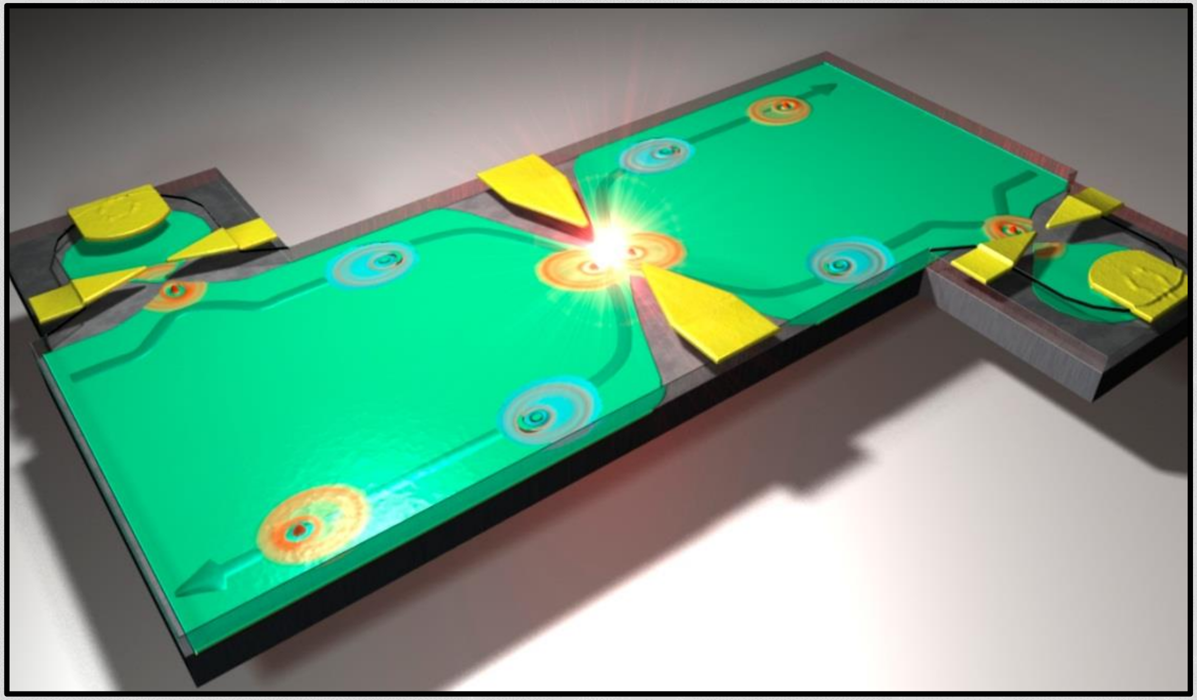
Electron quantum optics in quantum Hall edge channels

laboratoire pierre aigrain
électronique et photonique quantiques



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Motivation, electron optics

- coherence properties of light sources.

Wavelike description encoded in the first order coherence of the electromagnetic field

$$G^{(1)}(t, t+\tau) \propto \langle E(t)E(t+\tau)\rangle$$

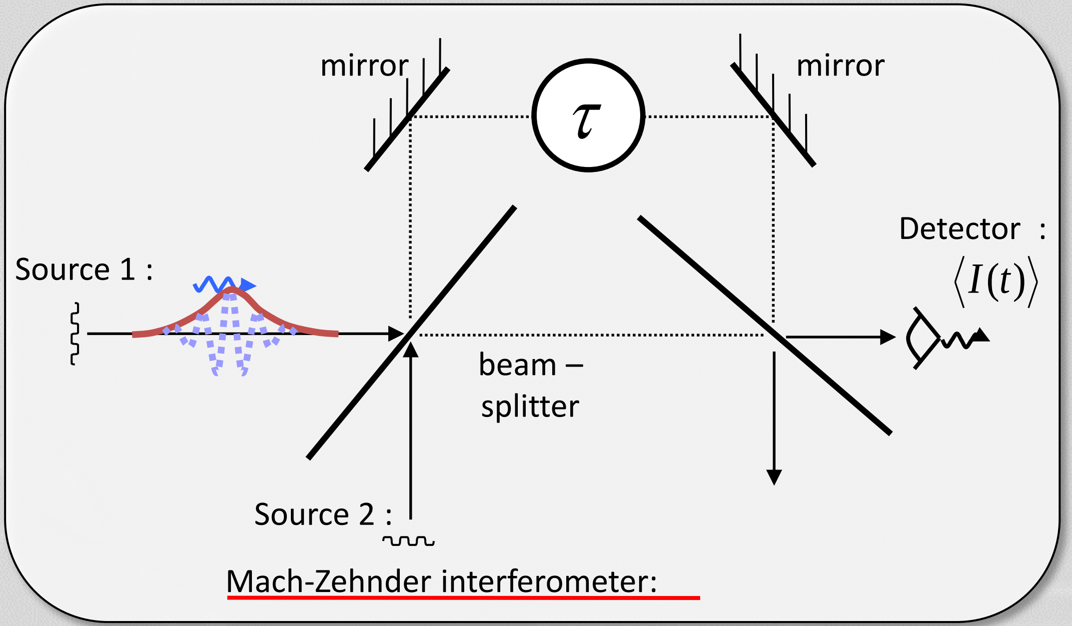
Mesured by interferometry, e.g, Mach-Zehnder.

