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DOCTORAL THESIS

**Quantum entanglement, Kondo  
effect, and electronic transport in  
quantum dots system**

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# *Abstract*

This thesis presents numerical renormalization group studies of quantum entanglement, Kondo effect and electronic transport through a system of quantum dots. The results are first presented for a triangular molecule built of coherently coupled quantum dots. Then, a single quantum dot with an assisted hopping is considered.

The first part of thesis presents studies of three electrons confined in a triple quantum dot with one of the dots connected to metallic electrodes which is modeled by a three-impurity Anderson Hamiltonian. It is focused on the pairwise quantum entanglement of a three-spin system and its relation to the thermodynamic and transport properties. It is shown that two many-body phenomena compete with each others, the Kondo effect and the inter-dot exchange interactions. In fact, coupling triple quantum dots to the electrodes results a formation of the Kondo singlet which can switch the entanglement due to the interplay between the interdot spin-spin correlations and various Kondo-like ground states. The quantum phase transition between unentangled and entangled states is studied quantitatively and the corresponding phase diagram is explained by exactly solvable four spin model. Although the work concentrates on the system of quantum dots, the model is more general and can be applied to the Kondo physics in molecules with a triangular symmetry.

The second part of thesis extends the transport properties of the triple quantum dot system for whole range of electron fillings. The study shows many-body feature of the ground states, which manifests itself in the conductance. Transport properties are also explained by an underlying Friedel-Luttinger sum rule which is applicable to both the regular- and singular-Fermi liquid ground states. The Friedel-Luttinger sum rule relates the conductance to the impurity charge and the Luttinger-integral. It is shown that the Luttinger integral is zero for the regular-Fermi liquid ground state and  $\pi/2$  for the singular-Fermi liquid phase ground state. The main attention is to electronic correlations, formation of many-body states and their role in electronic transport. Detail investigations of correlation

functions and conductance present the ground state characteristics specially their local magnetic moment formation, and a corresponding quantum phase transition which separates the regular- and singular-Fermi liquid ground states. More interestingly, it has been shown that one can obtain the underscreened Kondo effect related to partial screening of spin  $S = 1$ .

The last part of thesis studies conductance and thermopower of a single quantum dot coupled to the electrodes. The system is described by an extended single impurity Anderson model which takes into account the assisted hopping processes, *i.e.*, the occupancy-dependence of the tunneling rates. The gate-voltage and temperature dependencies are discussed in various regimes to show that the thermopower and the conductance are very sensitive probes of assisted hopping and Kondo correlations. It is found that the assisted hopping modifies the width and position of the levels, breaks the electron-hole symmetry, shifts the Kondo temperature, and strongly affects the conductance and the thermopower of a quantum dot. The assisted hopping can lead to anomalies in the mixed-valence regime, in particular with a very high Seebeck coefficient.