_{cycle} H\ dX = \pi H_0 X_0 \sin(\varphi) \qquad (2)$$

- Using the identity $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$ and remembering that:

$$X = X_0 \cos(\omega t - \varphi) = R.e(\chi H) = H_0(\chi' \cos(\omega t) + \chi'' \sin(\omega t))$$

we get, comparing coefficients of cos(ωt) and sin(ωt):

$$X_0 \cos(\varphi) = H_0 \chi' \qquad X_0 \sin(\varphi) = H_0 \chi''$$



[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

- Substituting the second of the above identities into Eq.2 we get [1]:

$$\boxed{W = \pi H_0^2 \chi''(\omega)}$$

- Dissipation in linear systems is controlled by the imaginary part of susceptibility.
- Expressing losses in terms of loss angle is not limited to linear systems.

$$W = \oint_{cycle} H \, dX = \pi H_0 X_0 \sin(\varphi) \qquad (2)$$

$$X_0 \cos(\varphi) = H_0 \chi' \qquad X_0 \sin(\varphi) = H_0 \chi''$$

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis - memory

- In a system exhibiting hysteresis future evolution depends on past history [1].
- In systems with memory the output at time t depends not only on the input at time t but also on previous inputs $H(t')$ at times t'.
- Nonpersistent memory

- input H varies for $t<t_0$ and remains constant for $t>t_0$; from Eq.1 we have:

$$X(t)=\chi_i H_0+\int_{-\infty}^{t_0}\Phi_d(t-t')H(t')dt'+H_0\int_0^{t-t_0}\Phi_d(t')dt'$$

As t increases beyond t_0:

$$\int_{-\infty}^{t_0}\Phi_d(t-t')H(t')dt'\rightarrow 0$$

- because $\Phi_d(t)\rightarrow 0 \quad as \quad t\rightarrow\infty$ the output reaches the limit:

$$X(t)=\left(\chi_i+\int_0^{\infty}\Phi_d(t')dt'\right)H_0=\chi'(0)H_0 \qquad \chi(\omega)=\chi_i+\int_0^{\infty}\Phi_d(t)e^{-i\omega t}dt$$

expression in parentheses equals Re[χ(ω)] for ω=0.

- in the limit the memory of input for $t<t_0$ is lost

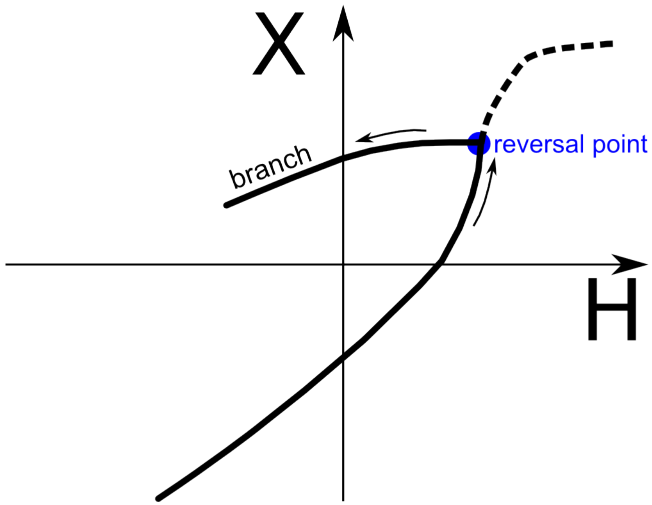
- Persistent memory – state of the system under constant input keeps on depending on the past history of the inputs even after all transients have died out [1]

For a given input H the system can occupy different states.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – memory, cont'd

- Branching – branch is generated when input ***H*** stops increasing and starts decreasing:



- Branching is an indication that the system is not in thermodynamic equilibrium
- Local memory – the values of ***H*** and ***X*** are enough to identify the state
- Nonlocal memory – different curves ***X***(***H***) can start from every (***H, X***) point; this is observed in many magnetic materials
- When thermodynamic equilibrium is reached any memory of the previous states is lost

- In **many magnetic systems** there is a input-rate interval where the rate dependence of hysteresis can be neglected [1]:
  - -rate must be slow enough not to introduce frequency dependents effects (like eddy currents in conducting magnets)
  - -rate must be high enough for thermal relaxation not to play a role

That is how most of the hysteresis curves shown in literature are measured – within that input-rate interval.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – bistable systems

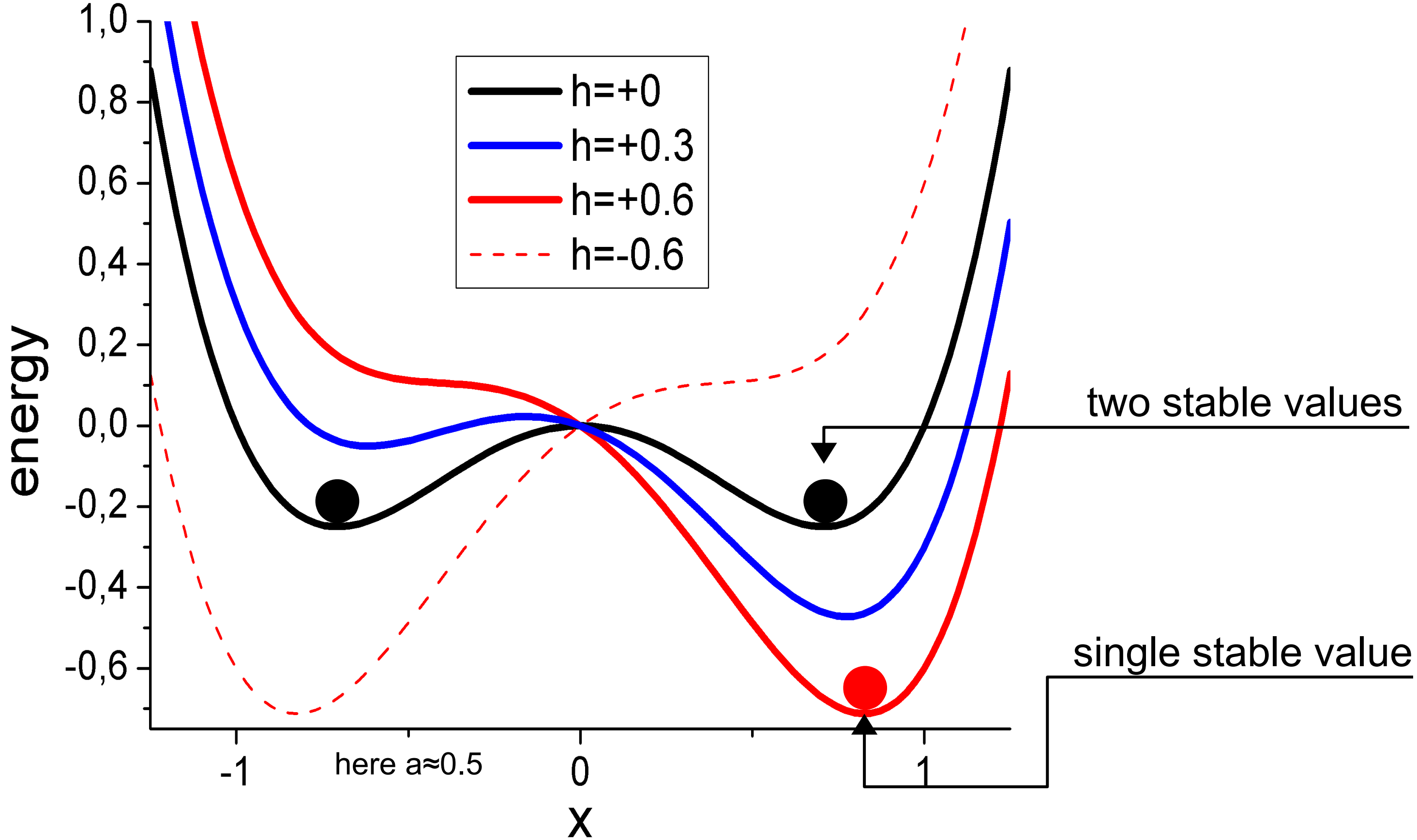
- Let us consider the system with local memory and energy given by [1]:

$$f(x)=x^4-2ax^2$$

- Introducing the input we can write (energy depends now on the input – for example magnetic field *h;* note the analogy with Stoner-Wolfarth model of lecture 3):

$$g(x)=x^4-2ax^2-hx$$

- Exemplary energy profiles show two local minima (metastable states) for small h values and one minimum for higher values:



[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

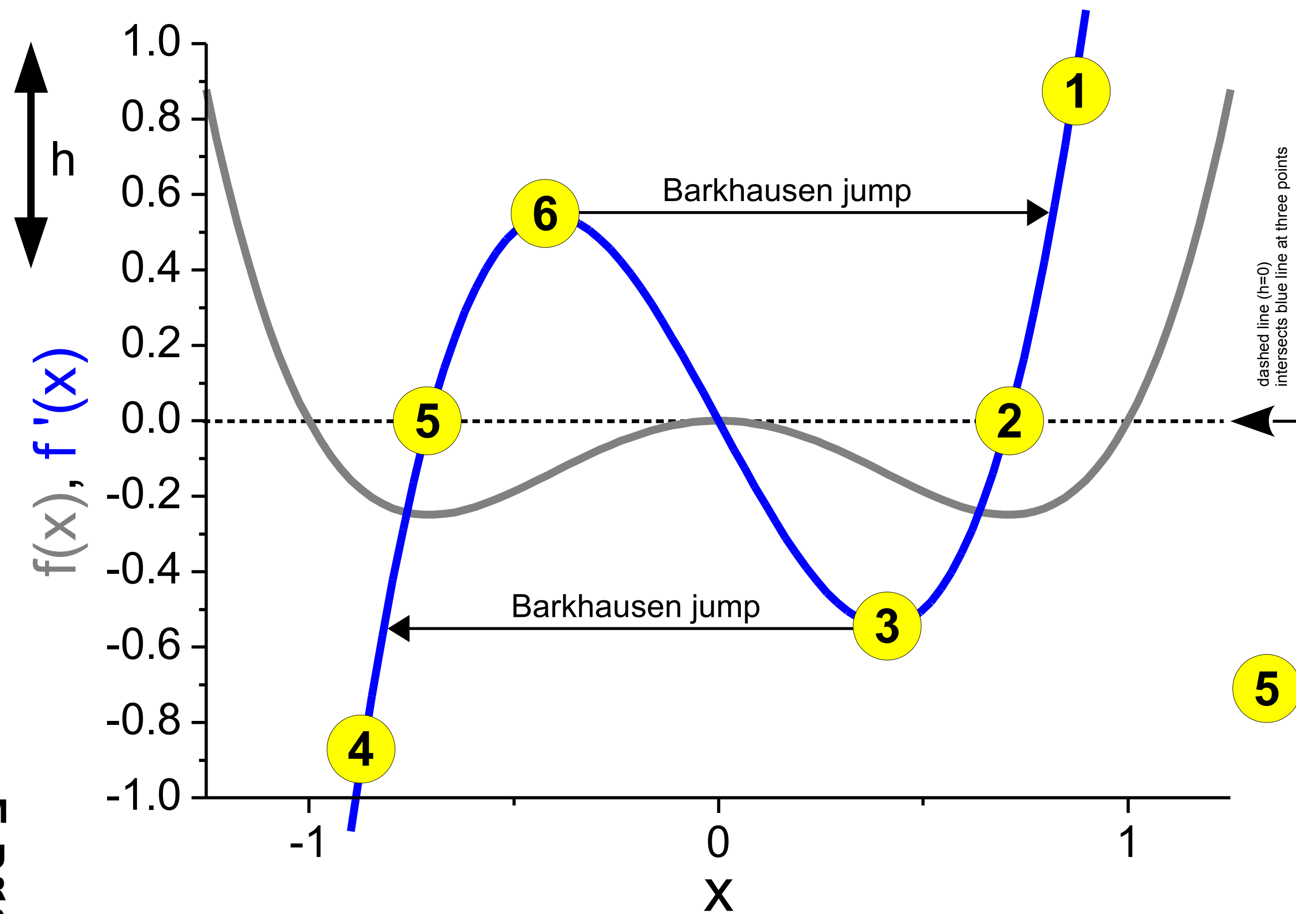
Hysteresis – bistable systems - cont'd

$$f(x) = x^4 - 2ax^2$$
$$g(x) = x^4 - 2ax^2 - hx$$

•From the condition for extremum of *g*(*x*) we have:

$$\frac{\partial}{\partial x} g(x) = \frac{\partial}{\partial x} f(x) - h = 0 \quad \leftarrow \quad g(x) = f(x) - hx$$

•Using $\partial/\partial x[f(x)] = h$ we can graphically trace hysteresis:



1 start at positive h (field)

2 At h=0 there are three extrema (one metastable)

3 At $h=h_C$ there is only one minimum – Barkhausen jump

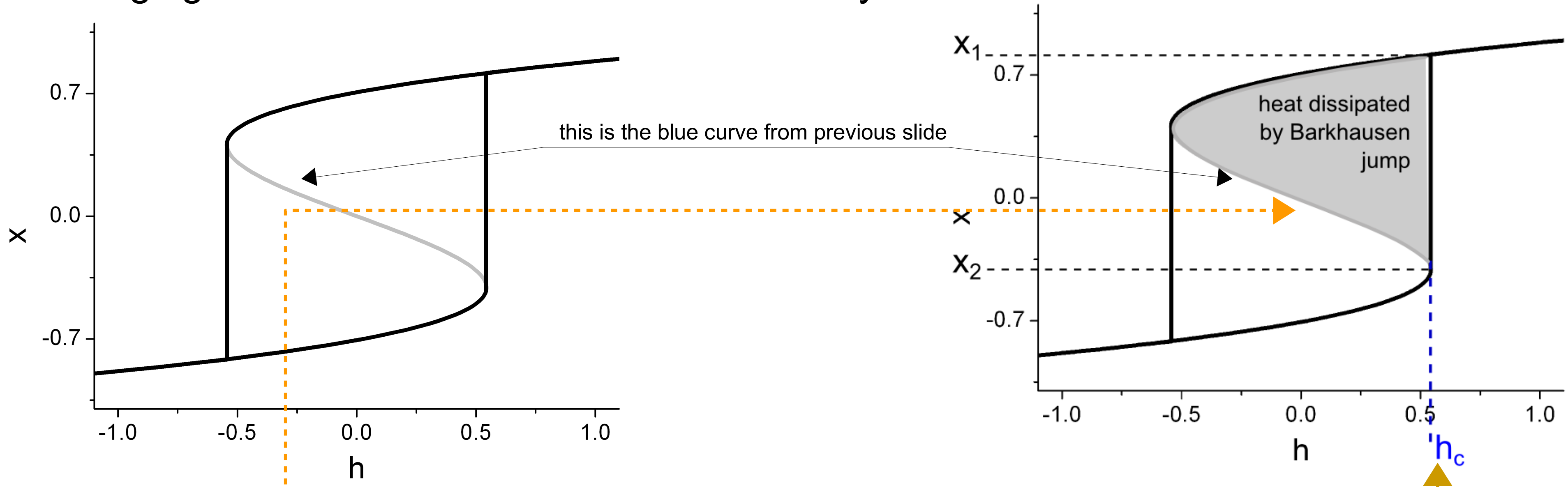
4 Field is increasing shifting the position of the single minimum