- statistical properties of light sources

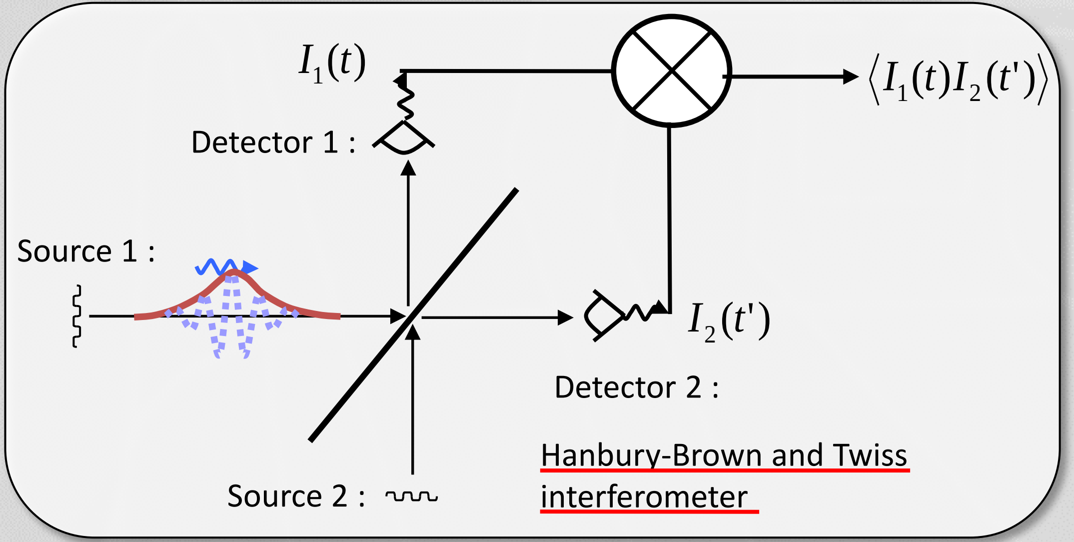
Corpuscular description encoded in intensity correlations.

$$G^{(2)}(t, t+\tau) \propto \langle I(t)I(t+\tau)\rangle$$

Mesured by Hanbury-Brown and Twiss (HBT) interferometry (HBT)



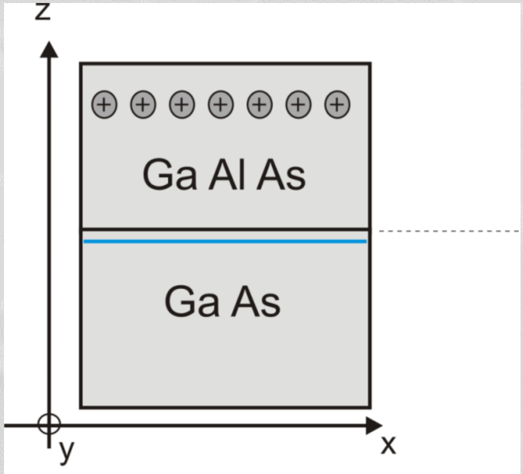
Mach-Zehnder interferometer:



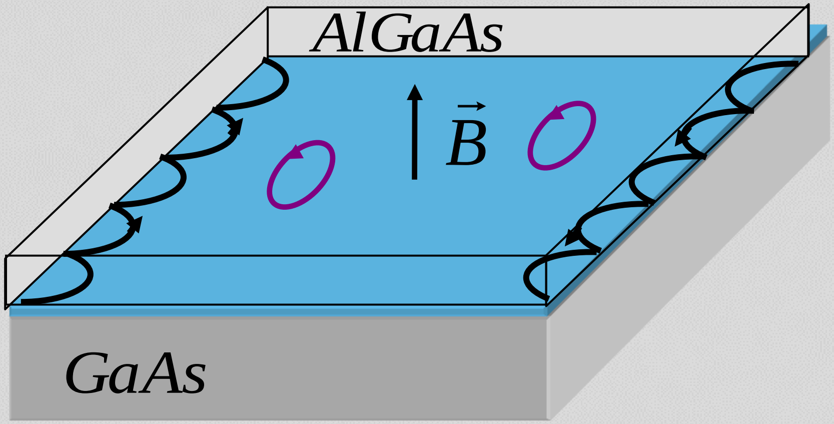
Hanbury-Brown and Twiss interferometer

One dimensional quantum conductor

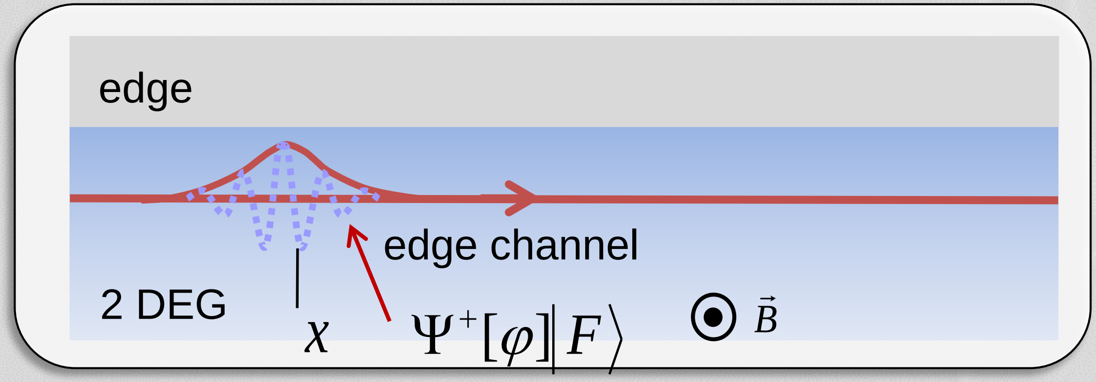
- 2 D electron gas (T<100 mK)



