Waiting times distribution of electrons flowing across mesoscopic conductors

M. ALBERT[1,2], G. Haack[2], C. FLINDT [2], P. DEVILLARD [3], M. BÜTTIKER [2]

[1]Institut Non-Linéaire de Nice - Nice,
[2]Département de Physique Théorique - Genève,
[3]Centre de Physique Théorique - Marseille

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES
Département de physique théorique

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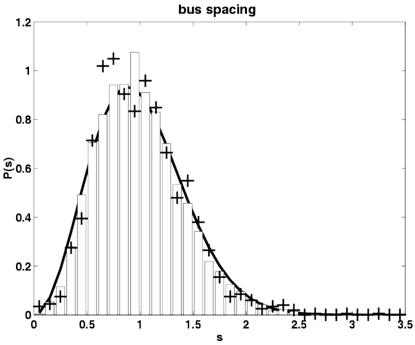
Université Nice Sophia Antipolis

QSIT

1. Motivations
2. WTD of a single electron source
3. Toward a quantum theory of waiting times
4. Conclusion and Outlook

Outline

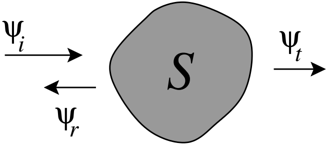


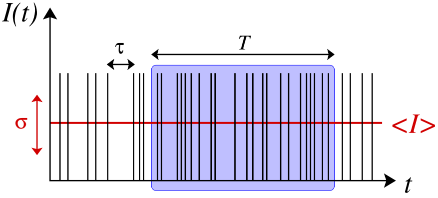
Example : City transport in Cuernavaca and Random Matrices
M. Krbálek and P. Seba J. Phys. A : Math. Gen. 33 L229 (2000).

- s is the waiting time between two buses.
- p(s) is the waiting time distribution.

Current fluctuations in mesoscopic conductors

Mesoscopic conductor : system size < coherence length. Quantum effects and fluctuations are non negligible.



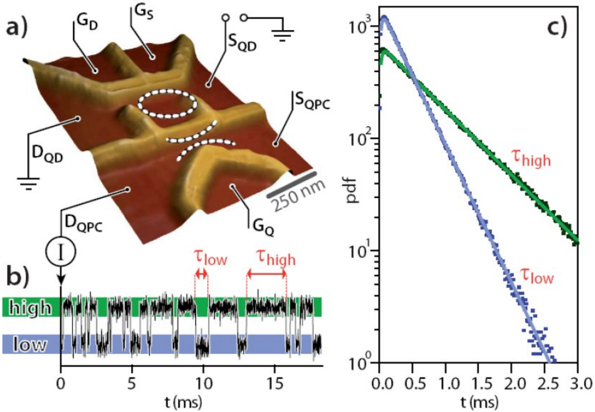


Examples : ballistic nano-wires, quantum dots, chaotic cavities, disordered wires, nanotubes etc...

Possible measures of fluctuations.

- Moments of the current distribution :
  - Noise : $S(t) = \langle I(t)I(0)\rangle - \langle I(t)\rangle^2$, $\quad \sigma = \sqrt{S(0)}$
  - Third cumulant $\langle I(t)I(t')I(0)\rangle$, etc...
- Full Counting Statistics (FCS) : $P(n,T)$: number of transferred charges over a long time window.
- Waiting time distribution (WTD) $\quad w(\tau)$

- Example in mesoscopic physics

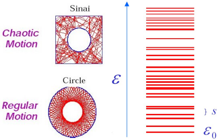


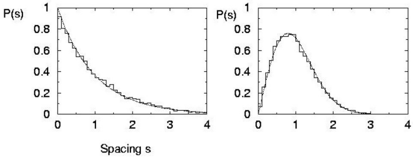
C. Flindt *et al* Proc. Natl. Acad. Sci. USA 106, 10116 (2009).

- Very general concept but rarely studied in mesoscopic physics

T. Brandes Ann. Phys (Berlin) 2008, Schriefl *et al* PRB 2005, S. Welack *et al* EPL 2009. M. Albert *et al* PRL 2011, 2012, K Thomas *et al* PRB 2013, L. Rajabi *et al* PRL 2013.