5 6 Field sweep direction is reversed

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – bistable systems - cont'd

•Exchanging axes we obtain a traditional form of hystersis:



•We want to know the area of the shaded region of the top right hysteresis (it should be half the work dissipated in hysteresis). The work can be expressed as (see previous slide) [1]:

$$\Delta W=\int_{x1}^{x2}\left[h_c-\frac{\partial f}{\partial x}\right]dx=-\int_{x1}^{x2}\left[\frac{\partial g}{\partial x}\right]_{h=h_c}dx=g(x1,h_c)-g(x_f,h_c) \quad \leftarrow \quad \frac{\partial}{\partial x}g(x)=\frac{\partial}{\partial x}f(x)-h=0$$

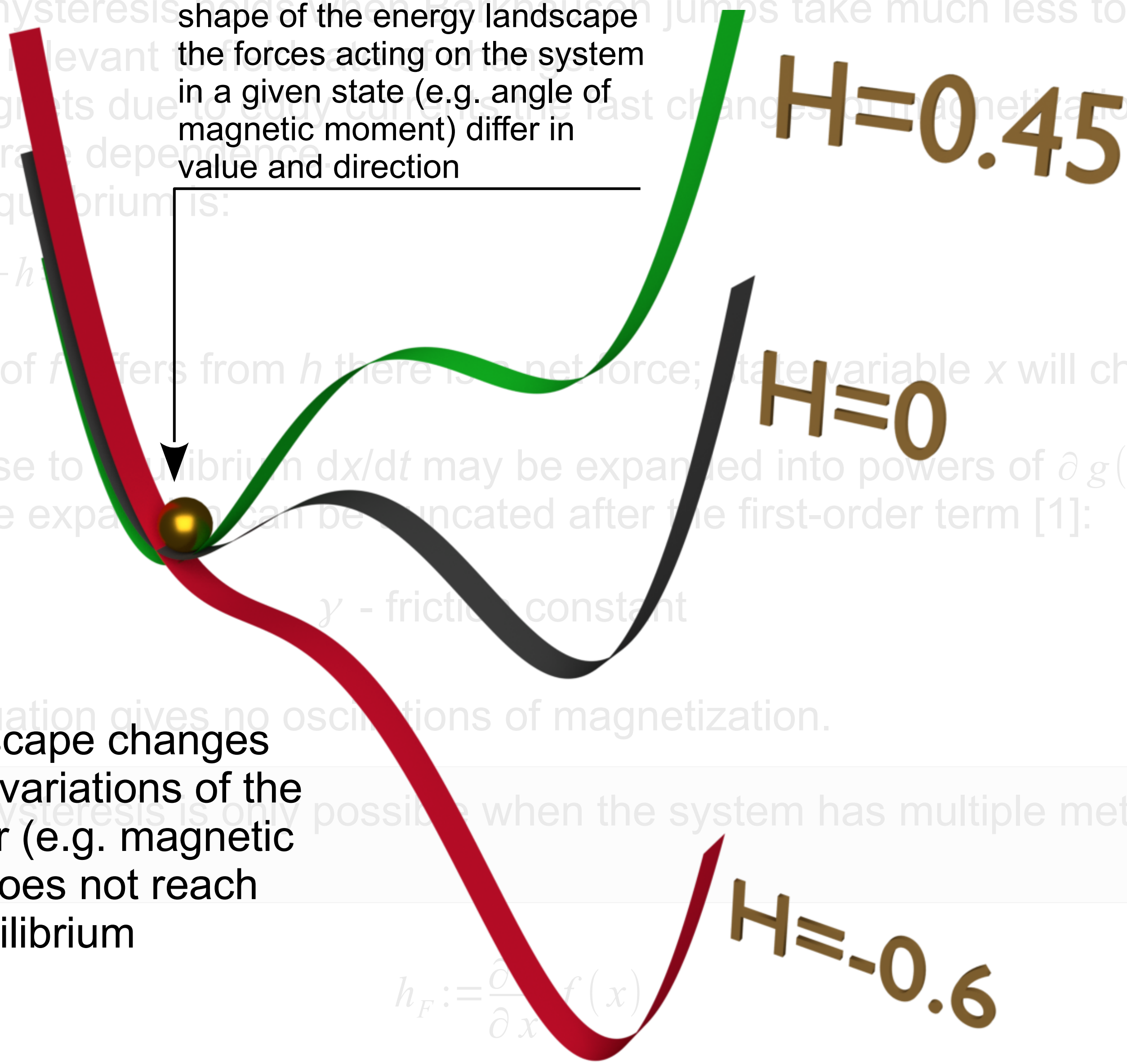
The work is exactly the energy decrease when the system makes Barkhausen jump

The above description is rate-independent: it was assumed that system is always in one of energy minima independently of the input rate of change.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – rate dependence

Depending on the instantaneous shape of the energy landscape the forces acting on the system in a given state (e.g. angle of magnetic moment) differ in value and direction



If the energy landscape changes rapidly, due to the variations of the external parameter (e.g. magnetic field) the system does not reach instantaneous equilibrium

If the field value was set to -0.6 for a long enough time the system (represented with a yellow sphere) would reach that minimum

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – rate dependence

- Rate independent hysteresis holds when Barkhausen jumps take much less to complete than the time scale relevant to field rate of change.
- In metallic ferromagnets due to eddy currents the fast changes of magnetization are damped leading to rate dependence.
- The condition for equilibrium is:

$$\frac{\partial}{\partial x} g(x) = \frac{\partial}{\partial x} f(x) - h = 0$$

- When the gradient of *f* differs from *h* there is a net force (state variable *x* will change with time):

$$F = \frac{\partial}{\partial x} f(x) - h$$

First term describes the force due to the system (internal forces, e.g. anisotropy energy) and the second the influence of the external field (e.g. magnetic)

- If the system is close to equilibrium d*x*/d*t* may be expanded into powers of $\partial g(x)/\partial x$. To simplify analysis the expansion can be truncated after the first-order term [1]:

$$\gamma \frac{dx}{dt} = -\frac{\partial g}{\partial x} \leftarrow \text{force}$$

γ - friction constant (like viscosity in a fluid)

- In this form the equation gives no oscillations of magnetization.

Rate independent hysteresis is only possible when the system has multiple metastable states

$$\gamma \frac{dx}{dt} = h - h_F$$

$$h_F := \frac{\partial}{\partial x} f(x)$$

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – rate dependence

- Assuming that the input (field) changes with time we can write (putting γ=1):

$$\frac{dx}{dt} = h - h_F$$

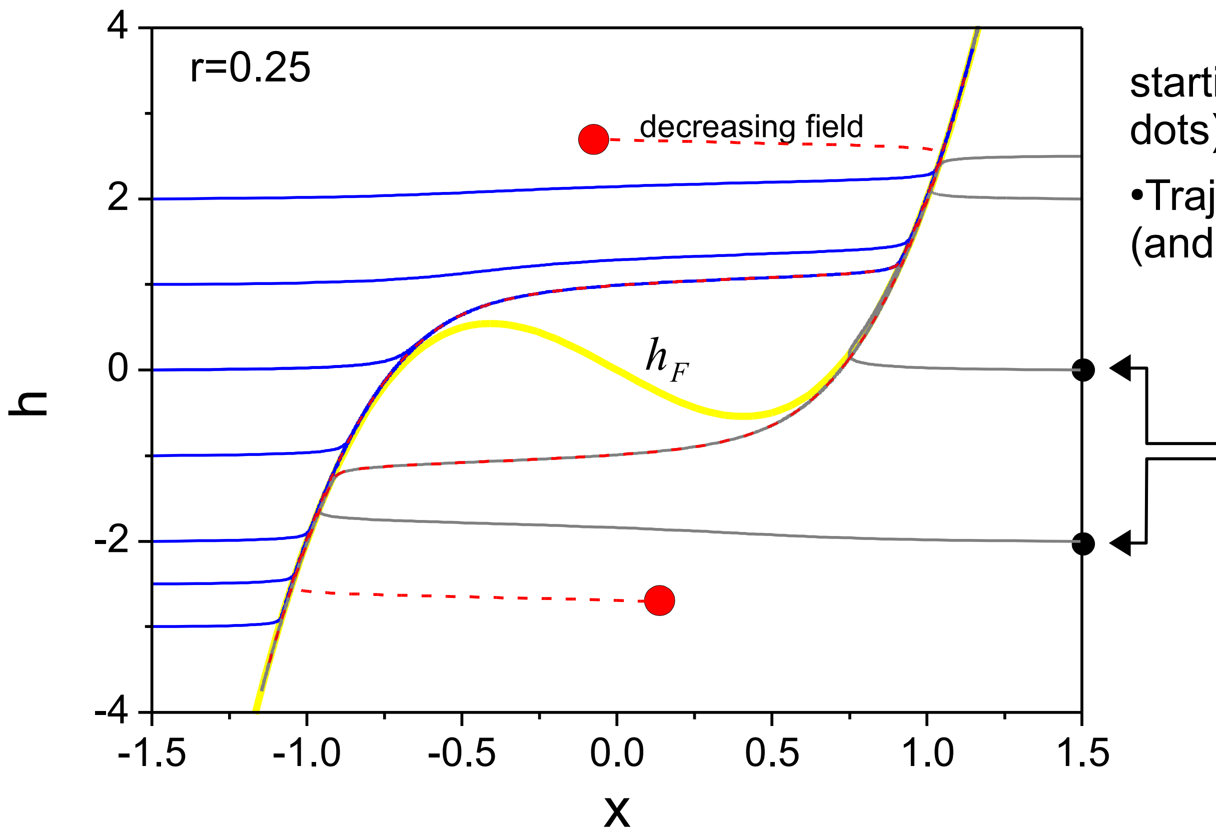
To numerically calculate the trajectory x(h(t)) we choose some initial point ($x(t_0)$,$h(t_0)$), then change x by Δt·(dx/dt), advance the time by Δt, change h accordingly (H depends linearly on time t) and so on...

$$h_F := \frac{\partial}{\partial x} f(x)$$

$$\frac{dh}{dt} = r(t)$$

- time varying field (eg field of VSM electromagnet)

- Exemplary phase portrait of above equation for $f=x^4-x^2$ and r=0.25 (and r=-0.25 – decreasing field).



starting points: x=-1.5 and 1.5 (two red dots)

•Trajectories for different starting values (and the same r) do not intersect [1].

some starting points of x(h) trajectories

Phase portrait of equation- sets of points (x(t),h(t)) on xh-plane for different starting values $x(t_0)$ and $h(t_0)$.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – rate dependence

- Assuming that the input (field) changes with time we can write (putting γ=1):

$$\frac{dx}{dt} = h - h_F \qquad h_F := \frac{\partial}{\partial x} f(x)$$

$$\frac{dh}{dt} = r(t)$$ - time varying field (e g field of VSM electromagnet)

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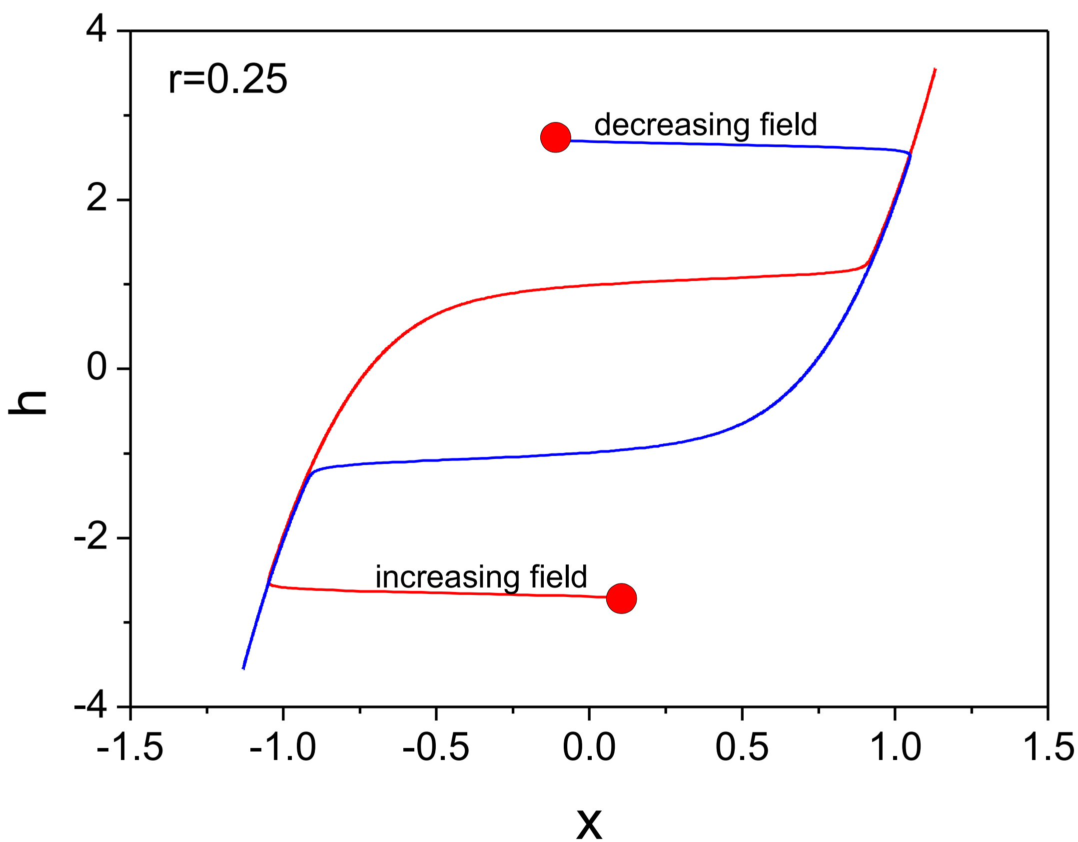


starting points: x=-1.5 and 1.5 (two red dots)

