Superconductivity in laminar systems

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1. Introduction

Influence of reduced dimensionality on superconductivity /SC/ remains one of the central topics in condensed matter physics. With the advent of high temperature /HTC/ superconductivity /SC/, which exhibit layered structure this problem become even more important. Unfortunately, till now main theoretical effort has been focused on search for mechanism responsible for the pair formation in the HTC SC systems. Many elaborate models of reasonable accuracy present finite domains of validity and often remain out of scope of the experimentalists and none of them explains satisfactorily all aspects of this phenomenon. Thus, despite an enormous theoretical effort over the years and quite a variety of treatments a complete theory of HTC SC still does not exist. Possibly there is coexistence of a few pairing mechanisms. This suggests that the theoretical description of the HTC systems should focus on more general properties rather than on microscopical mechanisms responsible for the pairing. Nevertheless, accumulated experimental data provide support for a widespread conjecture that superconductivity in general is a Bose-Einstein condensation of the charged Cooper pairs observed also in conventional superconductors [1]. In the following we will present an auxilairy approach, which bases on the dynamical properties of the electron/hole gas with no specific assumptions concerning the pairing. We don't intend to limit our consideration to the HTC materials. Our aim is to reevaluate our understanding of how geometry affects pair condensation in any laminar superconductors including thin films and SC superlattices fabricated of different materials.

Characteristic feature of the layered SC is that the electron gas responsible for the pair formation , due to the boundary conditions at interfaces or surfaces shows both anisotropy of mobility and anisotropy of concentration. Its is evident that quasi-2D mobility of the charge carriers is essential in formation of SC state. However, approximation of the Fermi gas in a quantum well (i.e. in a layered system) by a purely 2D or 3D system is seldom a reasonable choice. The purpose of the present paper is to formulate a simplified model of the Bose condensation in the intermediate region, when the *dynamical dimensionality* of the mobile charge carriers interpolates between 2D and 3D cases. In our approach we will treat the dimension of electron gas system as a continuous parameter. So let us before recall the concept of *fractional dimensionality* in the solid state physics.

2. Fractional spectral dimension

The concept "dimension of the system" may have several meanings. It may describe the number of coordinates to be dealt with e.g. in a problem of several quasi-particles. It can mean the dimension of the position (euclidean) space embedding the particles. In this work we shall be interested in another definition of dimensionality, which is related to the motion of quasi-particles within a solid (dynamical space, *spectral dimensionality*). Within quantum formalism the states of mobile quasi-particles within a finite solid (i.e. with periodic boundary conditions) are labeled by the **k**-wave-vectors, which form the reciprocal lattice. There is a widespread conjecture that dimensions of the position space (lattice) and of dynamical space (reciprocal lattice) should be both equal and integer. However, there is experimental evidence that in many laminar systems at least one of the abovementioned relations does not hold. In many low-dimensional systems like e.g. superlattices or overlayers, the vibrational as well the electron density of states, extracted from the experimental data correlates with those predicted for the systems of fractional dimension /FD/ [2], [3]. Laminar systems like Ag/Cu(001) overlayer or GaAs/$Al_xGa_{1-x}As$ quantum wells and superlattices as the layer thickness decreases [3] and references therein), show continuous dimensional crossover from 3D to almost 2D behaviour [3]. Generally, the dimension of these systems changes with the monolayer coverage, wire thickness or temperature. In the case of rough interfaces, a non-integer dimension of the stratified system can be interpreted in terms of fractal geometry (Haussdorff dimension) , but fractional dimensionality has been observed in a systems not having fractal structure. In principle, FD originates from restrained motion of mobile particles or quasi-particles in the stratified media. As it has been shown in [4], the observed FD of a given physical system is based on physical strength rather than on the geometrical effects. This can be easily understood, numerous physical problems involve basic objects, which are usually described by shrinking or stretching the shape of some characteristic functions. This fact modifies the energy spectrum of the mobile quasi-particles, which in turn determines the spectral dimension [3]. Invoking a FD space in description of such a systems offers a convenient alternative to computational techniques [5]. In this case single parameter - the spectral dimensionality- contains all of the information about the perturbation. We adopt the approach by He [3], who has shown that the anisotropic interactions in 3D space become isotropic ones in lower FD space, where the dimension is the Hausdorff dimension and is determined by the degree of anisotropy. Evidently, when the potential which causes the in-plane confinement is infinite, the system is purely 2D. However, in the case of finite quantum-wells the envelope functions of free electrons (holes) spread into the barrier region and partially restore the 3D character of the motion. Consequently, the system exhibits behavior, which is somewhere in between 2D and 3D.