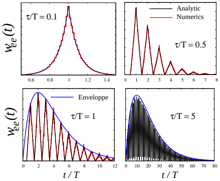
- Probe correlations on short time scales
  - Analogy with spectral statistics in regular and chaotic systems
    WTD=Level spacing distribution and FCS=Integrated density of states

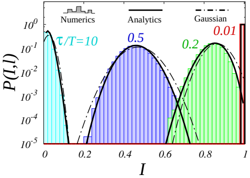




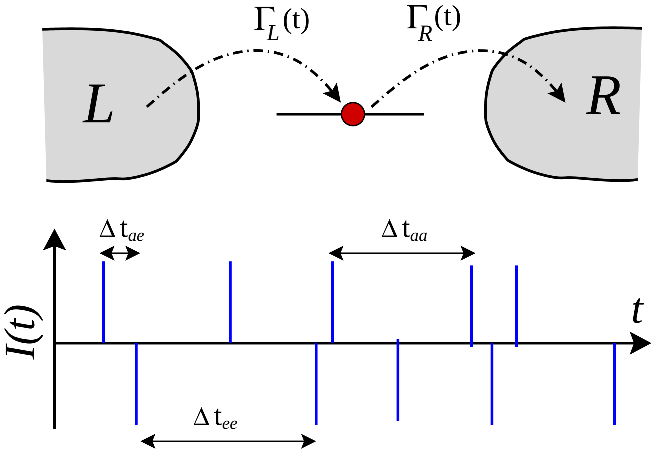
  - If two events cannot happen at the same time the WTD starts from 0.

- Measure the regularity of a source
- Give access to details that are hidden in other quantities





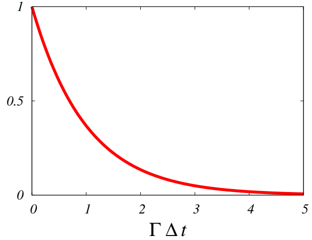
Single level system



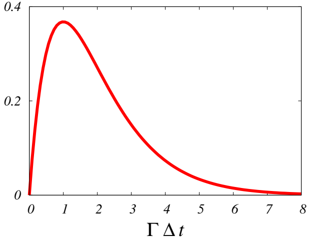
Let's consider a time independent system with $\Gamma_R = \Gamma_L = \Gamma$

$$\Rightarrow \quad w_{ea}(\Delta t) = w_{ae}(\Delta t) = \Gamma e^{-\Gamma \Delta t}, \quad w_{ee}(\Delta t) = \Gamma^2 \Delta t \, e^{-\Gamma \Delta t}$$

$w_{ea}(\Delta t)$



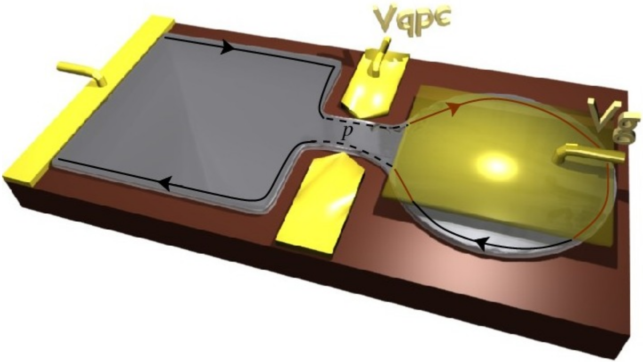
$w_{ee}(\Delta t)$



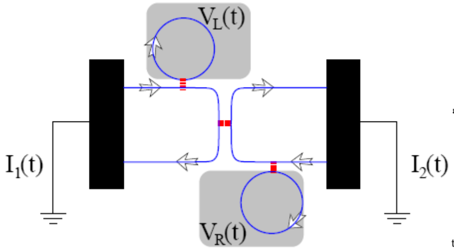
- Absorption and emission are independent events $\Rightarrow$ exponential distribution.
- Simultaneous emissions are prohibited $\Rightarrow$ hole in the WTD.

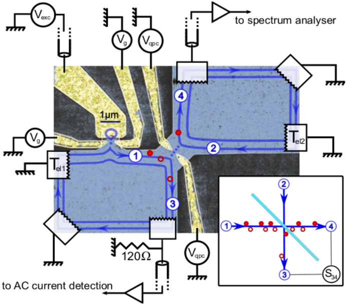
Outline

- 2D electron gas in the integer quantum Hall regime.
- Fermi sea coupled to a quantum cavity by a quantum point contact.
- Energy levels of the cavity modulated by an external voltage $V_g(t)$.