To obtain a hysteresis corresponding to given r one identifies trajectories for opposite r that intersect at desired peak input values of h.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

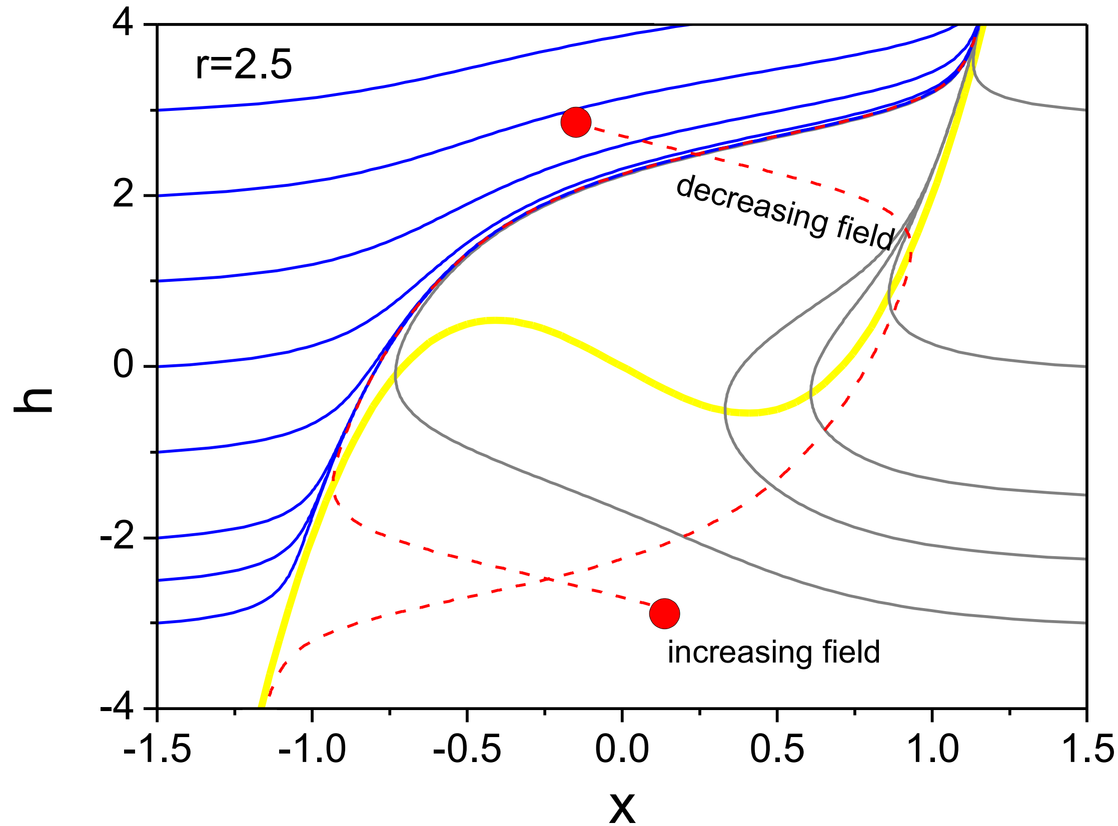
Hysteresis – rate dependence

- Assuming that the input(field) changes with time we can write (putting γ=1):

$$\frac{dx}{dt} = h - h_F$$
$$\frac{dh}{dt} = r(t)$$

Phase portrait of equation - sets of points (x(t),h(t)) on xh-plane for different starting values $x(t_0)$ and $h(t_0)$.

- Exemplary phase portrait of above equation for $f=x^4-x^2$ and **r=2.5** (and r=-2.5 – decreasing field).



starting points: x=-1.5 and 1.5 (plus two red dots)

To obtain a hysteresis corresponding to given r one identifies trajectories for opposite r that intersect at desired peak input values of h.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – rate dependence

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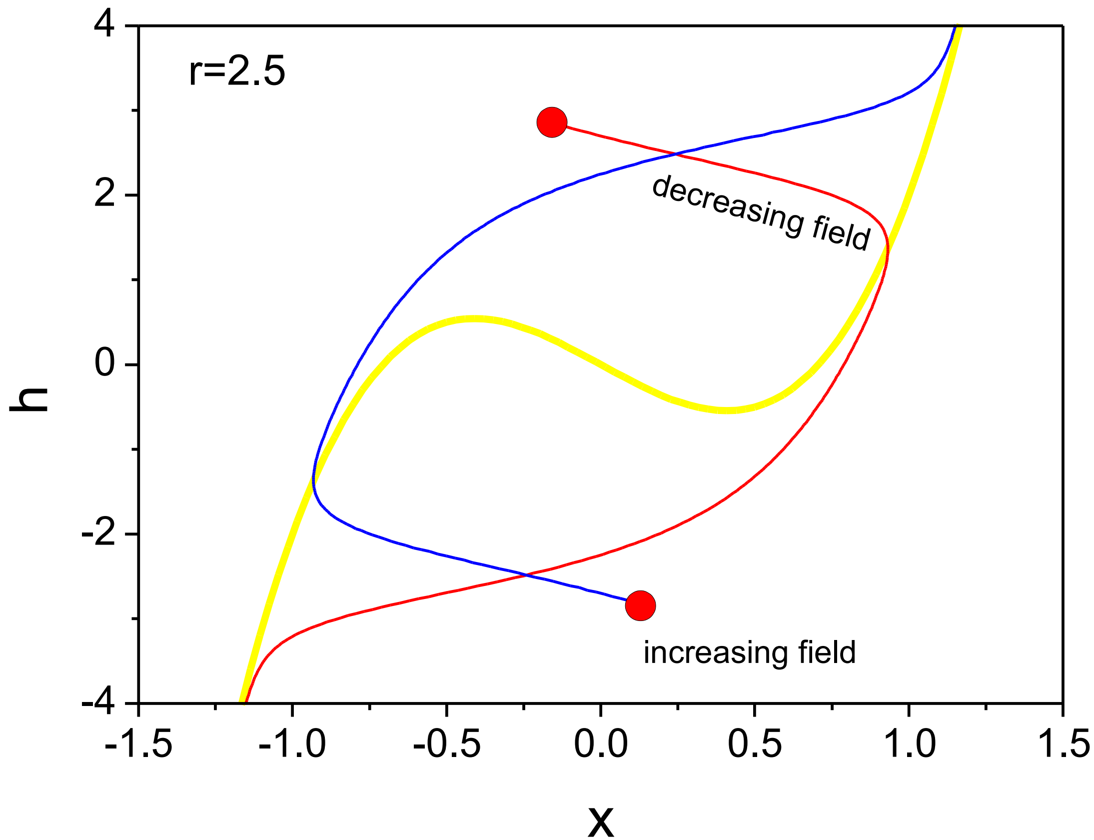

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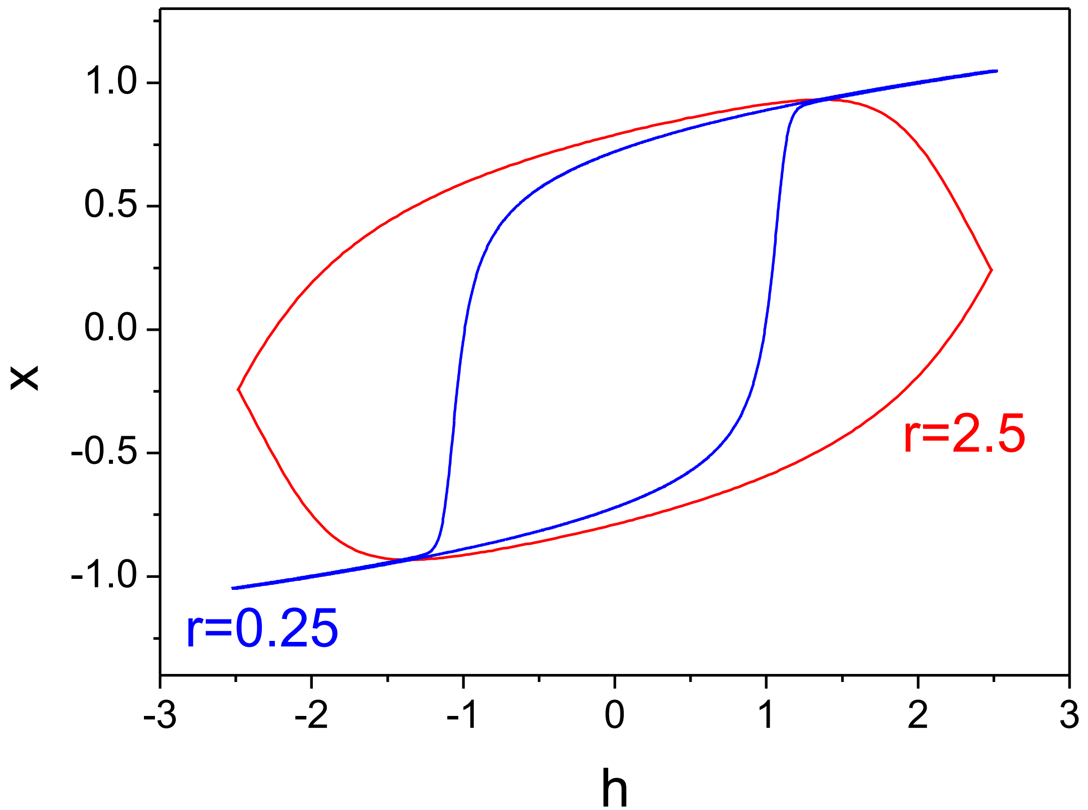
To obtain a hysteresis corresponding to given r one identifies trajectories for opposite r that intersect at desired peak input values of h.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – rate dependence

•Exemplary loops obtained for two input rates of change (see previous slides):



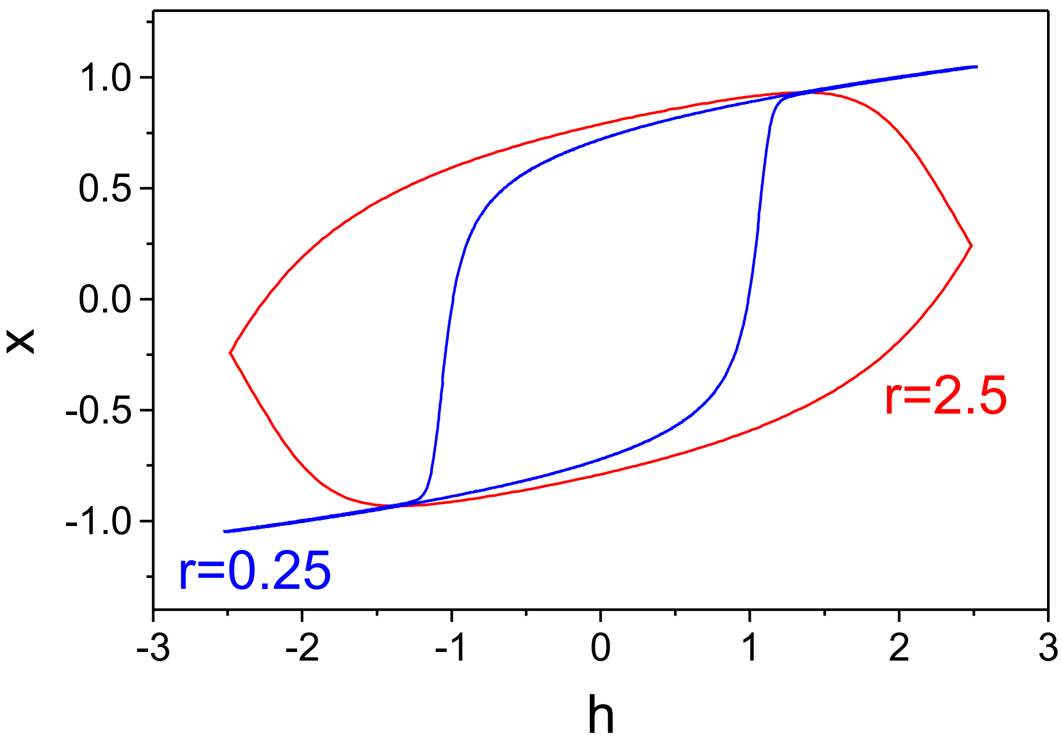
both curves obtained for the same field amplitude (h≈2.5)

- The shape of loop depends on the rate of change of input (*magnetic field*)
- With increasing frequency *f* **the area of the saturation loop increases** – this means that the amount of energy irreversibly turned into heat in on period increases with *f*.
- The shape of loop depends not only on the frequency of the input and its peak values (h_{min} and h_{max}) but also on the waveform applied, i.e., the $x(h)$ dependence is different for sinusoidally varying h from the one obtained for triangular excitations.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – rate dependence

•There is a qualitative agreement with experimental dependencies:



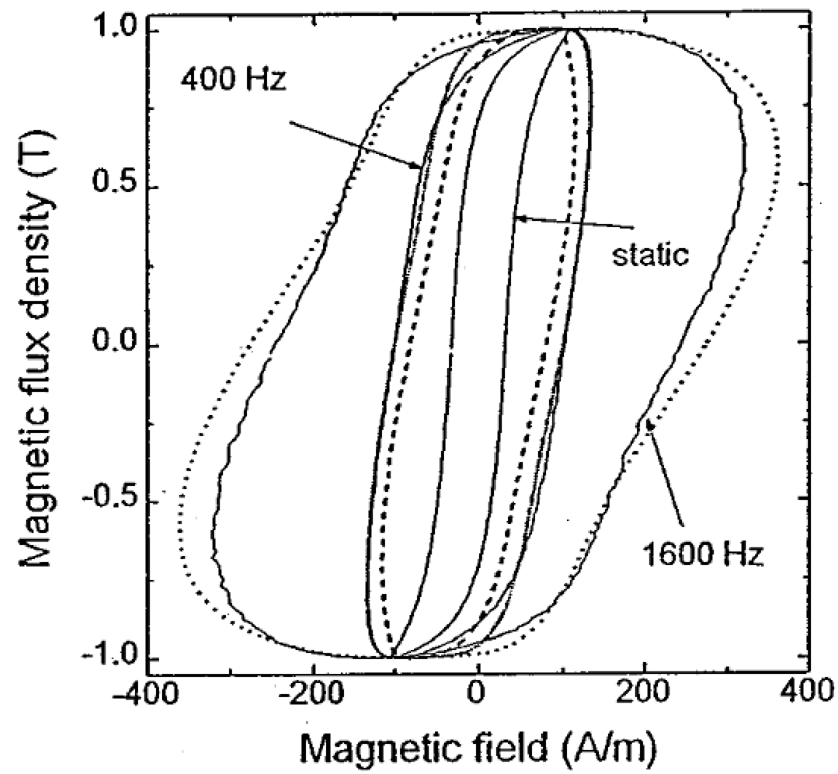
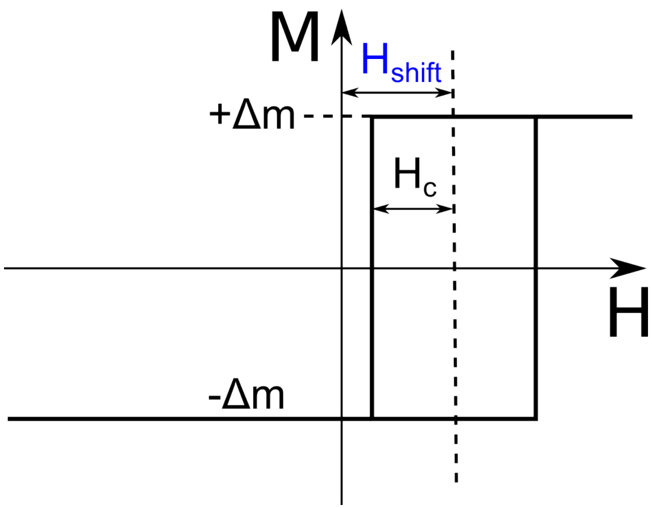


FIG. 3. Dynamic hysteresis loops at 1 T, for 0, 400, 1600 Hz. Continuous lines measurements. Dotted lines FEM prediction based on dynamic PM. Broken line. FEM prediction based on conventional PM.

V. Basso, G. Bertotti, O. Bottauscio, F. Fiorillo, M. Pasquale, M. Chiampi and M. Repetto, J. Appl. Phys. 81, 5606 (1997)

Hysteresis – Preisach model

•In many magnetic materials the hysteresis is a sequence of Barkhausen jumps
•The jump is associated with the system leaving metastable state in favor of state with lower energy
•Preisach proposed [2] to treat magnetic material as a set of units characterized by elementary rectangular hystereses with randomly distributed values of coercive field and shift field:



•In the analysis both *H* and *M* are usually treated as scalar quantities [1].
•It is assumed that rate dependent dissipation phenomena play no role in the magnetization reversal.
•It assumed too that thermal relaxation is absent.
•The above assumptions mean that the hysteresis is rate-independent and corresponds to zero temperature.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – Preisach model

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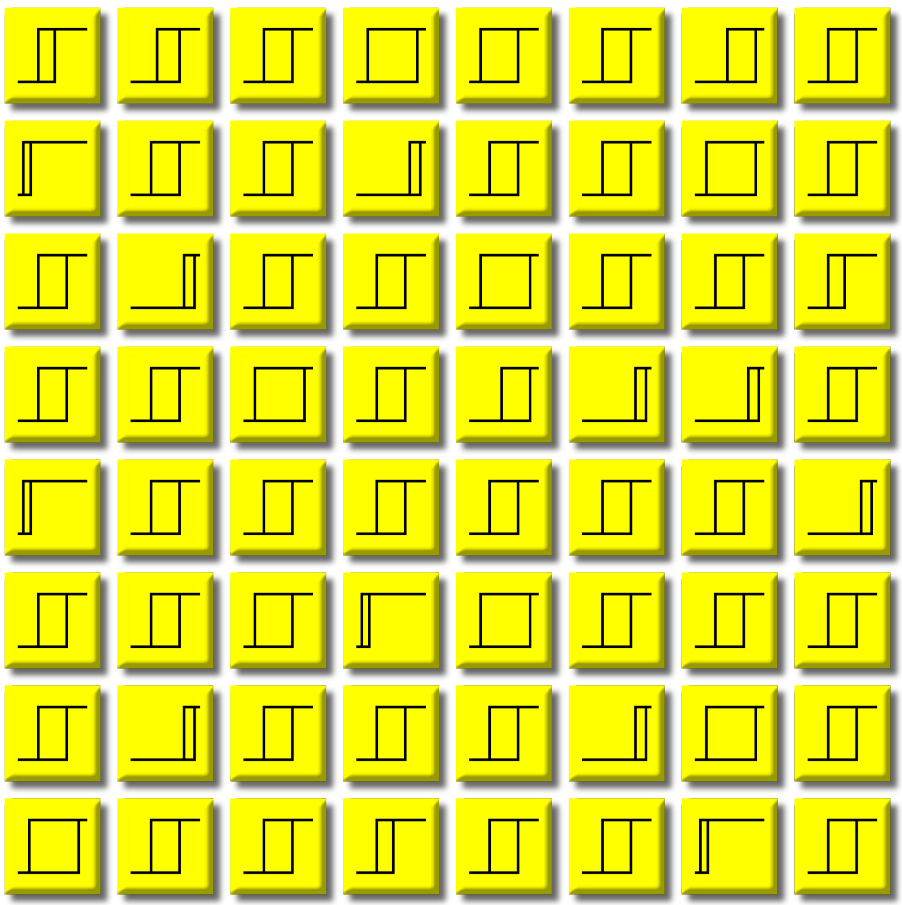
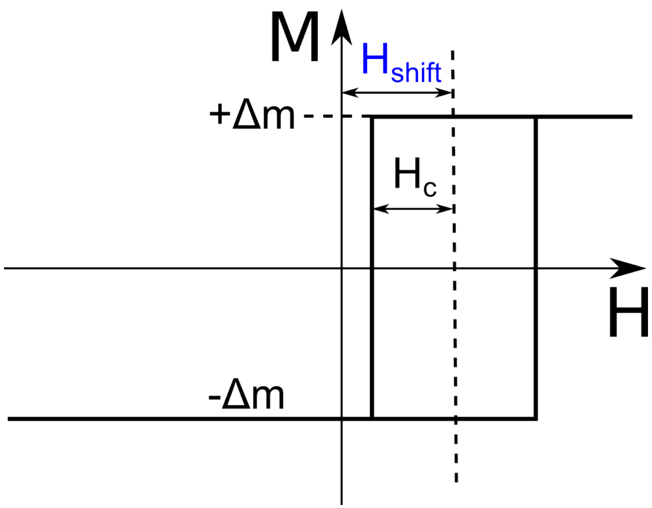


[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – Preisach model

- Each Preisach unit can be characterized by different associated magnetic moment (Δm)
- *Preisach distribution* $p(h_c,h_u)$– is a function describing relative abundance of Preisach units with given coercivity and shift field h_u.
- Preisach approach is expected to give approximate description of certain systems.
- Once Preisach distribution is known the magnetization reversal can be calculated.



- In ferromagnetic materials usually a input-output history {*H*(t),*M*(t)} implies that {-*H*(t),-*M*(t)} is also a admissible history for the system* [1]. It follows that:

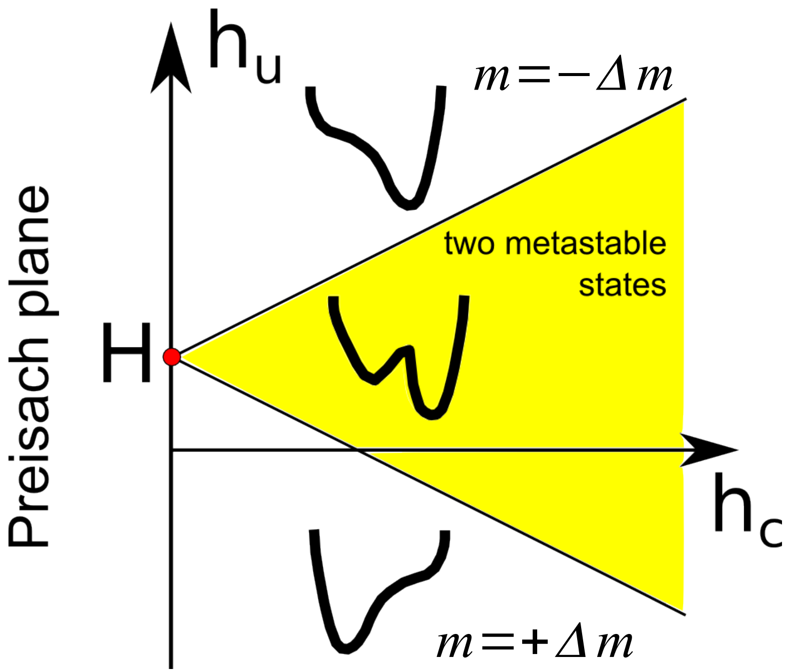
$$p(h_c, h_u) = p(h_c, -h_u)$$ in ferromagnetic materials

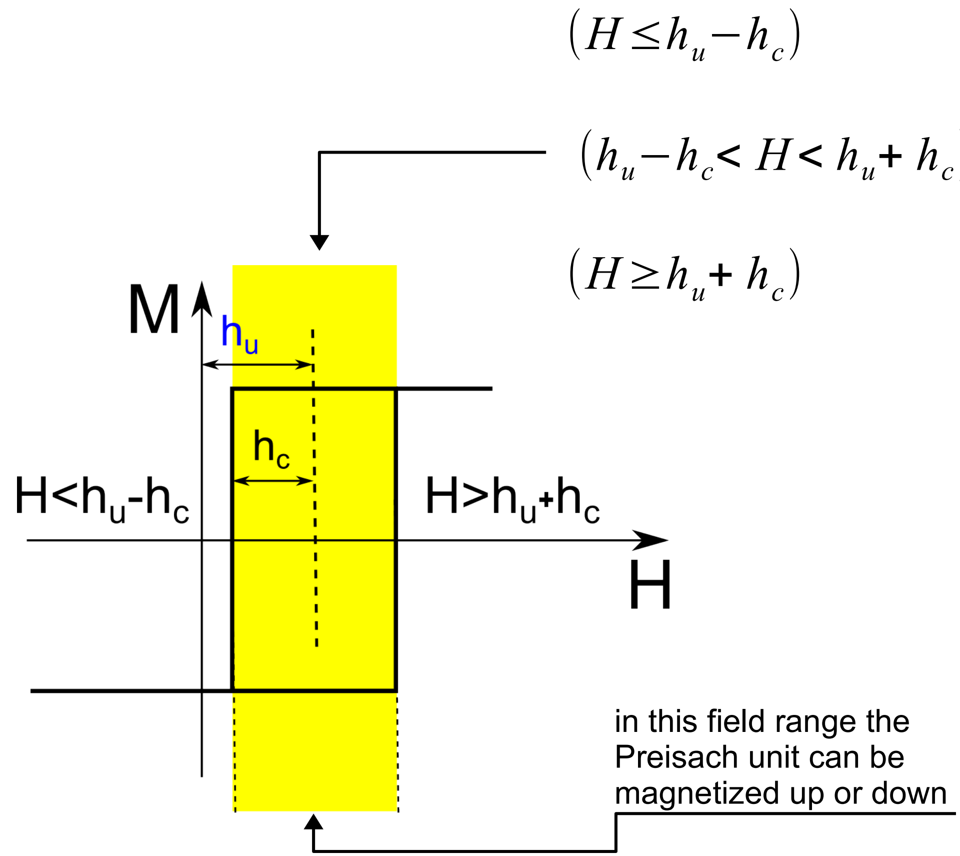
[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

*it may not be the case in systems with exchange anisotropy if the antiferromagnetic material is not saturated (minor loops)

Hysteresis – Preisach model

- Preisach plane:



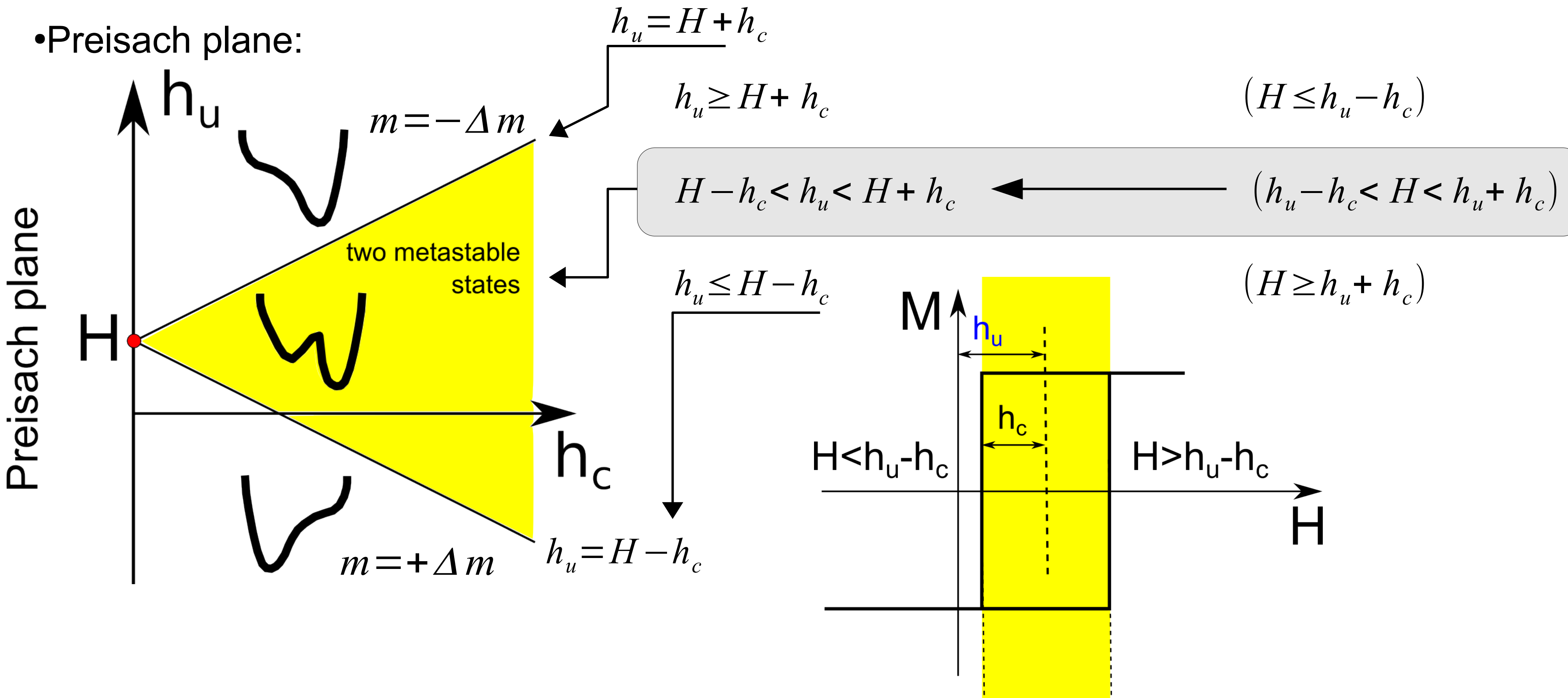


- The boundary of the cone (yellow region), position of which depends on the external field H, is a set of points where the Barkhausen jumps can take place.
- The different points on the Preisach plane correspond to elementary loops of different H_c and shift.
- The total number of metastable states is 2^N, where N is the number of Preisach units in yellow region.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Hysteresis – Preisach model

•Preisach plane:



•The boundary of the cone (yellow region), position of which depends on the external field H, is a set of points where the Barkhausen jumps can take place.

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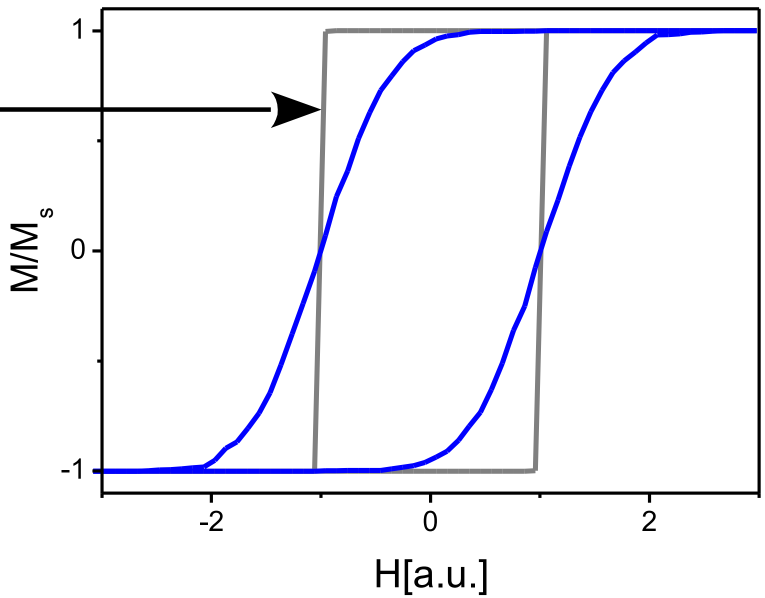
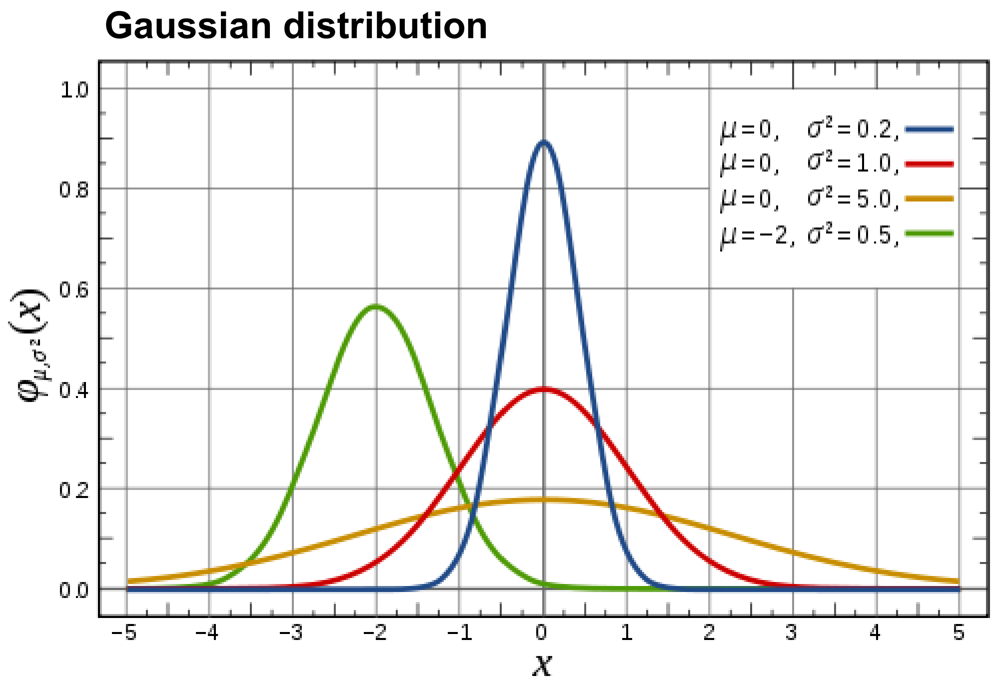
•The total number of metastable states is 2^N, where N is the number of Preisach units in yellow region.

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

•Exemplary hystereses from Preisach model

-1000 Preisach units, h_c – Gaussian distribution with average μ=1 and σ^2=0.01;
h_u – Gaussian distribution with μ=0 and σ^2=0.01:





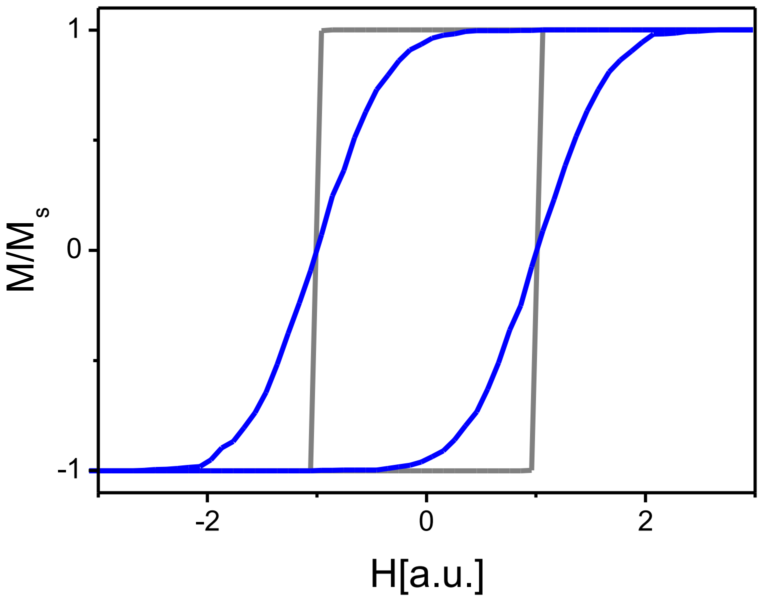
•For narrow distribution of Preisach units on Preisach plane we essentially recover the elementary square loop of a single element.

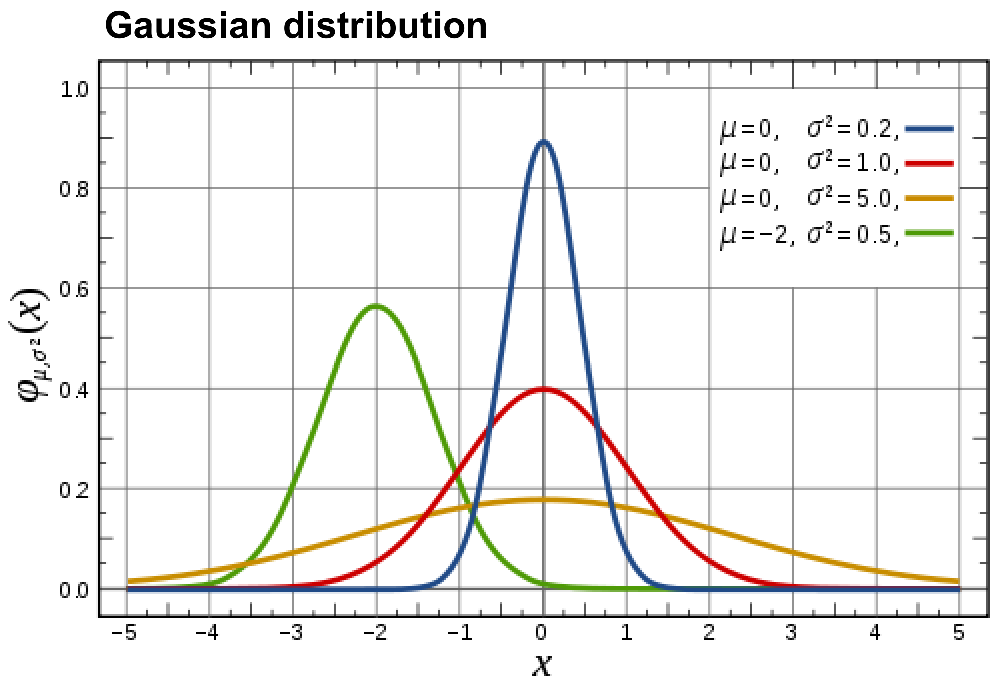
[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

image source:
Wikimedia Commons; author: Inductiveload

Hysteresis – Preisach model

•Exemplary hystereses from Preisach model
-1000 Preisach units, h_c – Gaussian distribution with average $\mu=1$ and $\sigma^2=0.5$;
h_u – Gaussian distribution with $\mu=0$ and $\sigma^2=0.01$:





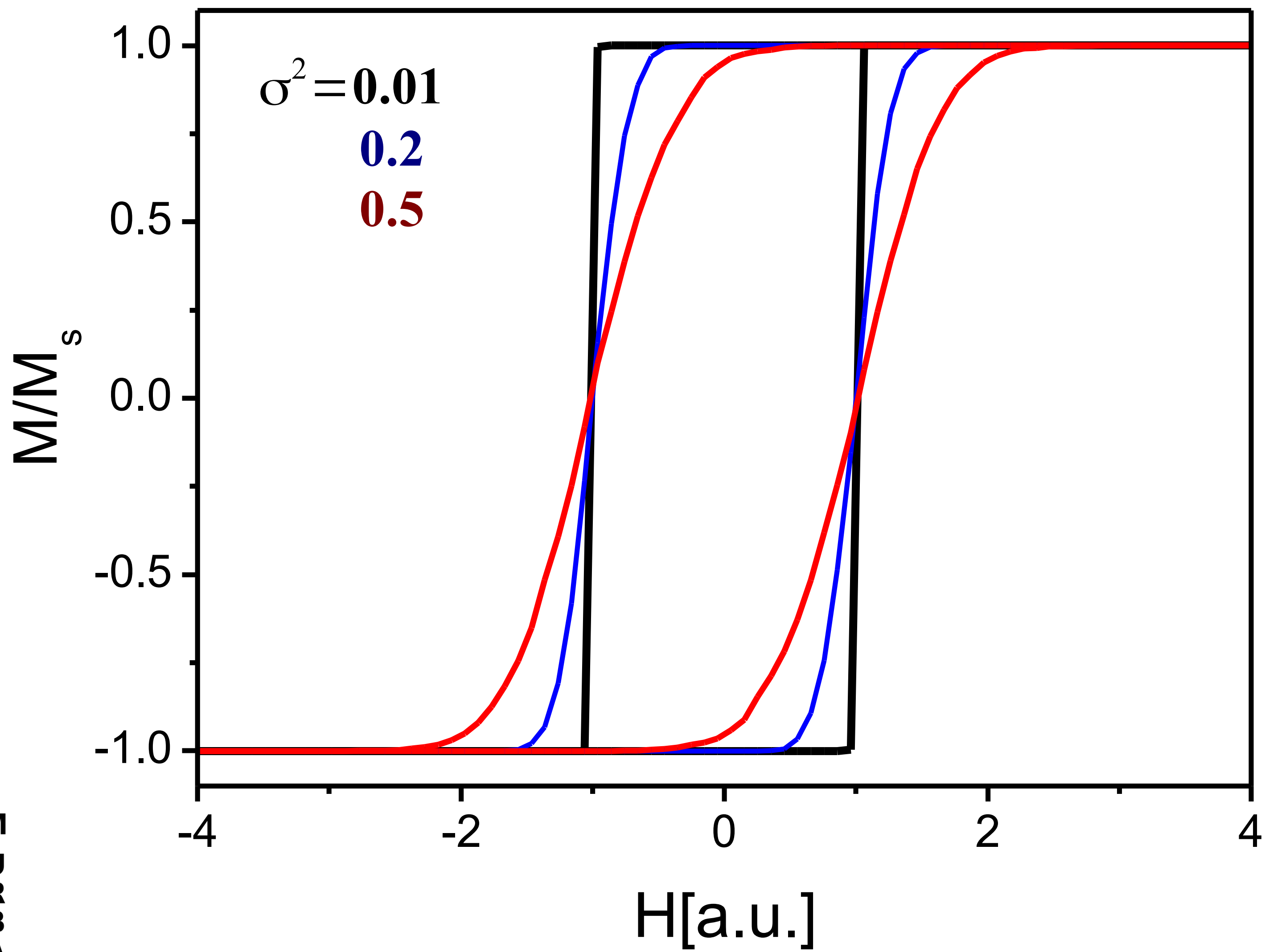
•The shapes of the hysteresis depend mainly (assuming zero average shift h_u) on the width of the distribution
•In systems composed of individual physical entities (grains etc.) one often uses factorization (Preisach distribution which is a product of distributions for h_c and h_u):

$$p(h_c, h_u) = f(h_c)\, g(h_u)$$

image source:
Wikimedia Commons; author: Inductiveload

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

•Evolution of hysteresis with increasing spread of coercive fields
-2000 Preisach units, h_c – Gaussian distribution with average $\mu=1$ and changing σ^2 *;
h_u – Gaussian distribution with $\mu=0$ and $\sigma^2=0.01$:

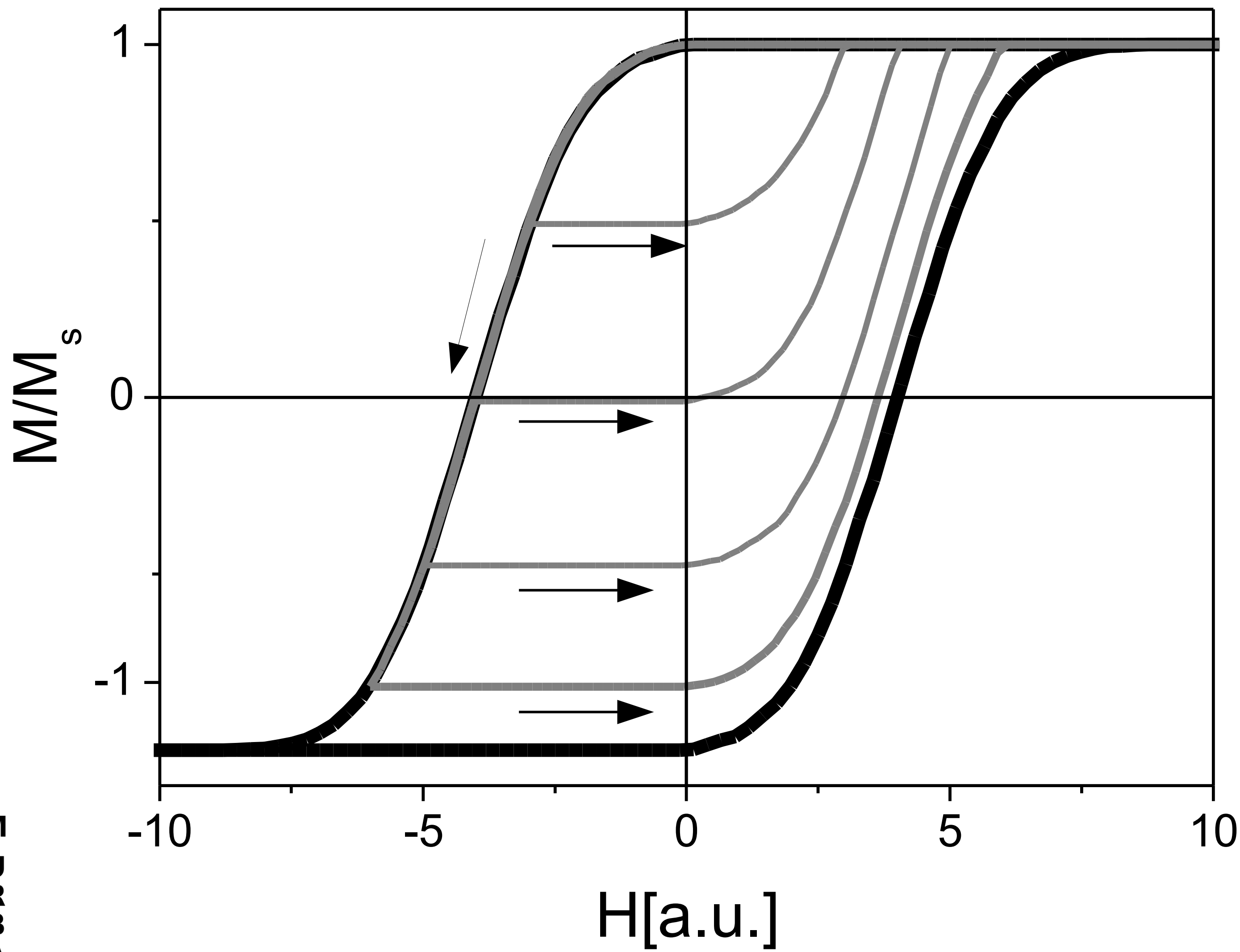


•In systems with structural imperfections one often observes a spread of coercivities.

*care must be take not to have negative coercivities in the distribution

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

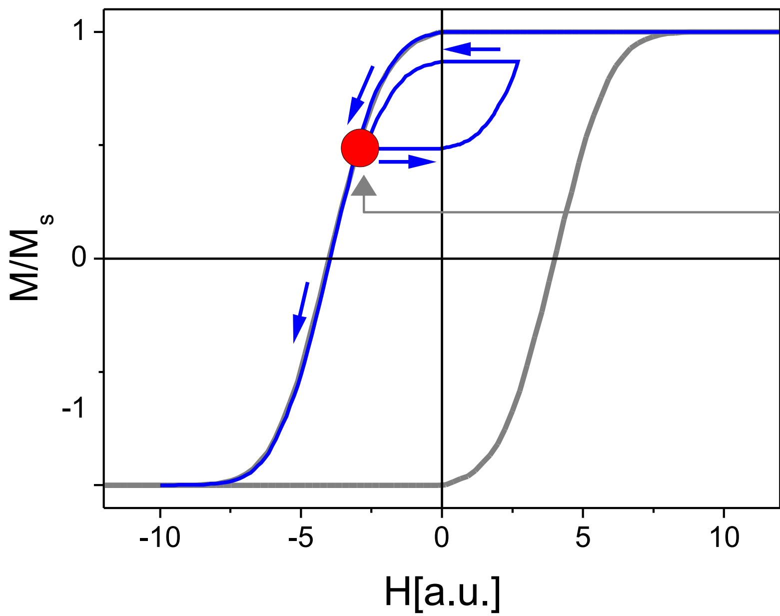
•Exemplary major and corresponding **minor hystereses**
-5000 Preisach units, h_c – absolute values from Gaussian distribution with average μ=4 and σ^2 =1.5; h_u – Gaussian distribution with μ=0 and σ^2=0.01:



[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

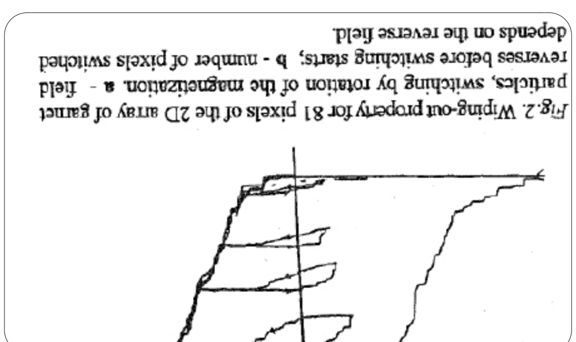
Preisach model - applicability

- A necessary condition for the Preisach model to be applicable is that the system exhibits **return-point memory** [1]



When the field returns back to H=H_1 the system returns back to the exactly same state it occupied when the point H=H_1 was reached for the first time.

5000 Preisach units, h_c – absolute values from Gaussian distribution with average μ=4 and σ^2 =1.5; h_u – Gaussian distribution with μ=0 and σ^2=0.01.



rotation by 180 Deg

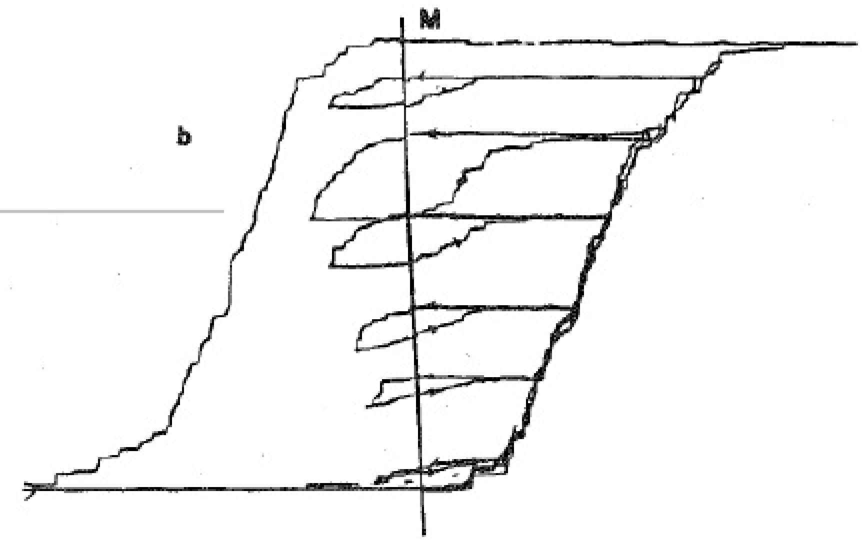


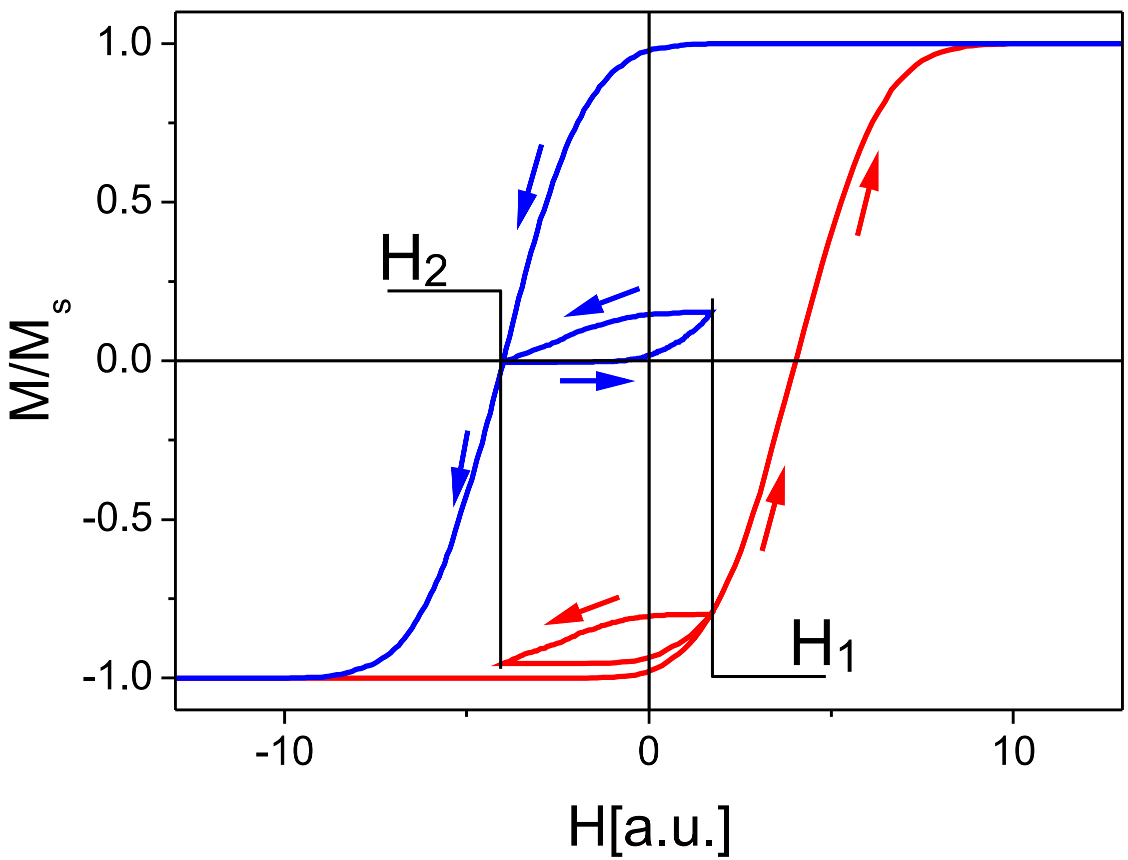
Fig. 2. Wiping-out property for 81 pixels of the 2D array of garnet particles, switching by rotation of the magnetization. **a** - field reverses before switching starts; **b** - number of pixels switched depends on the reverse field.

image source: M. Pardavi-Horvath and G. Vertesy, IEEE Trans. Magn. 33 3975 (1997)

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Preisach model - applicability

•A necessary condition for the Preisach model to be applicable is that the system exhibits **congruency** [1]



5000 Preisach units, h_c – absolute values from Gaussian distribution with average μ=4 and σ^2=1.5; h_u – Gaussian distribution with μ=0 and σ^2=1.

Field cycling between field values H_1 and H_2 produces the minor hysteresis loop of the same geometrical shape (**congruent**).

