Floquet Theory of Electron Waiting Times in Quantum-Coherent Conductors

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Outline

Electron Waiting Times
- Waiting time distribution
- Why a theory of WTDs?
- Quantum formalism

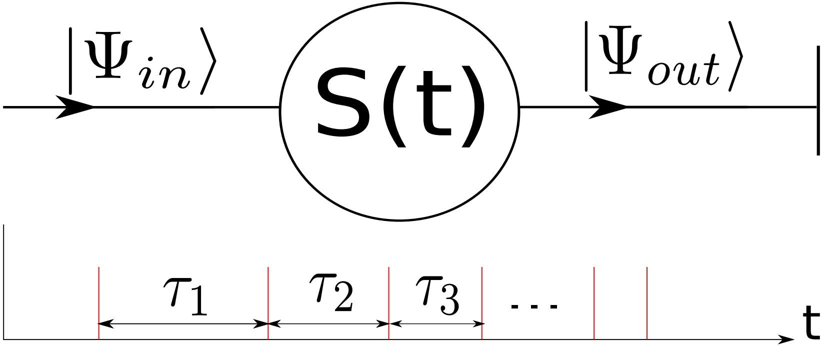
Floquet scattering theory
- Formalism
- Evaluating the ITP

Applications
- Quantum point contact with modulated transmission
- Lorentzian voltage pulses

Conclusions

Electron Waiting Times

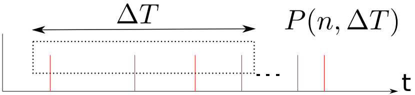
The waiting time distribution (WTD)



Electron Waiting Times

Why a theory of WTDs?

Full counting statistics (FCS) typically evaluated in the long time limit



Electron Waiting Times

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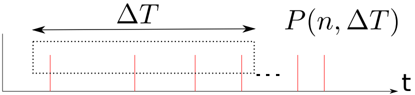
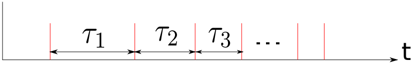
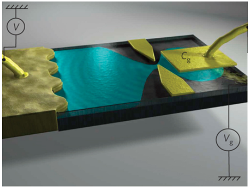
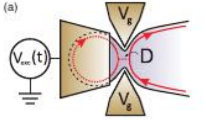


Driven quantum systems (e.g. single electron sources, quantum pumps): Characterization on short time scales important







Pictures: Mora et al., Nat. Phys. 6, 697 (2010); Parmentier et al., PRB 85, 165438 (2012)

Electron Waiting Times

Quantum formalism

- Idle time probability (ITP): probability for *no* detection in the time interval $[t_0, t_0 + \tau]$

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WTD and ITP

$$\mathcal{W}(\tau) = \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} \partial_\tau^2 \Pi(t_0, \tau) \mathrm{d} t_0$$

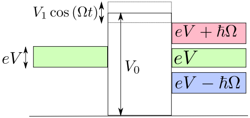
Albert et al., PRL 108, 186806 (2012); Vyas et al., PRA 38, 2423 (1988)

Outline

Floquet scattering theory

Formalism

Periodically time-dependent scattering problems: Particles can absorb or emit modulation quanta $\hbar\Omega$



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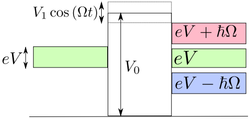

Floquet scattering matrix

$$\hat{b}(E) = \sum_{E_n} \mathcal{S}(E, E_n) \hat{a}_\beta(E_n)$$

$$E_n = E + n\hbar\Omega$$

connecting incoming ($\hat{a}$) and outgoing ($\hat{b}$) annihilation operators

Floquet scattering theory

Evaluating the ITP

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Evaluate in outgoing state

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- Evaluate in outgoing state
- Use Floquet S-matrix to map onto evaluation of an equilibrium average

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Applications

Quantum point contact with modulated transmission

"Frozen" transmission probability

$$T(t) = T_0[1 - \varepsilon \sin(\Omega t)]^2$$

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"Frozen" transmission probability