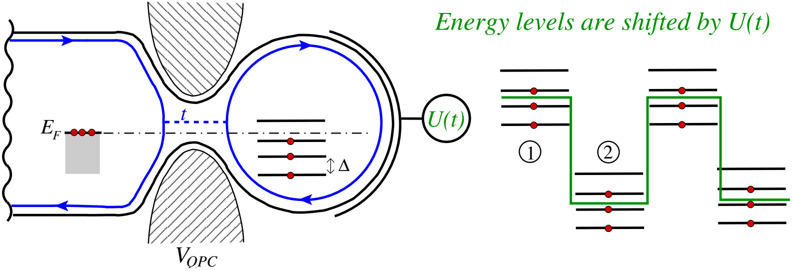
Buttiker *et al* Phys. Lett. A **180**, 364 (1993), Gabelli *et al* Science **313**, 499 (2006), Fève *et al* Science **316**, 1169 (2007), Mahé *et al* PRB **82**, 201309(R) (2010)





- Samuelsson *et al*, PRL **92**, 026805 (2004)
- Ol'khovskaya *et al*, PRL **101**, 166802 (2008)
- Splettstoesser *et al*, PRL **103**, 076804 (2009)
- Moskalets *et al*, PRB **83**, 035316 (2011)

- Haack *et al*, PRB **84**, 081303(R) (2011)
- Grenier *et al*, Mod. Phys. Lett. B **25**, 1053-1073 (2011)
- Grenier *et al*, NJP **13**, 093007 (2010)
- Bocquillon *et al*, PRL **108**, 196803 (2012)
- Jonckheere *et al*, PRB **86**, 125425 (2012)
- Bocquillon *et al*, Science, 1232572 (2013)

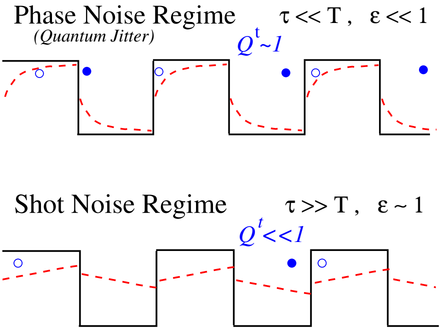


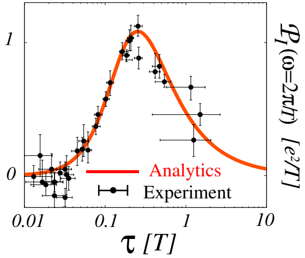
If the amplitude of excitation is equal to the level spacing ⇒ Single electron source
$P_1(t)$ is the probability to have 1 charge on the dot.

The model at zero temperature

$$\partial_t P_1(t) = \begin{cases} \Gamma[1 - P_1(t)] & ① \\ -\Gamma P_1(t) & ② \end{cases} \quad , \quad \Gamma = \frac{1}{\tau_0} \ln[1/(1 - |t|^2)]$$

Mahé *et al* PRB **82**, 201309(R) (2010), Albert *et al* PRB **82**, 041407(R) (2010).



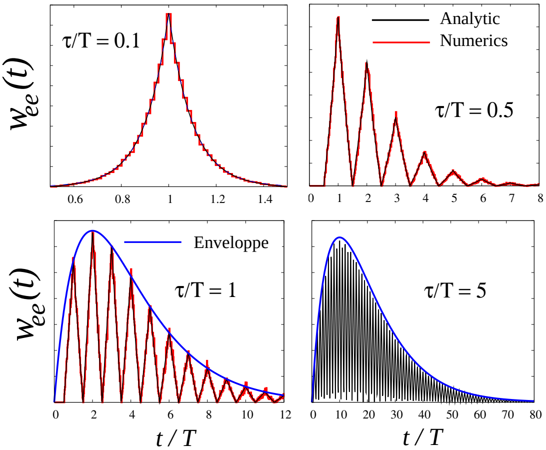


$P_I(\omega) = \int dt_0 \overline{\langle \delta I(t) \delta I(t+t_0) \rangle}^t \, e^{i\omega t_0}$

$\tau = \Gamma^{-1}$ is the escape time, T the period of driving and Q^t the transferred charge per period.

- $\tau \ll T$ phase noise regime : $Q^t \simeq 1$.
- $\tau \gg T$ shot noise regime : $Q^t \simeq 0$.