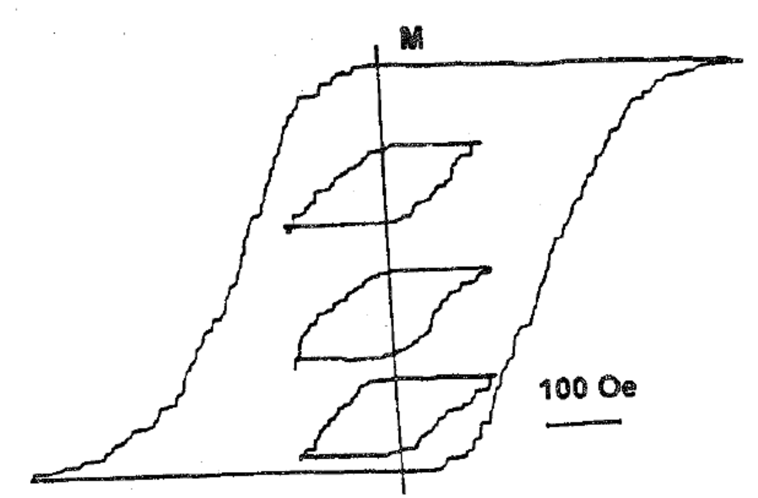


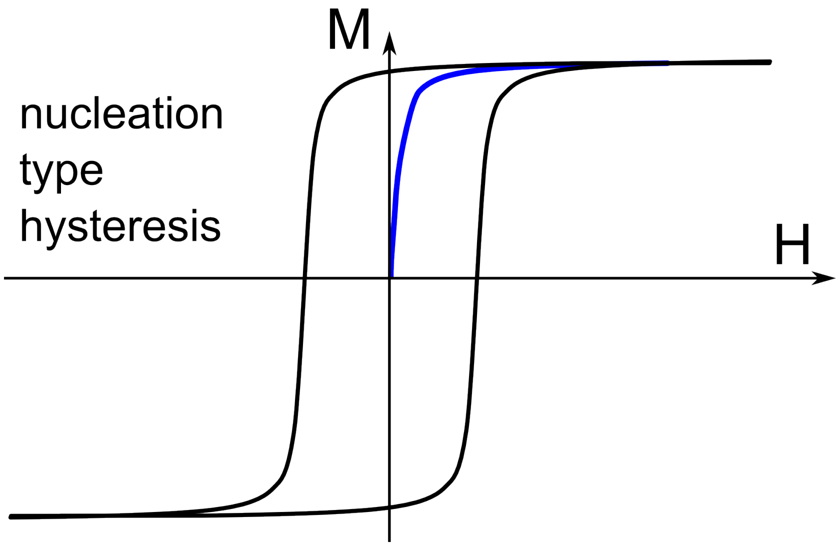
Fig.3. Congruency property for the same 81 pixels, upon cycling the field around H=0.

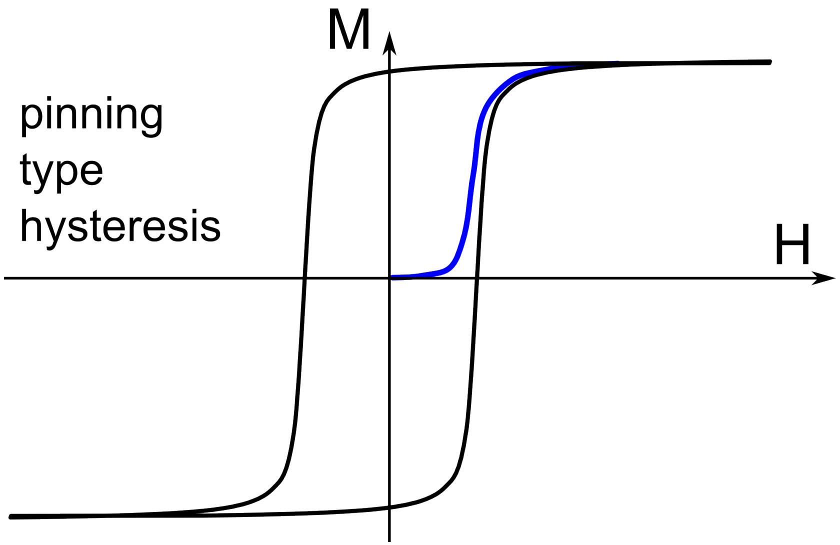
image source: M. Pardavi-Horvath and G. Vertesy, IEEE Trans. Magn. 33 3975 (1997)

[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Nucleation and pinning-type magnets

- In nucleation-type magnets, the virgin curve (obtained after *thermal demagnetization of the sample*) is steep and saturation is reached in fields small compared to coercive field of the saturation loop:
  - -domain walls are present in virgin state
  - -the formation of reverse domains is difficult and the demagnetization curve (second quadrant) is characterized by high coercivity
- In pinning-type magnets domain wall pinning is substantial also in the virgin state





[1] G. Bertotti, Hysteresis in Magnetism, Academic Press, 1998

Reentrant hysteresis

- The nucleation of walls often requires much higher fields than those required to sustain the domain wall motion.
- If one measures the hysteresis keeping the rate of change of magnetization very slow the so called *reentrant loop* can be obtained [1,3].

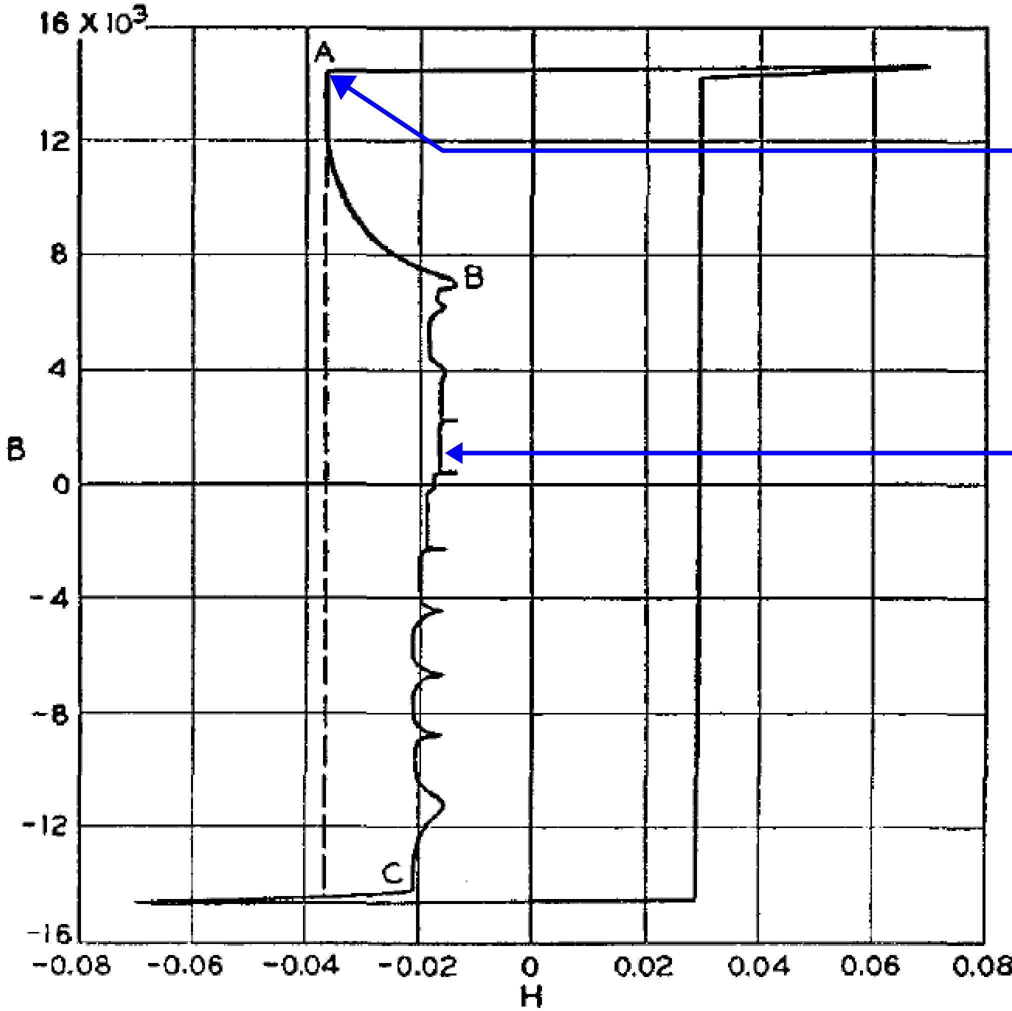


FIG. 4. Hysteresis loop of Perminvar after magnetic anneal. *A*—Starting field; *A–B*—rapid decrease of applied field; *B–C*—traced under equilibrium conditions. Dotted line indicates path described if field had not been reduced to critical field.

H. J. Williams and Matilda Goertz, J. Appl. Phys. 23, 316 (1952)

Field induced spin-flop phase transition

- The spin-flop transition* can be induced in an uniaxial antiferromagnet with low anisotropy by the field applied parallelly to anisotropy axis:

Spin-flop in bulk antiferromagnet

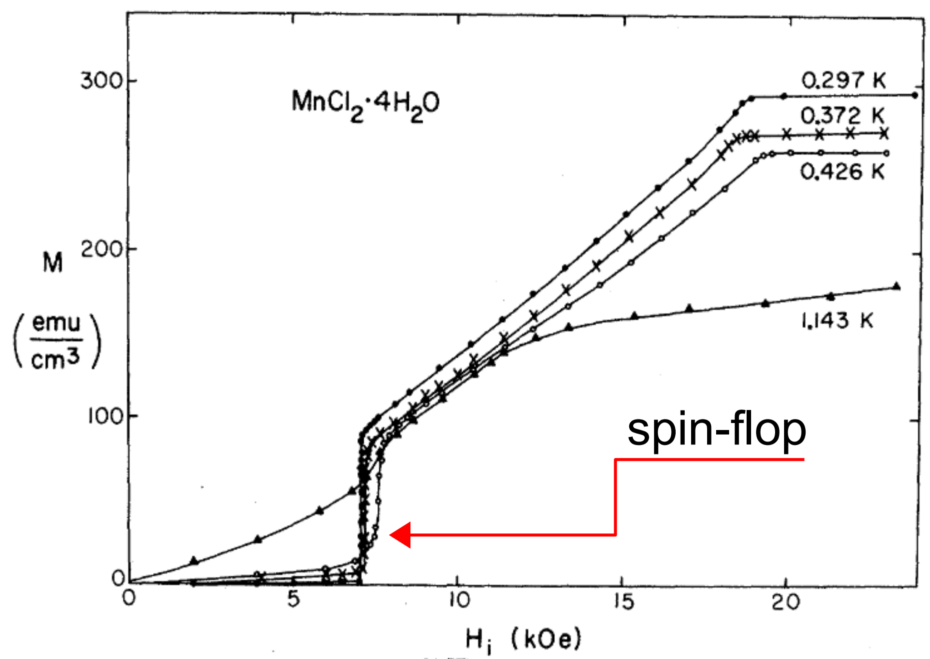
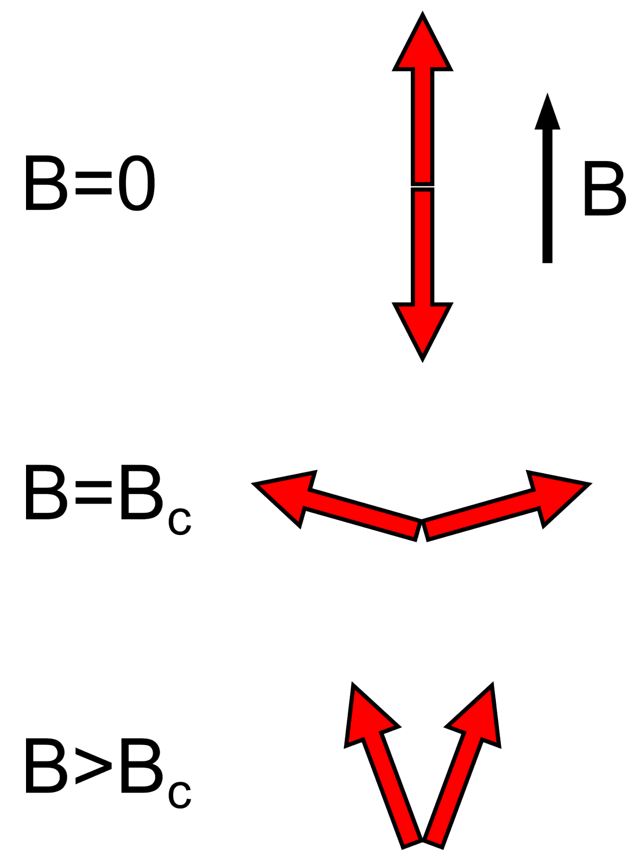


FIG. 5. Magnetization of $MnCl_2 \cdot 4H_2O$ as a function of internal field for several temperatures with the external field along the preferred direction.



J. E. Rives and V. Benedict, Phys. Rev. B 12, 1908 (1975)

*an example of a matamagnetic transition. "*A metamagnetic transition is a general term for (qualitative) changes in magnetic order due to the application of a magnetic field*", Kai Fauth, University of Würzburg