- quantum Hall effect



- Ballistic, 1 dimensional, chiral, spin polarized propagation along one edge channel



$$\Psi^{+}[\varphi]|F\rangle = \int dx\, \varphi(x)\, \Psi^{+}(x)|F\rangle$$

Electron/photon : analogies/differences

- analogies

$$\boxed{\Psi(t) \leftrightarrow E^{+}(t) \qquad \Psi^{+}(t) \leftrightarrow E^{-}(t)}$$

- first order coherence function

$$\underline{G^{(1)}(t,t') = \left\langle \Psi^{+}(t)\Psi(t') \right\rangle} \qquad \underline{\tilde{G}^{(1)}(\varepsilon,\varepsilon') = \left\langle a^{+}(\varepsilon)a(\varepsilon') \right\rangle}$$

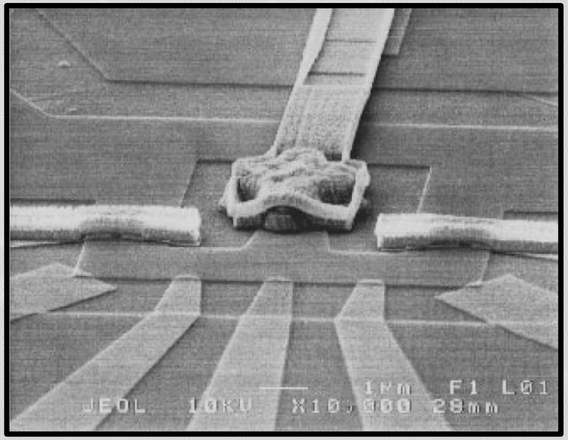
- electrical current/ light intensity

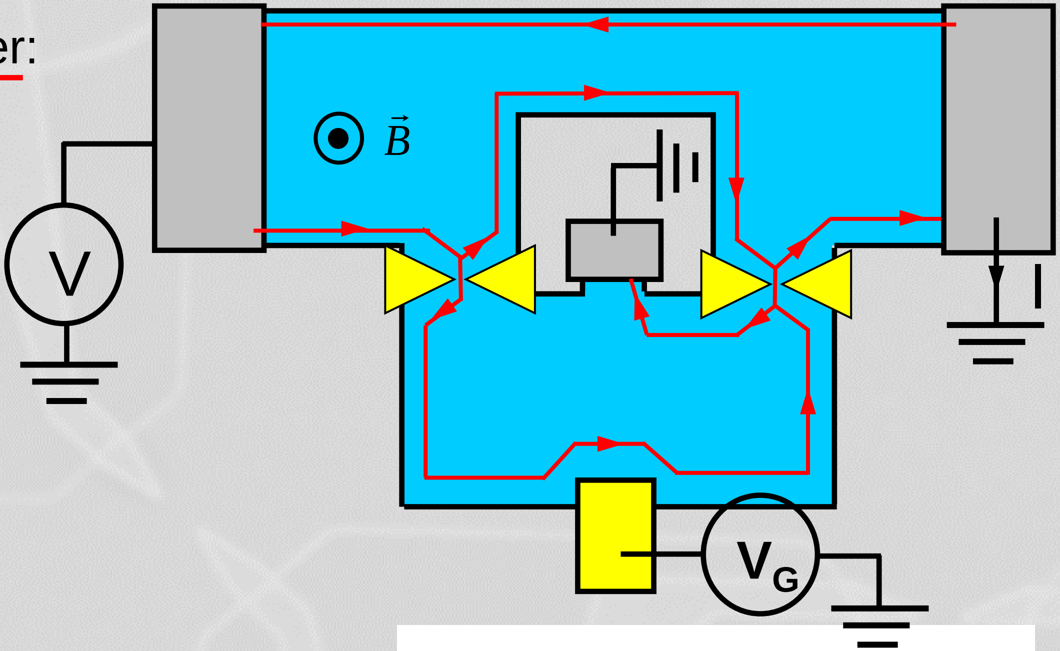
$$\underline{I(t) = e\Psi^{+}(t)\Psi(t)} \leftrightarrow \underline{I_{ph}(t) \propto E^{-}(t)E^{+}(t)}$$

- but important differences
  - Fermionic vs Bosonic statistics $\left|F\right\rangle \neq \left|0\right\rangle$
  - Coulomb interactions

Measurement of electronic coherence: the Mach-Zehnder interferometer

- The Mach-Zehnder interferometer:

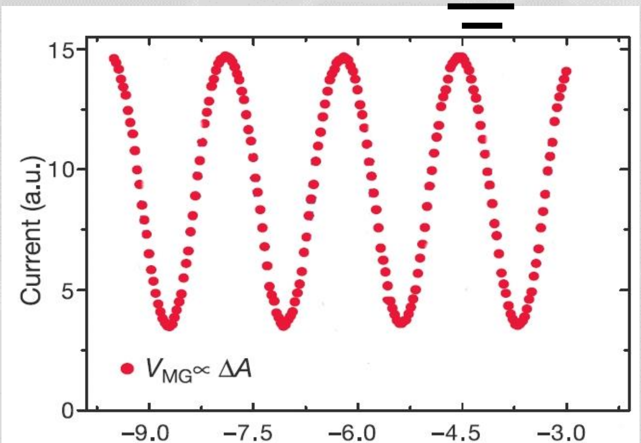




- Measurement of $g^{(1)}(\tau)$

$$I=\frac{e^2V}{2h}\left[1-\mathrm{Re}\left(g^{(1)}(\tau)\right)\right]$$

$$I=\frac{e^2V}{2h}\left[1-\cos(\varepsilon_f\tau/\hbar)\sin c(eV\tau/2\hbar)\right]$$



Y. Ji et al., Nature **422**, 415 (2003)
P. Roulleau et al., PRL **101**, 186803 (2008)

Electron optics experiments: stationary sources

Most experiments : DC sources

- Coherence: Mach-Zehnder interferometers

Y. Ji et al., Nature 422, 415 (2003)
L. V. Litvin et al., Phys. Rev. B 75, 033315 (2007).
P. Roulleau et al., Phys. Rev. Lett. 101, 186803 (2008)