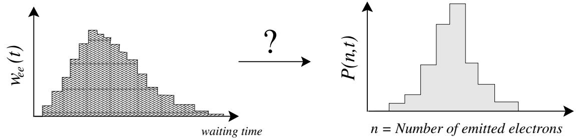
Mahé *et al*, PRB **82**, 201309(R) (2010), Albert *et al*, PRB **82**, 041407(R) (2010)
Parmentier *et al*, PRB **85**, 165438 (2012), Jonckheere *et al*, PRB **85**, 045321 (2012).



$\tau = \Gamma^{-1}$ is the escape time, T the period of driving and Q^t the transferred charge per period.

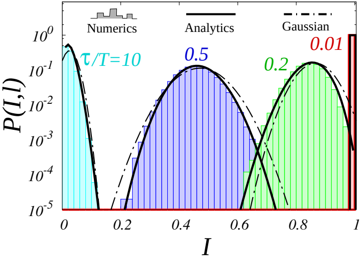
- $\tau \ll T$ phase noise regime : $Q^t \simeq 1$.
- $\tau \gg T$ shot noise regime : $Q^t \simeq 0$. T becomes irrelevant, $w_{ee}(t) \sim t\, e^{-\Gamma t}$.

From waiting time distribution to FCS



$$p(n,t) = \int d\tau_1 \cdots d\tau_n \, w_{ee}(\tau_1) \cdots w_{ee}(t-\tau_n) \, \delta\Big(\sum_{i=1}^{n} \tau_i - t\Big).$$

- The pole equation (in Laplace space) is exactly solvable.
- Same result than the one already obtained with another method.
  Albert *et al* PRB(R) (2010), Pistolesi PRB (2004).
- $I = n/\ell$, ℓ is the number of periods and $\tau = \Gamma^{-1}$ is the escape time.



Brandes Ann. Phys. (Berlin) **17** 477 (2008), Albert *et al* PRL **107**, 086805 (2011)

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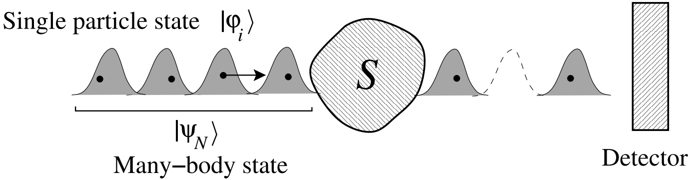


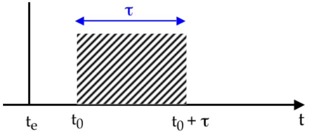
- Quantum particles are described by wave packets.
  $\rightarrow$ Quantum jitter !
- Classical measurement $\rightarrow$ detect a spike.
- The WTD probes both the structure of the many-body state and the fluctuations generated by the scatterer.

Problems !

- Energy-time Heisenberg inequality.
- The vaccum is not "empty" : Fermi sea.
- We have to include the detection process in the theory.

Ideal situation : $T = 0$, free fermions, no Fermi-Sea, time-independent scatterer, stationary process and ideal detector.

- Idle time probability $\Pi^0(\tau)$: prob to detect nothing in a time slot τ.



$$\Pi^0(\tau) = \frac{1}{\langle\tau\rangle}\int_{t_e}^{\infty} dt_0 \left(1 - \int_{t_e}^{t_0+\tau} w(t)dt\right)$$

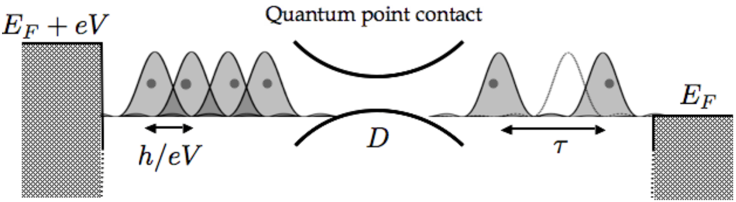
$$w(\tau) = \langle\tau\rangle \frac{d^2\Pi^0(\tau)}{d\tau^2}$$

- Transmission operator over a finite time window $\mathbf{Q}(\tau)$
  - For 1 electron : $\Pi^0(\tau) = \langle\varphi|\mathbb{1} - \mathbf{Q}(\tau)|\varphi\rangle$
  - For N electrons : $\Pi^0(\tau) = \langle\Psi_N|\prod_1^N[\mathbb{1} - \mathbf{Q}(\tau)]|\Psi_N\rangle$

If $|\Psi_N\rangle$ is a Slater determinant we get the determinant formula :

$$\Pi^0(\tau) = \det\langle\varphi_n|\mathbb{1} - \mathbf{Q}(\tau)|\varphi_m\rangle$$