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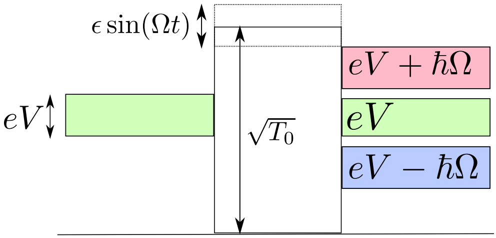
Compared to the static case, get two side bands

$$S_F(E_n, E) = \sqrt{T_0}[\delta_{n,p} + i\varepsilon(\delta_{n,p-1} - \delta_{n,p+1})/2]$$

where $eV = p\hbar\Omega$ for simplicity



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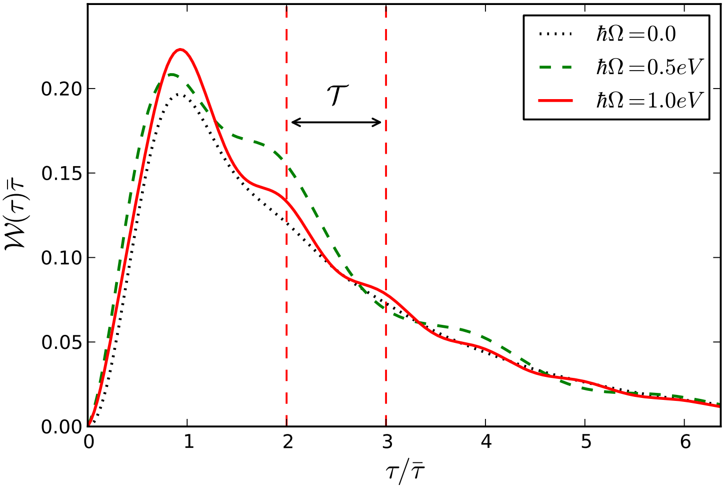
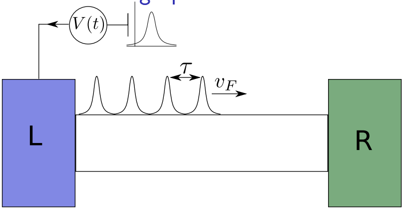


$\bar{\tau} = h/eV$

Applications

Lorentzian voltage pulses



$\mathcal{T}$: period

τ: waiting time

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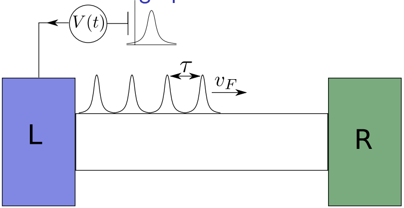

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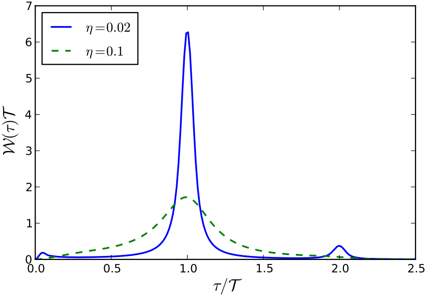
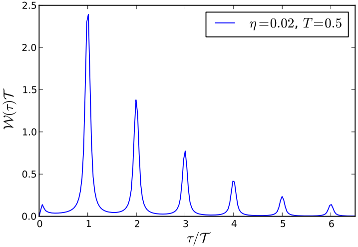


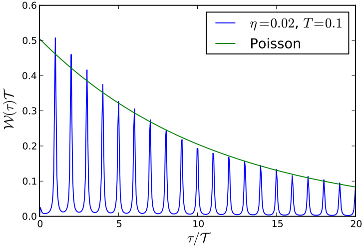
η: pulse width divided by period

Applications

Lorentzian voltage pulses

Add a QPC with transmission T:





Crossover to Poissonian statistics

Compare Albert et al., PRL 107, 086805 (2011) for classical analogue

Conclusions

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- DD, C. Flindt, M. Büttiker (in prep.)

Floquet scattering theory

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One-particle operator measuring occupation probability of the spatial interval $[x_0, x_0 + v_F \tau]$:

$$Q_\tau = \int_{x_0}^{x_0+v_F\tau} |x\rangle \langle x| \, \mathrm{d}x$$

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Second quantized version of the operator that applies $1 - Q_\tau$ to all particles in the Slater determinant at once:

$$:\exp(-Q_\tau) := \bigotimes_{n=1}^{N} (1 - Q_\tau)$$

See also Levitov et al., J. Math. Phys. 37, 4845 (1996) and Hassler et al., PRB 78, 165330 (2008)