- Statistics: Hanbury-Brown & Twiss experiments

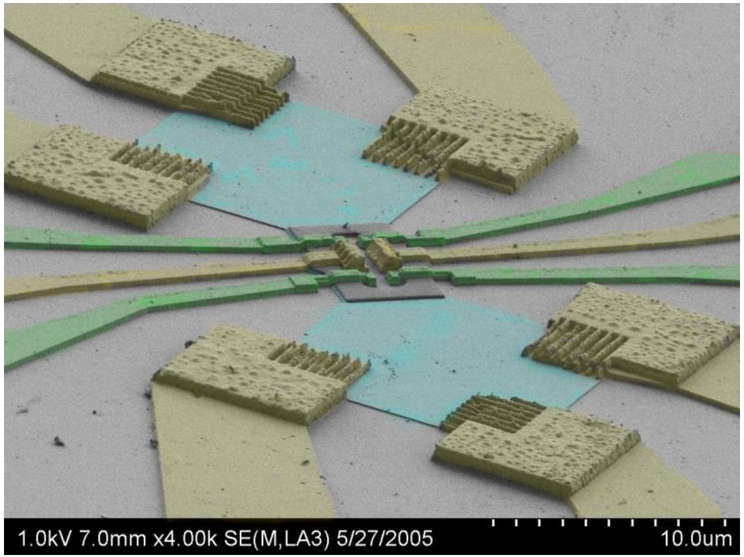
W. Oliver et al., Science 284, (5412), 299-301, (1999)
M. Henny et al., Science 284 (5412), 296 (1999)

- Spectroscopy

C. Altimiras et al., Nature Physics 6, 34 (2010)

- Two electron interferences

R. C. Liu et al., Nature 391, 263 (1997).
P. Samuelsson et al., Phys. Rev. Lett. 92, 02685 (2004)
I. Neder et al., Nature 448, 333 (2007)



MZ interferometer, CEA Saclay

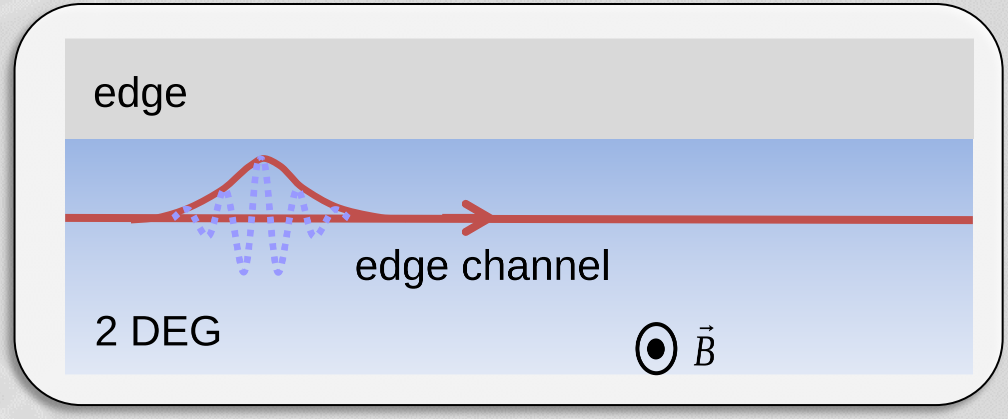
Emission not triggered

No single particle control

Triggerred electron emitters

- Electron pumps

M.D. Blumenthal et al., Nature Physics 3, 343 (2007)
P. Mirovsky et al., APL 97, 252104 (2010)
F. Hohls, Phys. Rev. Lett. 109, 056802 (2012)