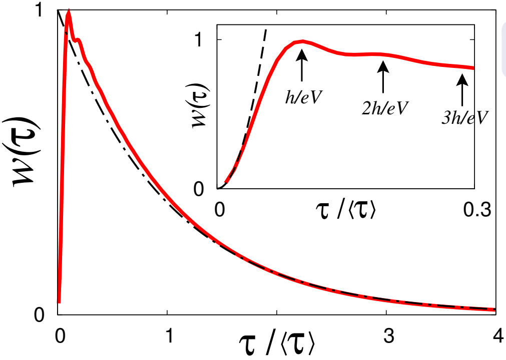
We take the $\mathbf{N} \to \infty$ limit to mimic a stationary process. Hassler *et al* PRB 2008.



- Quantum mechanical time scale h/eV.
- Intuitive picture : The Pauli principle leads to the formation of a train of wave packets. T. Martin and R. Landauer PRB 1992.

- FCS in the long time limit : Binomial process with time step h/eV and probability D. Levitov and Lesovik JETP 1993.
  For $D \ll 1$: poissonian statistics (uncorrelated transport).
- The noise $S(\omega) = \int e^{i\omega t} \langle \delta I(t) \delta I(0) \rangle dt = 0$ at $D = 1$!!!
  $\rightarrow$ No access to the structure of the wave function !

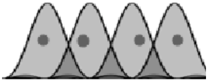
The WTD should exhibit the quantum jitter !

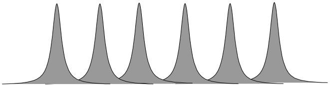


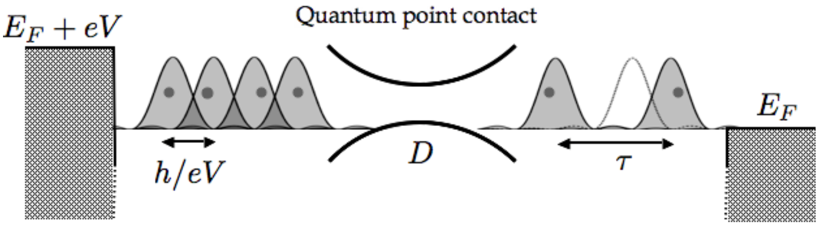
$D = 0.1\,, \quad \langle\tau\rangle = \frac{h}{eV\,D}$

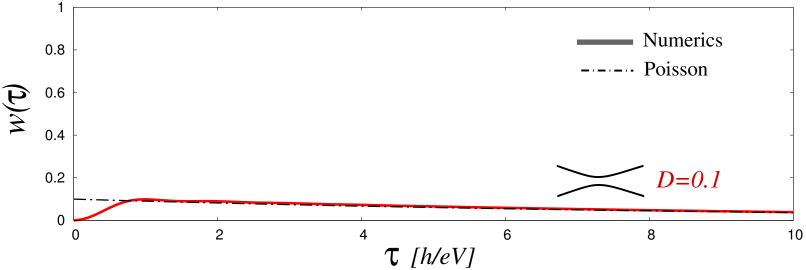
- Almost uncorrelated
  → exponential WTD.
- Pauli exclusion principle
  → hole at $\tau = 0$.
- Quantum oscillations with period h/eV!!

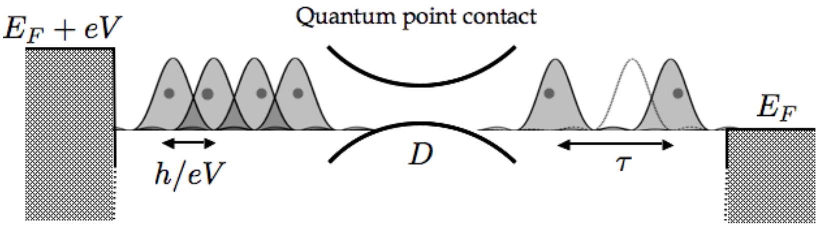
- Liquid like correlations due to the strong overlap of the wave packets. Here the particles have to fill the quantum channel.
- Solid like correlations would be observable with a triggered source. J. Keeling, I. Klich and L. Levitov PRL 2006.