The method by He [3] postulates that the electron quantum states are homogenously distributed in the αD **k**-space and a surface of constant energy is an αD spherical shell. Suppose further that the energy dispersion is parabolic ($E-E_o$) ~ k^2 we obtain the expression for the density of states in αD **k-**space as [3]

$$n(E)dE \approx (E - E_0)^{\alpha/2-1} dE \qquad (1)$$

where E_o is the band-gap. This means although the ionic (mass) distribution *position space* of dimensionality β shows no peculiarities, the density of free particle eigenstates shows (sometimes fractional) power law scaling (with effective spectral dimension $\alpha \neq \beta$) [3]. The effective spectral dimensionality of laminar system can be easily determined provided that energy spectrum of mobile particles within the layer is known. In principle it is enough if the density of states fulfills relation (1) in a small energy window close to the Fermi energy. Extensive analytical discussion of how the effective spectral dimensionality is associated with the number of the free electron modes can be found in [6].

3. Superconductivity

As we have shown above the dynamical states of mobile charge carriers in some laminar systems can be described properly with help of a **k**-space having fractional dimension. This concern also the YbaCuO compounds for which the FD (e.g. αD =2,03 [7]) has been postulated . Most of the theoretical approaches to the superconductivity rely of the **k**-space pairing, thus it is reasonable to consider the problem of SC in a system of (spectral) FD. For the use of further considerations it is not necessary to specify any peculiar mechanism of pairing. Experiments confirm that spectral FD case arise in various laminar systems involving polarons [8] (and thus bipolaronic SC , excitons [5], phonons [8], or magnons [9]. This variety of quasi-particles and interactions covers almost all mechanisms postulated for description of HTC SC (provided that real space pairing theories are excluded). Concluding we assume that the Hamiltonian that responsible for the Cooper pair formation is given by

$$H = \sum_{ks} (\varepsilon_k - \mu) c^{+}_{k,s} c_{ks} + \sum_{k,k_1} V_{k,k_1} c^{+}_{k,\uparrow} c_{-k\downarrow} c^{+}_{k\uparrow_1} c_{k_{1,\downarrow}} \tag{2}$$

where c^{+}_{ks} the fermion creation operator labeled by k and spin s. The only difference when compared to conventional approaches is that the **k**-states fill the space of non-integral dimensionality. As we have mentioned above, we assume the SC transition as the Bose-Einstein condensation of preexisting boson pairs. It is well known fact that Bose-Einstein condensation produces a non-zero absolute temperature T_c, below which a macroscopic condensation emerges, only if D >2. The conventional theory of boson condensation derived for systems of integral dimensionality [10] can be easily extended onto systems, which exhibit fractional spectral dimension α. The total number of bosons $N_B(T)$ in the system consists of the $N_{B,0}(T)$ ones that occupy the ground state ε_0, (ε_0 =0 in the thermodynamic limit), while the others are distributed over higher energy levels. In view of this we have:

$$N_B = N_{B,0}(T) + \sum_{k \neq 0} \frac{1}{e^{\beta(\varepsilon_k - \mu_B)} - 1} \tag{3}$$

where $\beta = 1/k_B T$ and $\mu_B < 0$ is the chemical potential. Similarly as in Eq. (3) we assume that summation goes over the **k** -states filling the fractional αD space. The sum over k in (3) can be converted to an integral over positive k , where **k** fills the αD space with use of special formula of integration over αD space (with $2 < \alpha < 3$) [6]. Applying it to Eq. (3) we obtain

$$\frac{N_{B,0}(T)}{N_{B,0}(0)} = 1 - \frac{V_\alpha}{N_B} \left(\frac{m^*}{2\pi\beta\hbar^2} \right)^{\alpha/2} \varsigma(\alpha/2) \tag{4}$$