- Electrons flying on SAW

R. McNeil et al., Nature 477 (7365), 439 (2011)
S. Hermelin et al., Nature 477 (7365), 435 (2011)

- Lorentzian voltage pulse

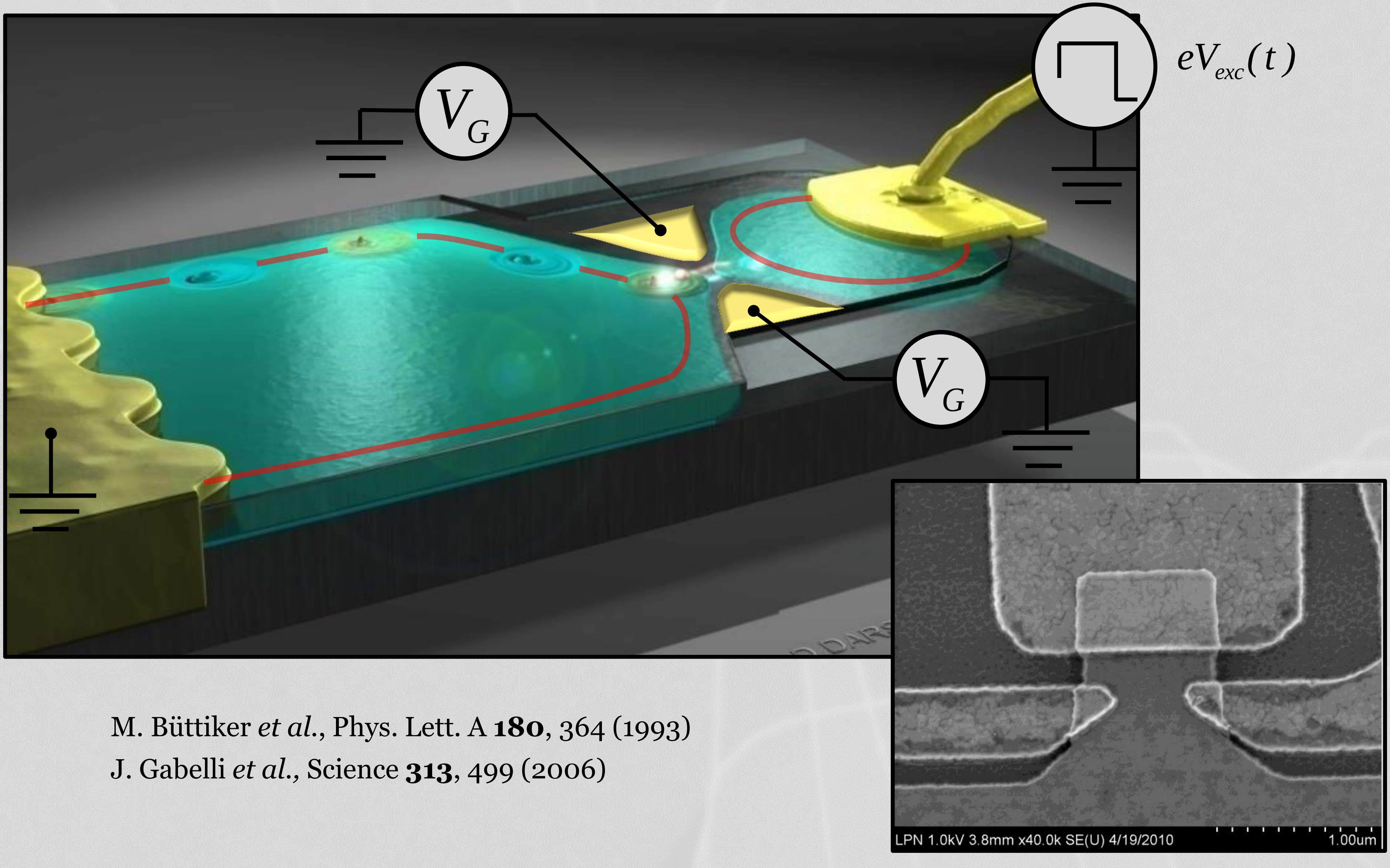
D. A. Ivanov, et al., Phys. Rev B 56, 6839 (1997)
J. Dubois et al., Phys. Rev. B 88, 085301 (2013)
Ch. Grenier et al. Phys. Rev. B **88**, 085302 (2013)

Talk of David Dasenbrook yesterday

- Mesoscopic capacitor (purely AC)

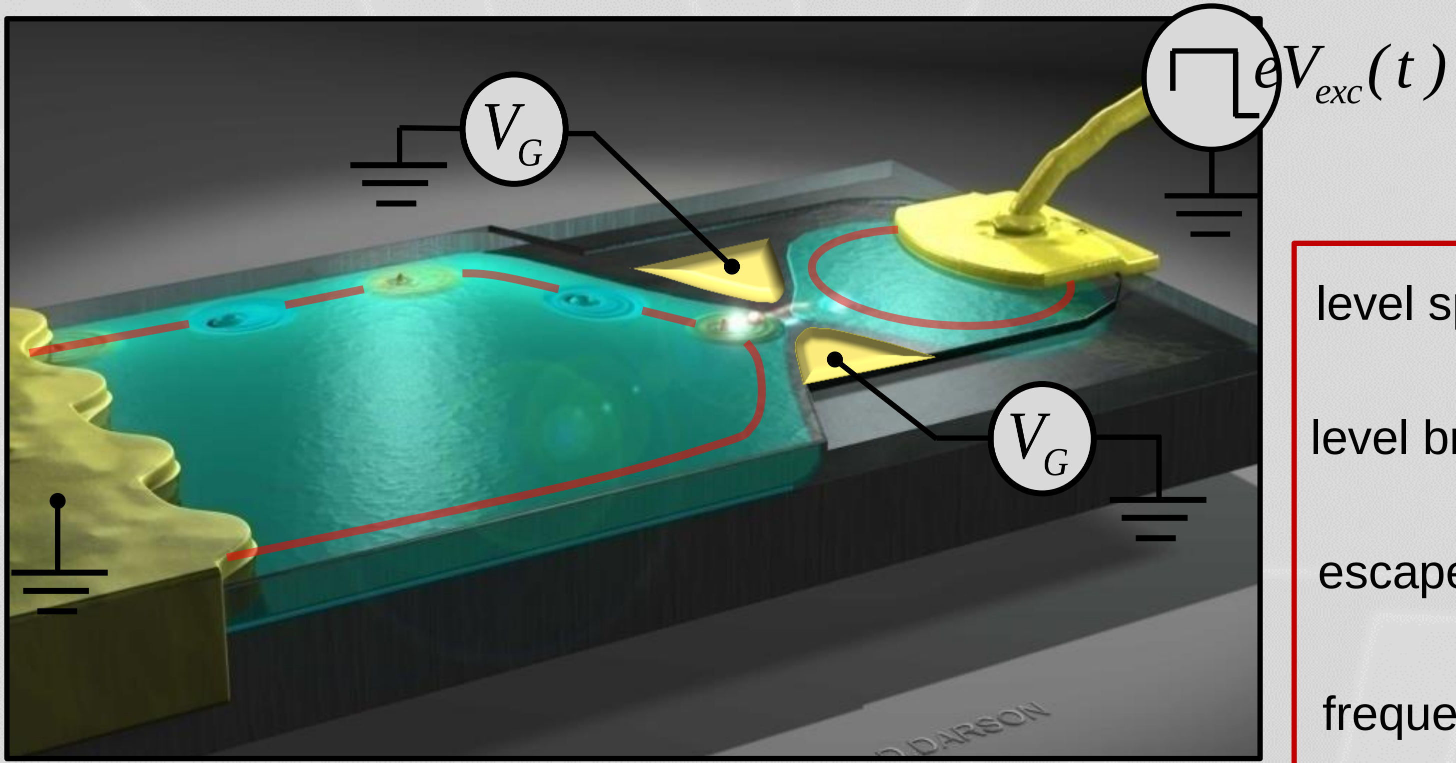
G. Fève et al., Science 316, 1169 (2007)
Could be turned to a dc source by separating the electron and hole stream :
F. Battista and P. Samuelsson, Phys. Rev. B 83, 125324 (2011)