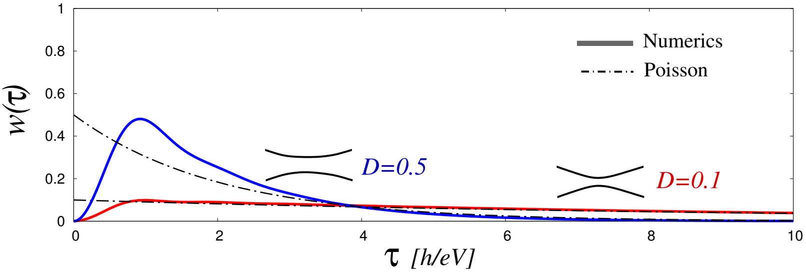



$E_F + eV$
Quantum point contact
E_F
h/eV
D
τ

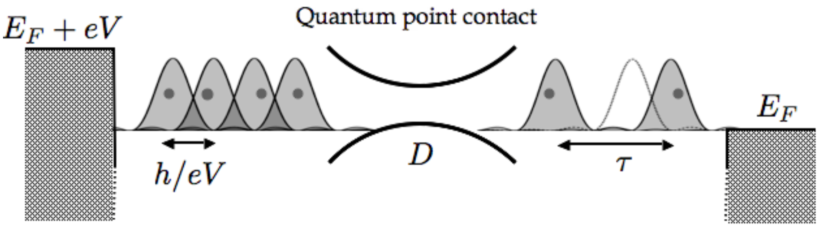

Numerics
Poisson
D=0.1
$w(\tau)$
τ [h/eV]
0
0.2
0.4
0.6
0.8
1
2
4
6
8
10

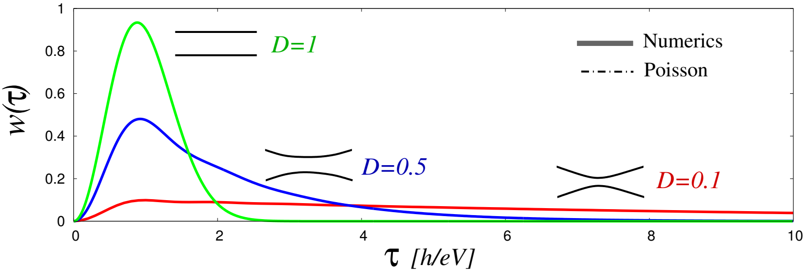


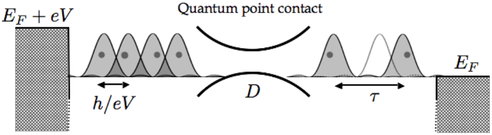
$E_F + eV$
Quantum point contact
E_F
h/eV
D
τ

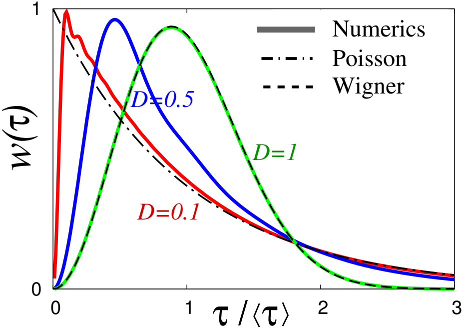


Numerics
Poisson
D=0.5
D=0.1
$w(\tau)$
τ [h/eV]
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10

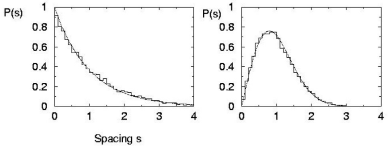

$E_F + eV$
Quantum point contact
E_F
h/eV
D
τ


1
0.8
0.6
0.4
0.2
0
$w(\tau)$
0
2
4
6
8
10
τ [h/eV]
D=1
D=0.5
D=0.1
Numerics
Poisson





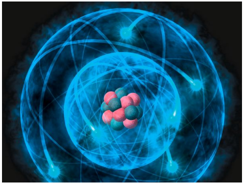
- Average waiting time $\langle\tau\rangle = \frac{h}{eVD}$.
- Crossover from Poisson to Wigner-Dyson (GUE).

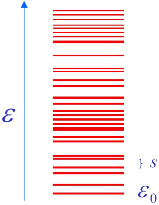


$p(s) = e^{-s}$ $\quad$ $p(s) = \frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi} s^2}$

- Large fluctuations even at $D = 1$.
  $\rightarrow$ Quantum jitter !