Field induced spin-flop phase transition

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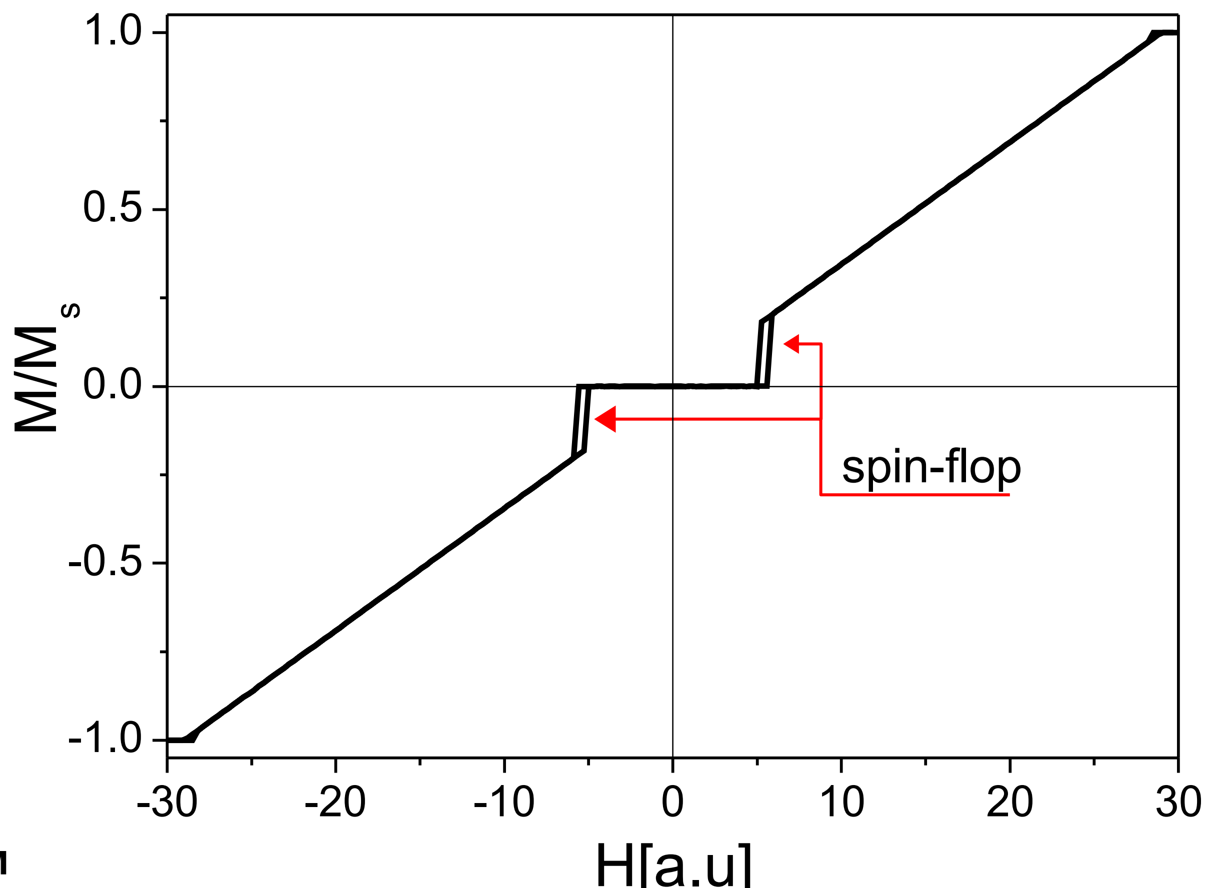
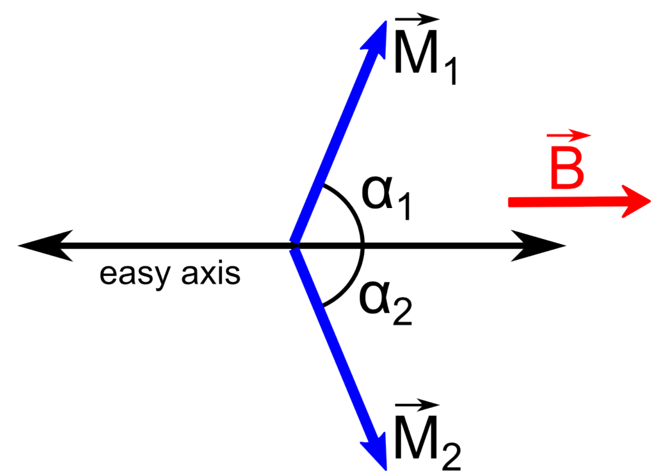


Stoner-Wohlfarth like model:

$$E=-BM_1\cos(\alpha_1)-BM_2\cos(\alpha_2)-K_a\cos^2(\alpha_1)-K_a\cos^2(\alpha_2)-J_{coupling}\cos[\alpha_1-\alpha_2]$$



M(***H***) dependence is obtained by minimizing the above energy expression with respect to α-s for consecutive values of ***H***.

Field induced spin-flop phase transition

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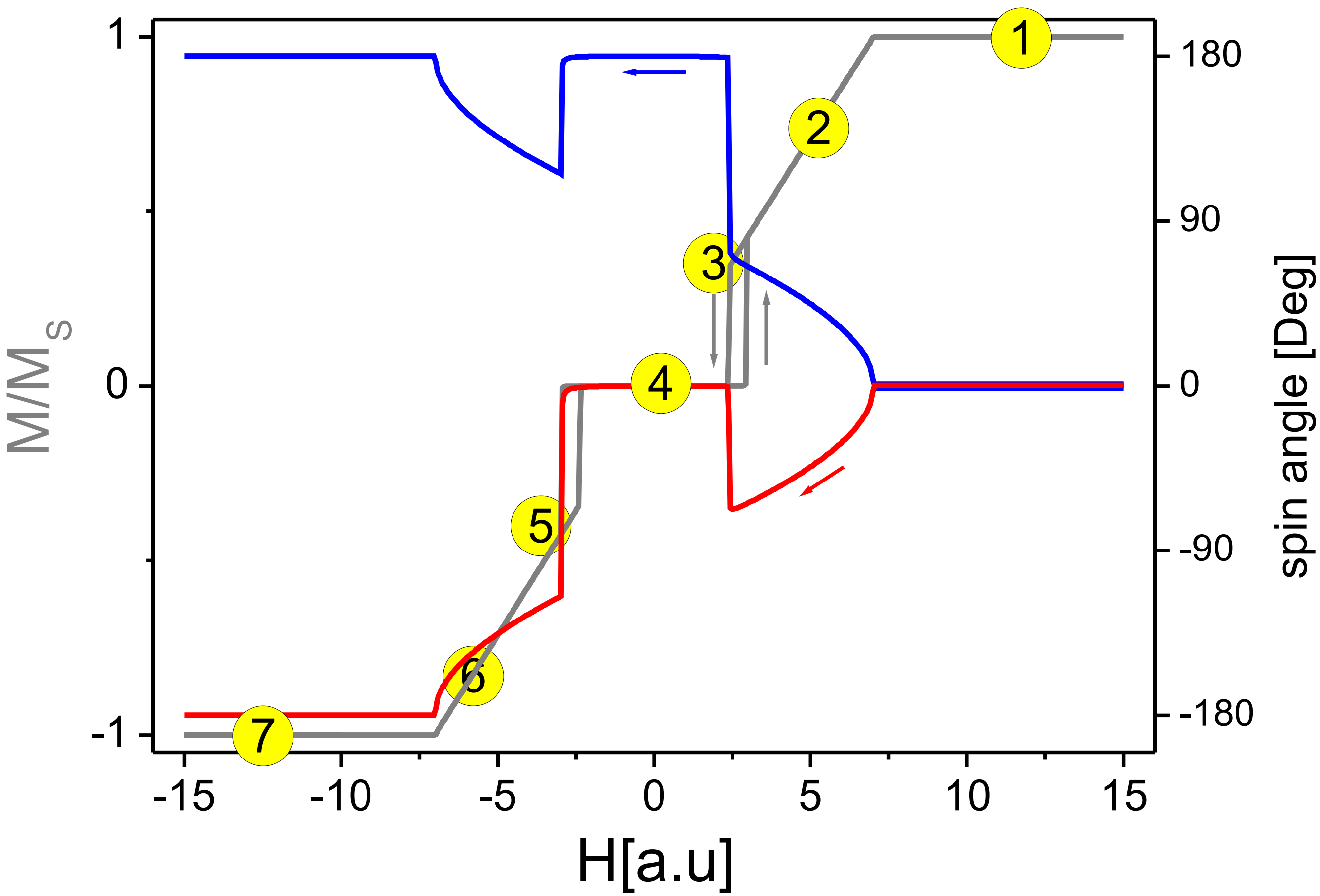
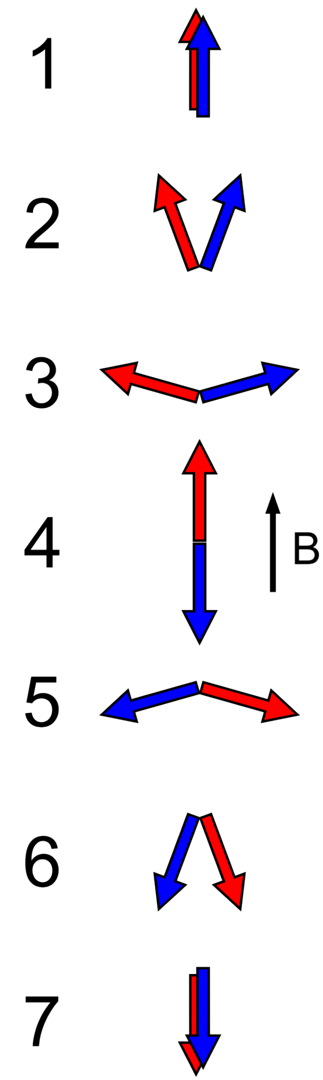
Spin-flop

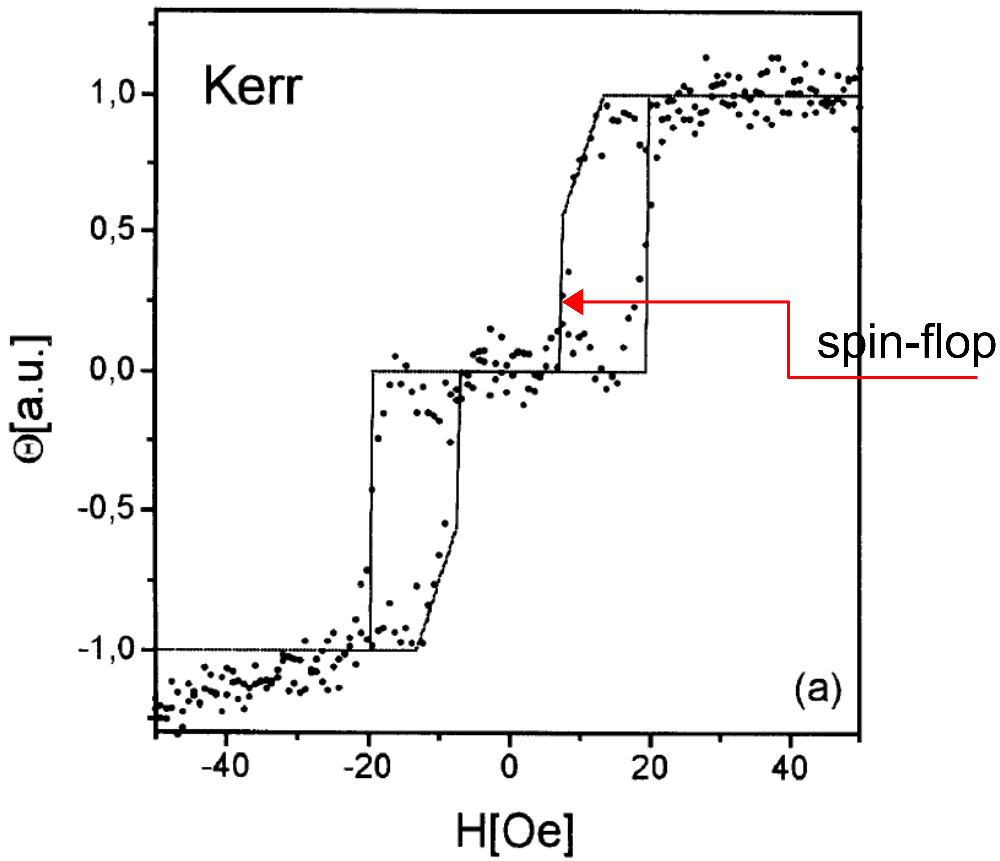




Field induced spin-flop phase transition

- The spin-flop transition can be induced in an uniaxial antiferromagnet with low anisotropy by the field applied parallelly to anisotropy axis:

Spin-flop in $[NiFe/Cu]_N$ thin film



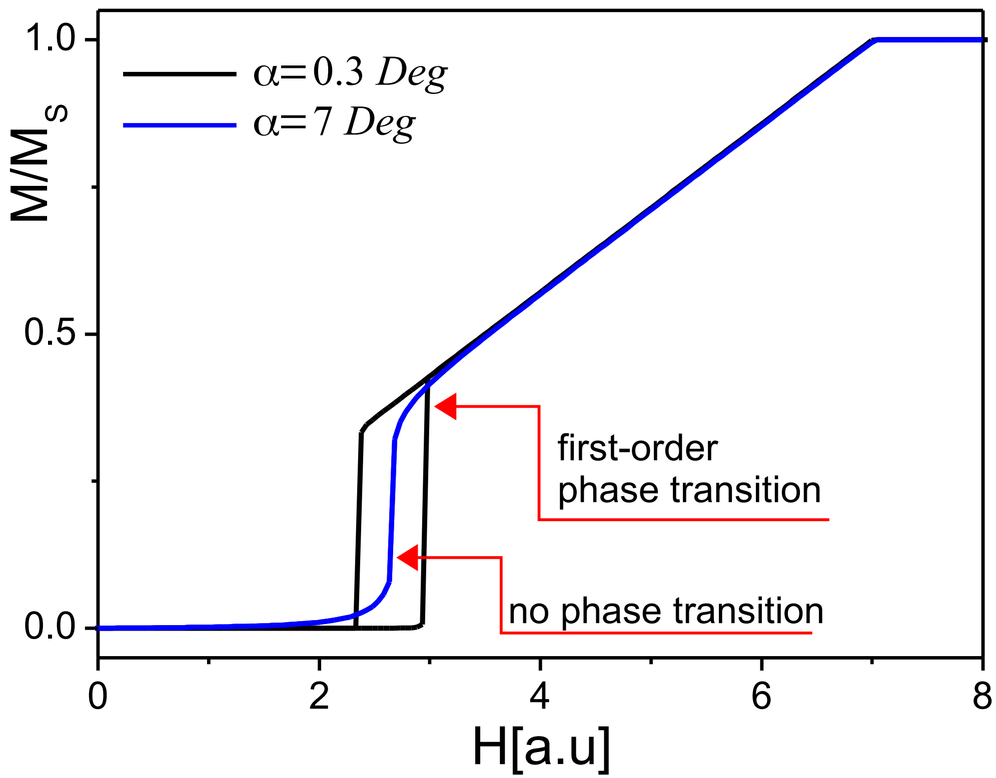
Stoner-Wohlfarth like model:

$$E = -B M_1 \cos(\alpha_1) - B M_2 \cos(\alpha_2) - K_a \cos^2(\alpha_1) - K_a \cos^2(\alpha_2) - J_{coupling} \cos[\alpha_1 - \alpha_2]$$

- In $[NiFe/Cu]_N$ multilayer each NiFe layer can be treated as a macrospin
- The NiFe sublayers are coupled by RKKY-like coupling

Field induced spin-flop phase transition

- The spin-flop transition is classified as first order field induced phase transition because of discontinuous change of magnetization of the sample*.
- When the angle between the external field and the easy axis of ferromagnet (α) exceeds some critical value the phase transition is suppressed and the magnetization changes continuously



- The critical angle is very sensitive to demagnetizing fields: "the critical angle of a cylinder is nearly three times that of a disk"*.

*K.W. Blazey, H. Rohrer and R. Webster, Phys.Rev. B **4**, 2287 (1971)

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