The single electron source : a mesoscopic capacitor



M. Büttiker *et al.*, Phys. Lett. A **180**, 364 (1993)
J. Gabelli *et al.*, Science **313**, 499 (2006)

The single electron source : a mesoscopic capacitor

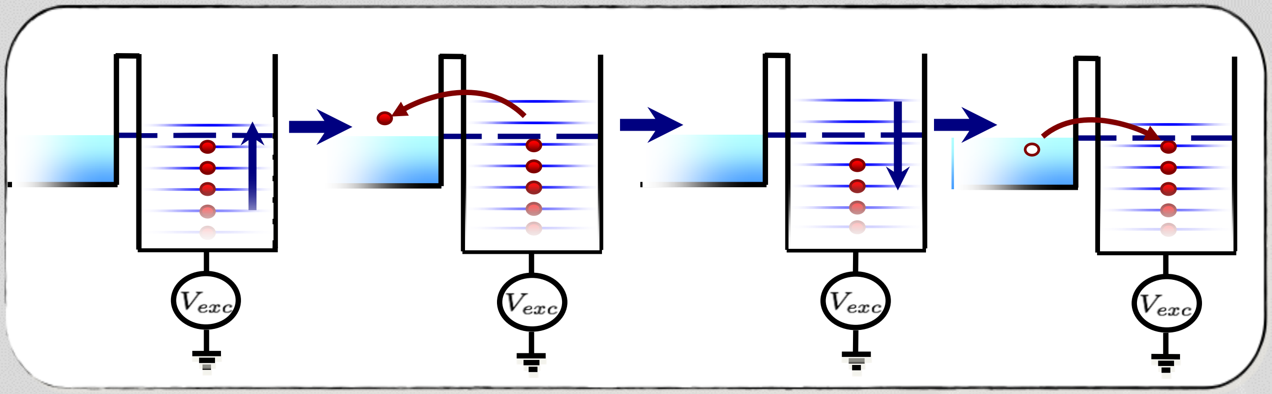


level spacing $\Delta \simeq 2.1K$

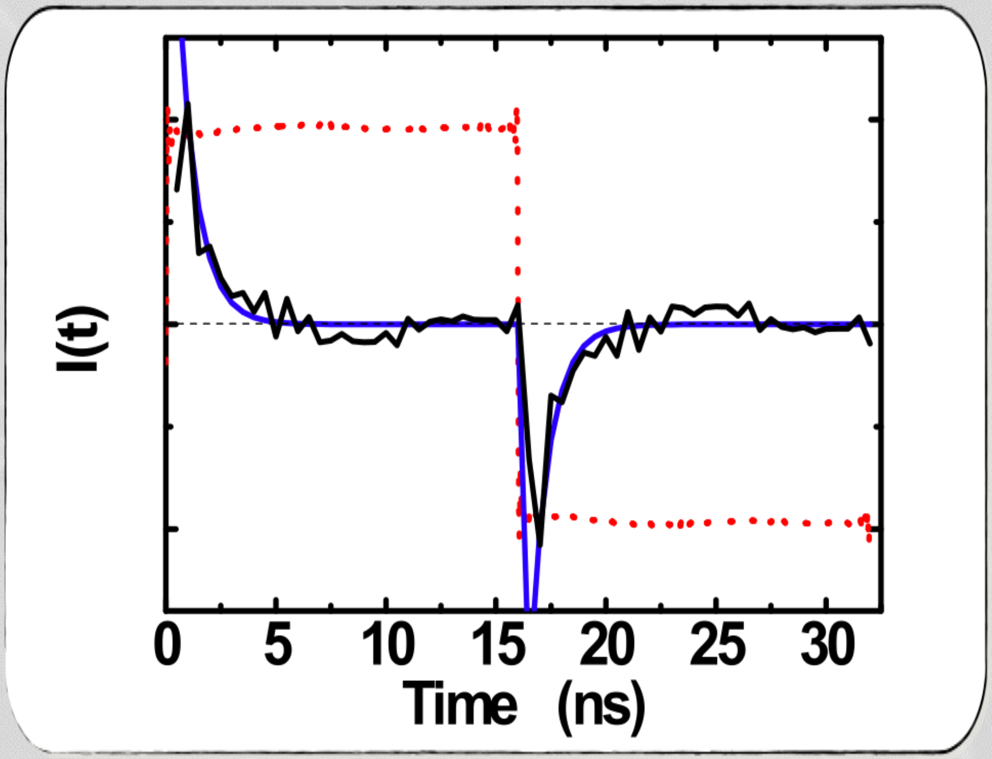
level broadening $D\Delta$

escape time $\tau \simeq \frac{h}{D\Delta}$

