Transport relaxation time of spin-polarized electrons

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The transport of polarized electrons through n-dimensional systems (n=1, 2, 3) is one of the most important problems in modern electronics (spintronics), where spin of conduction electrons plays crucial role [1]. Particularly important interaction for spintronics is the spin-orbit (SO) interaction which causes the rotation of electron spin and gives rise to the spin relaxation phenomena, which are very important for industrial applications. The spin-orbit interaction results from the coupling of electron spin with the static electric field. If a charged impurity is the source of such electric field then the SO interaction is called the Elliott-Yafet spin scattering mechanism [2]. This type of SO scattering plays a significant role in coherent transport in disordered systems where it can dramatically change the properties of the system in the case of weak localization [3]. The transport of unpolarized electrons in 3D disordered metallic systems was treated in our previous paper [4]. Grimaldi [5] calculated the transport properties of 2D electron gas polarized in-plane by a magnetic field. The purpose of this paper is to consider polarized electrons in 3D disordered system, which interact with rarely distributed metal impurities or other artificial structures. We assume that polarized electrons are injected into the metallic system or heavily doped semiconductor. We neglect the interaction of electrons with positively charged matrix which is equivalent to the jellium model. Our description involves the spin-orbit interaction as an additional scattering mechanism. We first give the theoretical background of presented model and point out its limitations. Next we calculate the transport relaxation time and finally we discuss numerical results.

We consider the spin dependent scattering of the conduction electrons from the impurity described by the spherically symmetric atomic potential $u_a(r)$. The degree of disorder is low and the Ioffe-Regel criterion [6] is assumed to be valid. It means that electrons can be treated as free particles propagating as plane waves between collisions [6, 7]. The nonmagnetic impurity introduced into the system produces the perturbation of conduction electrons in the form:

$$U(r) = u_a(r) + a_{SO}\hat{\sigma} \cdot \left[\nabla u_a(r) \times \hat{\mathbf{p}}\right], \tag{1}$$

where $\hat{\mathbf{p}}$ is the momentum operator, $\hat{\sigma}$ denotes the Pauli spin matrices, $a_{SO} = \hbar/(2mc)^2$, $\hbar$ and c have their usual meaning. The first term in Eq. (1) represents the ordinary scattering and it does not affect the spin variable, and the second one represents the spin-orbit scattering. The problem of the electron scattering by the potential can be described by the Lippman-Schwinger (LS) equation [8]. Finding the exact solution of the LS equation is impossible in general, however we solved it using the Born approximation. Using this method we can find the scattering amplitude $\hat{\mathbf{F}}(\theta)$ which is directly related to the differential cross-section by the formula

$$\frac{d\sigma}{d\Omega} = \left| < \mathbf{k}', s' \left| \hat{\mathbf{F}}(\theta) \right| \mathbf{k}, s > \right|^2, \tag{2}$$

where $\hat{\mathbf{F}}(\theta)$ is a 2×2 matrix in spin space, and the indices $s, s' = \uparrow, \downarrow$ (referred to z – components of the electron spin). The standard interpretation of the elements of this matrix can be based on the well known literature [9] . Its diagonal elements represent the scattering without spin-flip, and the off-diagonal ones describe the spin-flip processes during the scattering. The scattering amplitude can be decomposed into three parts: two diagonal ones and one off-diagonal as follows:

$$\hat{\mathbf{F}}(\theta) = \hat{\mathbf{F}}_0(\theta) + \hat{\mathbf{F}}_{nsf}(\theta) + \hat{\mathbf{F}}_{sf}(\theta), \tag{3}$$

where $\hat{\mathbf{F}}_0(\theta)$ is normal scattering amplitude, $\hat{\mathbf{F}}_{nsf}(\theta)$ is non-spin-flip one, and $\hat{\mathbf{F}}_{sf}(\theta)$ is spin-flip one. We calculate the scattering amplitude using the lowest Born approximation. The use of this approximation means that the scattering amplitude is proportional to the matrix element of the perturbation potential $U(r)$ taken between the plane waves representing the conduction electron before and after the scattering. We assume a current of totally polarized electrons. The total differential cross-section for 'up'-polarized electrons in the first Born approximation now has the form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} + \frac{d\sigma_{\uparrow\uparrow}}{d\Omega} + \frac{d\sigma_{\downarrow\uparrow}}{d\Omega}\,. \tag{4}$$

To calculate the transport relaxation time we assume the Coulomb screened potential in (1) as a realistic approximation for the scattering impurities in the form

$$u_a(r) = u_0\,\frac{\exp(-\lambda r)}{r} \tag{5}$$

where u_0 is a strength of the potential, and λ is the inverse of screening length of the bare Coulomb potential. If the concentration of the impurities is low, then they can be considered as independent and we can reckon the scattering rate for a single impurity. The more realistic model of disorder should include some correlations between impurities. One of the simple realizations of this task is introducing of the structure factor [10] to the scattering rate as it was presented in Ref. [4] . In the considered case, we assume the delta-like structure factor and therefore the impurities are treated as independent ones.

It is not difficult to calculate the matrix element in (2) for the Coulomb screened potential and the adequate differential cross section has the form

$$\frac{d\sigma_0}{d\Omega} = \left(\frac{2mu_0}{\hbar^2}\right)^2 \frac{1}{\left(\lambda^2 + q^2\right)^2}\,. \tag{6}$$

Calculation of the matrix elements for the spin-orbit part gives rise to more complicated integrals. Assume the current of electrons polarized along the z axis and denote the angle between this axis and vector $\mathbf{k}\times\mathbf{k}'$ normal to the scattering plane, by ξ. Then the differential cross-sections for these two kinds of scattering are

$$\frac{d\sigma_{\uparrow\uparrow}}{d\Omega} = \left(\frac{b_{SO}mu_0}{\hbar^2}\right)^2 \frac{\lambda^{-4}q^4}{\left(1+\lambda^{-2}q^2\right)^2}\cot^2\left(\frac{\theta}{2}\right)\cos^2\xi \tag{7}$$

and

$$\frac{d\sigma_{\downarrow\uparrow}}{d\Omega} = \left(\frac{b_{SO} m u_0}{\hbar^2}\right)^2 \frac{\lambda^{-4} q^4}{\left(1+\lambda^{-2} q^2\right)^2} \cot^2\left(\frac{\theta}{2}\right) \sin^2 \xi \tag{8}$$

where $b_{SO} = \hbar a_{SO}$. The transport relaxation time is given by the well-known formula

$$\tau_{tr}^{-1} = 2\pi \frac{v_F}{V} \int_0^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega} \left(1 - \cos\theta\right), \tag{9}$$

where v_F is the Fermi velocity, V is the volume of the system, and θ is the scattering angle.

It is interesting to consider two special cases. If the axis of quantization is chosen parallel to **k** and the incident electrons are fully polarized then $\xi = \pi/2$ and all spins are reversed due to scattering. This result is identical with the one presented in Grimaldi work [5] but he considered 2D problem and we solved 3D one here. For other orientations of the z-axis we can observe both reversed and non-reversed spins. If the z-axis is perpendicular to **k** the angle ξ can take any value and we have to average both cross sections over the whole range, *i.e.* 2π. It results in equal probability for both spin-flip and non-spin-flip scattering. This is the main result of our work.

We also obtained some numerical results which will be published in a full paper.

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