Connection with Random Matrix Theory (RMT)





$$\mathbf{H} = \begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$

$$P(\mathbf{H}) \sim \exp\left[-\mathrm{Tr}\, V(\mathbf{H})\right]$$

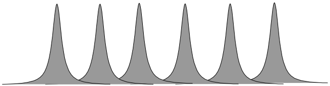
$$P(E_1,\ldots,E_N) \sim \prod_{n>m} |E_n - E_m|^{\beta} \exp\left[-\sum_n V(E_n)\right] = |\Psi(E_1,\ldots,E_N)|^2$$

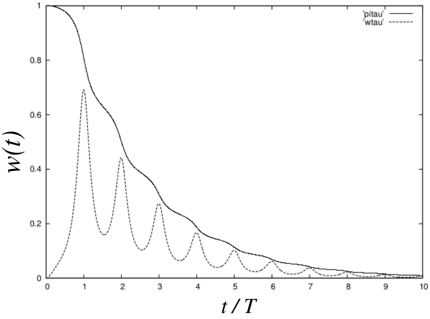
The level repulsion $(E_n - E_m)^{\beta}$ depends on symmetries ($\beta = 1, 2, 4$ for orthogonal, unitary and symplectic ensembles).

$\Psi(E_1,\ldots,E_N)$ is the ground state of the Calogero-Sutherland Model :

$$\hat{H} = -\frac{1}{2}\sum_n \frac{\partial^2}{\partial E_n^2} + \frac{\beta}{2}\left(\frac{\beta}{2} - 1\right)\sum_{n>m} \frac{1}{(E_n - E_m)^2}$$

$\beta = 2$: free fermions $\Rightarrow$ mapping between RMT and free fermions in 1D. All the correlation functions are identical. $E_n \leftrightarrow x_n$





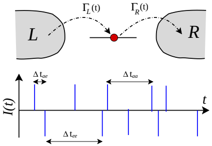
$\xi = 0.2T, D = 0.4$

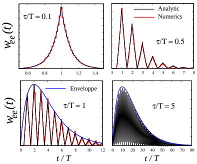
Lorentzian pulses with $n = 1$
J. Keeling, I. Klich and L. Levitov PRL 2006.
J. Dubois *et al* PRB 2013.

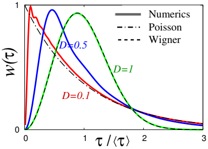
- Tunable aspect ratio : liquid to solid crossover.
- $\xi \ll T$: thin peaks reflecting the shape of the wave packet.
- $\xi \gg T$: constant bias limit $eV = h/\xi$.
- $D = 1$ is no longer given by RMT.

- See David Dasenbrook talk !

Conclusion and outlook

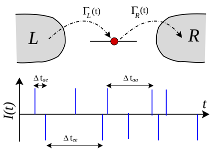


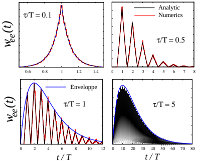


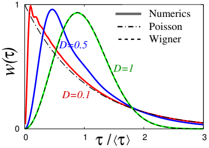


$$\begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$

- Waiting time distribution as a tool to probe accuracy and correlations .
- Connection between WTD and FCS.
- Quantum capacitor and quantum dots as experimentally relevant examples.
- Quantum theory of WTD for non-interacting electrons. Link with RMT.
- Open questions : more general systems, arbitrary time dependence, interaction effects, universality classes ?







$$\begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$


M. Büttiker


C. Flindt


G. Haack

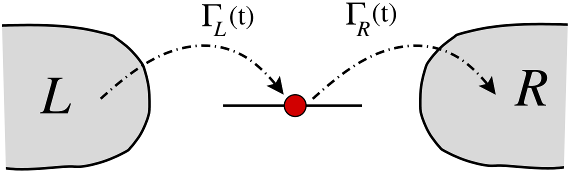

M. Albert


P. Devillard

Thank you for your attention !

For further reading see

- Albert *et al* PRB **82**, 041407(R) (2010)
- Albert *et al* PRL **107**, 086805 (2011)
- Brandes Ann. Phys. (Berlin) **17** 477 (2008)
- Albert *et al*, PRL **108**, 186806 (2012)



Master equation for $P_1(t) = \langle Q(t) \rangle$

$$\partial_t P_1(t) = -[\Gamma_L(t) + \Gamma_R(t)]P_1(t) + \Gamma_L(t)$$

- Incoming current $\Rightarrow \quad \langle I_{in}(t) \rangle = \Gamma_L(t)[1 - P_1(t)]$
- Outgoing current $\Rightarrow \quad \langle I_{out}(t) \rangle = \Gamma_R(t) P_1(t)$

How to calculate the waiting time distribution ?