frequency $f = \frac{1}{T} \simeq \mathrm{GHz}$



The single electron source : a mesoscopic capacitor



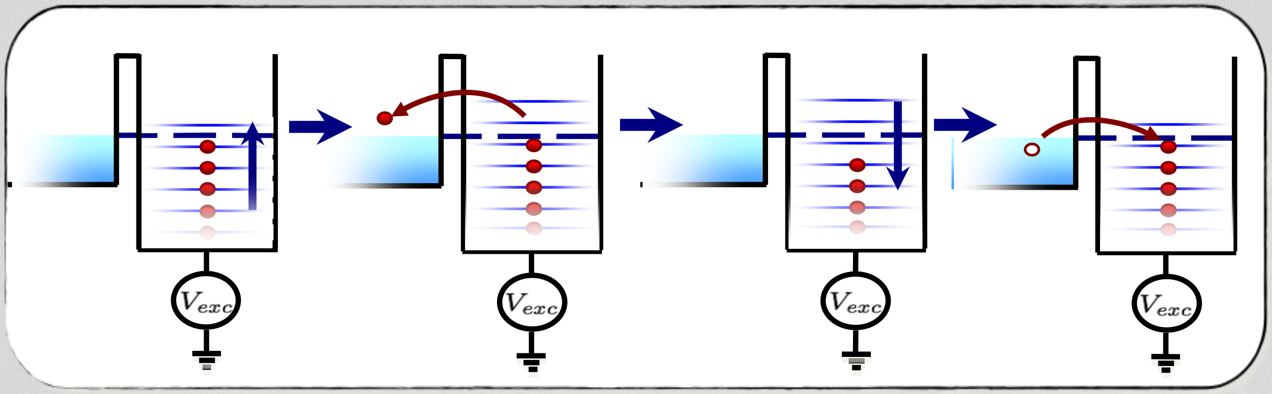
$$D \approx 0.02 \quad \tau = 0.9\ ns$$

$$\tau \ll T/2$$

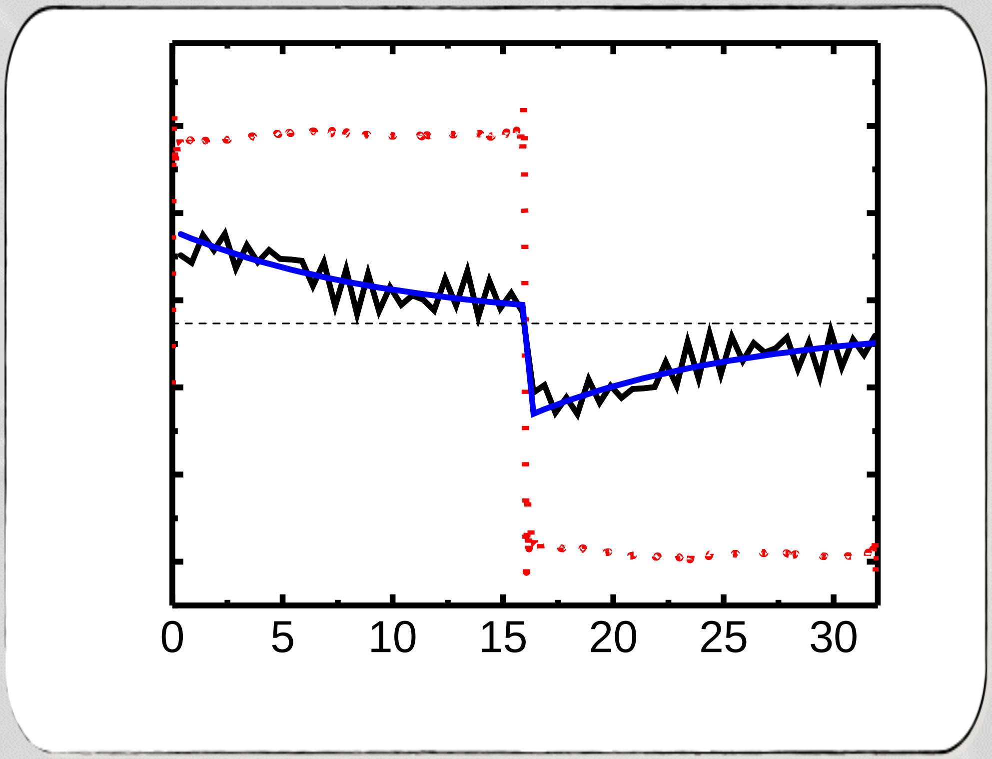
$$\Rightarrow Q^t = e$$

⇒ **single charge emission on average**

M. Moskalets *et al.*, PRL **100** (8) (2008
A. Mahé et al., JLTP **53**, 339 (2008)
G. Fève et al., Science **316**, 1169 (2007)