where we have accounted for the fact that at T=0 all boson pairs form the condensate i.e. $N_{B,0}(T=0)=N_{B,0}(0)=N_B$, ζ – is the Riemann Zeta ζ function . In the calculations the parabolic energy spectrum of the quasi-particles is assumed. In conventional theories the ratio V_α/N_B is treated as the inverse boson pair concentration n_B^{-1}. Such interpretation is justified provided that spectral dimension α and dimension of real space β (position space) are equal. However, in systems of FD such interpretation is not valid. Suppose, that in the system under consideration we have some characteristic length L, then the volume $V_\alpha \sim L^\alpha \sim (k_F)^{-\alpha}$. Simultaneously the volume of the system, i.e. volume filled with quasi-particles (boson pairs) can be expressed as $V_\beta \sim L^\beta$. In view of this, concentration n_B being the real space quantity reads as $n_B = N_B/V_\beta$. Distinction between this different notions of dimensionality is often missed, but as it will be shown below crucial in proper description of dimensional effects in SC. The condensate fraction falls off when the temperature is increased and eventually at T_c, the condensate vanishes i.e. $N_{B,0}(T)/N_{B,0}(0)=0$. From this condition we can derive the formula for the critical temperature T_c, as a function of the effective spectral dimension α. Inserting relation $V_\alpha \sim L^\alpha$ into Eq. (4) we have [11]:

$$T_{c,\alpha} = \frac{m^* L^2}{2\pi k_B \hbar^2} \left(\frac{\varsigma(\alpha/2)}{N_B} \right)^{2/\alpha} . \tag{5}$$

Let us consider a FD system in two states, which exhibit FD α and α' respectively. Moreover let us assume that number of preexisting boson pairs is constant during this dimensional crossover. In view of Eq. (5) the hypothetical critical temperatures in both states fulfill the relation [11]

$$\frac{T_{c,\alpha'}}{T_{c,\alpha}} = \frac{m^*_{\alpha'}}{m^*_{\alpha}} \frac{\varsigma(\alpha/2)^{2/\alpha}}{\varsigma(\alpha/2)^{2/\alpha'}} N_B^{(2/\alpha-2/\alpha')} \tag{6}$$

Let us study the variation of the critical temperature $T_{c,\alpha}$ associated with the continuous dimensional crossover. We assume that in (6) $\alpha = 3$ i.e. take the 3D case as the reference system, First of all let us note that ratio $m^*_{\alpha}{}'/ m^*_{\alpha}$ and $\zeta(\alpha'/2)/ \zeta(\alpha/2)$ are factors of order of unity. The factor that shows strongest influence on the ratio (6) of critical temperatures in different states of the system under consideration (i.e. in states which exhibit different values of effective spectral dimension) is the last term namely$^{)}$. $(N_B)^{(2/\alpha-2/\alpha')}$. In the case $\alpha= 3$, $\alpha'= 2{,}8$ and $N_B = 10^{20}$ this factor can be estimated as $(N_B)^{(2/\alpha-2/\alpha')} =10^{-1}$, while for $\alpha = 3$, $\alpha'= 2{,}5$ it takes value $(N_B)^{(2/\alpha-2/\alpha')} =10^{-3}$. This means that when the effective dimension is decreased the critical temperature decreases in a very rapid manner. Contrary to the previous remark if $\alpha = 3$ and $\alpha' > 3$ one would expect an elevated critical temperature. This point is important per se independently of quantitative predictions since it allows us to draw general conclusions concerning the role of dimension in formation of SC phase. In connection with previous remarks there arises a question why the copper oxides, which are commonly believed to be quasi 2D superconductors with parabolic dispersion, exhibit so high critical temperatures?. The possible explanation is that the effective spectral dimension of the copper oxide system is higher than three. At first sight conclusion that the dimension of **k**-space for the boson gas confined within layers can be higher than three appears to be counter-intuitive. However, it has been proven that quasicrystals [12] and mobile quasiparticles within quantum wells [13], [14] exhibit spectral dimension $\alpha > 3$.

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