$w_{ea}(t, t+\Delta t)$ is proportional to the probability that an electron is absorbed a time t and emitted at time $t+\Delta t$.

- Absorption at time $t \sim |\langle I_{in}(t)\rangle|$
- **Next emission** at time $t+\Delta t \sim |\langle I^s_{out}\rangle| = \Gamma_R P^s_1$

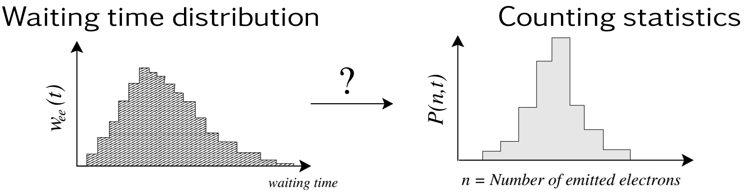
Master equation for the survival probability

$$\partial_t P^s_1(t+\Delta t) = -\Gamma_R(t+\Delta t) P^s_1(t+\Delta t) \quad \text{with } P^s_1(t) = 1$$

$$\Rightarrow \quad w_{ea}(t, t+\Delta t) = \mathcal{N} |\langle I_{in}(t)\rangle \langle I^s_{out}(t+\Delta t)\rangle|$$

In general the initial time is arbitrary and one should sum over all the possibilities.

$$\overline{w}_{ea}(\Delta t) = \int_0^T \frac{dt}{T} w_{ea}(t, t+\Delta t)$$



$$p(n,t) = \int d\tau_1 \cdots d\tau_n \, w_{ee}(0,\tau_1) \cdots w_{ee}(t-\tau_n,t)\, \delta\left(\textstyle\sum_{i=1}^{n} \tau_i - t\right).$$

In the long time limit we make the following approximation

$$p(n,t) \simeq \int d\tau_1 \cdots d\tau_n \, \overline{w}_{ee}(\tau_1) \cdots \overline{w}_{ee}(\tau_n)\, \delta\left(\textstyle\sum_{i=1}^{n} \tau_i - t\right).$$

After some manipulations in Fourier space in the long time limit

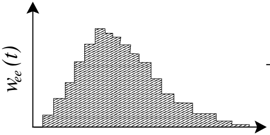
$$\Rightarrow \quad \mathcal{G}_{ee}(\mathcal{S}(\chi,t)) + i\chi = 0$$

$\mathcal{G}_{ee}(z) = \ln \int_0^\infty d\tau \overline{w}_{ee}(\tau)\, e^{-z\tau}$ and $\mathcal{S}(\chi,t) = \ln \sum_{n=0}^{\infty} p(n,t)\, e^{i\chi n}$

CGF of waiting times — CGF of the FCS

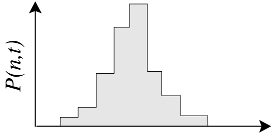
This equation was previously derived in Brandes Ann. Phys. 2008 for time independent systems.

Waiting time distribution



?

Counting statistics



Equation relating the two cumulant generating functions

$$\Rightarrow \quad \mathcal{G}_{ee}(\mathcal{S}(\chi,t)) + i\chi = 0$$

$$\mathcal{G}_{ee}(z) = \ln \int_0^\infty d\tau \overline{w}_{ee}(\tau)\, e^{-z\tau} \quad \text{and} \quad \mathcal{S}(\chi,t) = \ln \sum_{n=0}^\infty p(n,t)\, e^{i\chi n}$$

CGF of waiting times — CGF of the FCS

From this very simple relation one can extract some relations between cumulants

$$I = \frac{\langle\langle n \rangle\rangle}{t} = \frac{1}{\langle\langle \tau \rangle\rangle} \quad , \quad F_2 = \frac{\langle\langle n^2 \rangle\rangle}{\langle\langle n \rangle\rangle} = \frac{\langle\langle \tau^2 \rangle\rangle}{\langle\langle \tau \rangle\rangle^2},$$

$$F_3 = \frac{\langle\langle n^3 \rangle\rangle}{\langle\langle n \rangle\rangle} = 3\frac{\langle\langle \tau^2 \rangle\rangle^2}{\langle\langle \tau \rangle\rangle^4} - \frac{\langle\langle \tau^3 \rangle\rangle}{\langle\langle \tau \rangle\rangle^3}$$