The single electron source : a mesoscopic capacitor



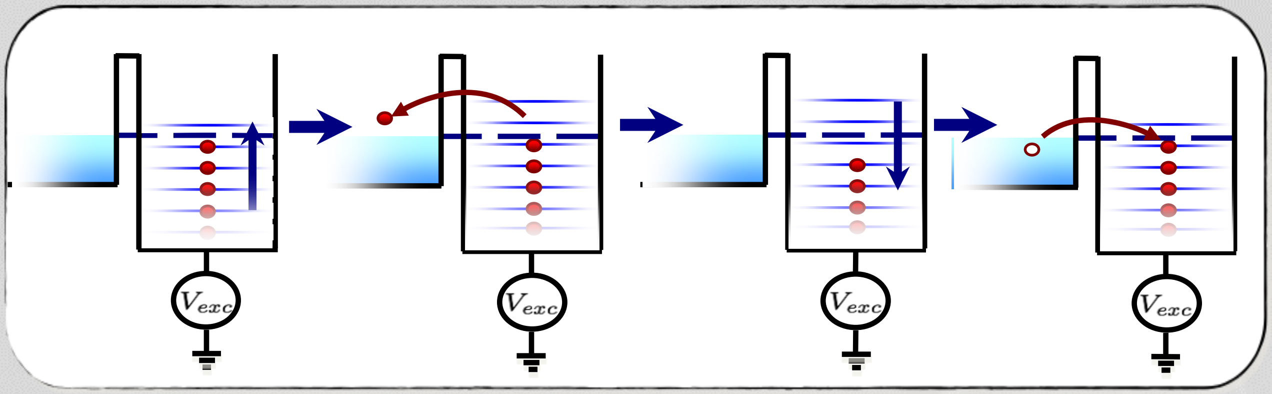
$$D \approx 0.002 \quad \tau = 10\ ns$$

$$\tau \geq T/2$$

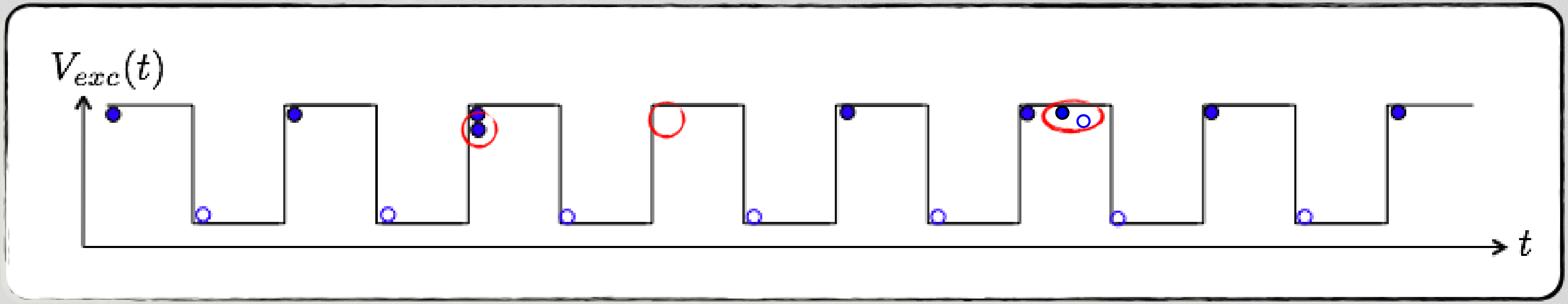
$$\Rightarrow Q^t < e$$

⇒probability of single charge emission

M. Moskalets *et al.*, PRL **100** (8) (2008)
A. Mahé et al., JLTP **53**, 339 (2008)
G. Fève et al., Science **316**, 1169 (2007)



Beyond average current : ac sources



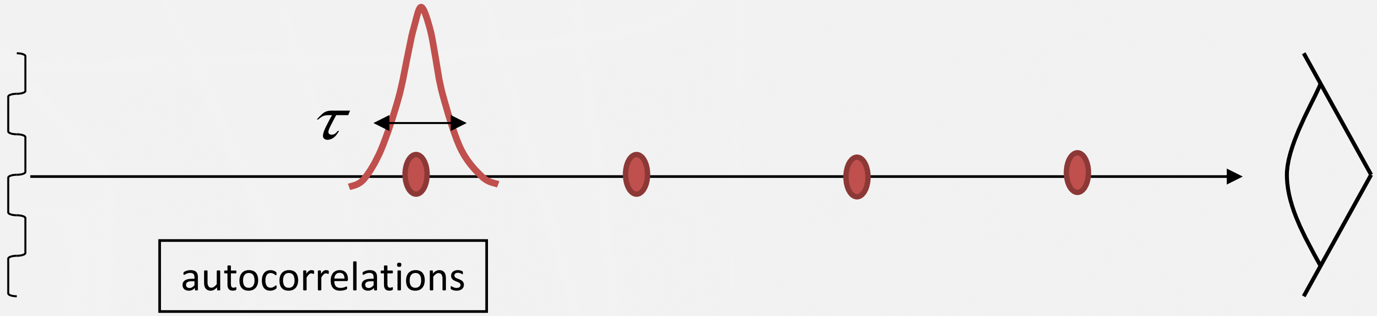
No DC current, **no fluctuations** at zero frequency

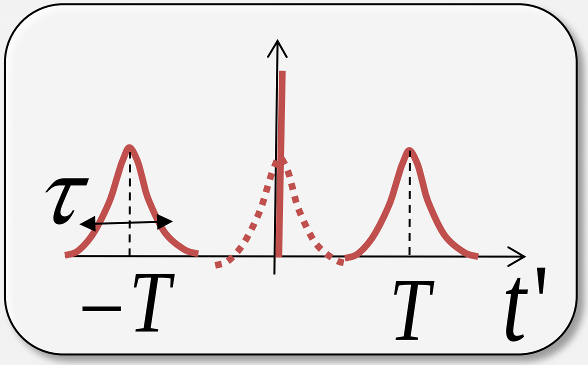
$$\boxed{\begin{array}{l} Q = e(N_e - N_h) = 0 \\ \langle \delta Q^2 \rangle = 0 \qquad S(\omega = 0) = 0 \end{array}}$$

AC current $< I(t) >$

Short time current-current correlations $\overline{< \delta I(t) \delta I(t+t') >}^{t}$, $S(\omega \neq 0)$

Current correlations (noise)





$$\overline{<I(t)I(t+t')>} = \frac{e^2}{T}\delta(t')$$

$$\overline{<I(t)><I(t+t')>} = \frac{e^2}{T}\frac{e^{-|t'|/\tau}}{\tau}$$

$$S_{II}(\omega) = 2\int \overline{<\delta I(t)\delta I(t+t')>}^{t} e^{i\omega t'}dt'$$

$$S_{II}(\omega) = 2\frac{e^2}{T}\frac{(\omega\tau)^2}{1+(\omega\tau)^2}$$

$$\omega \approx 2\pi\, 1.5\, GHz$$

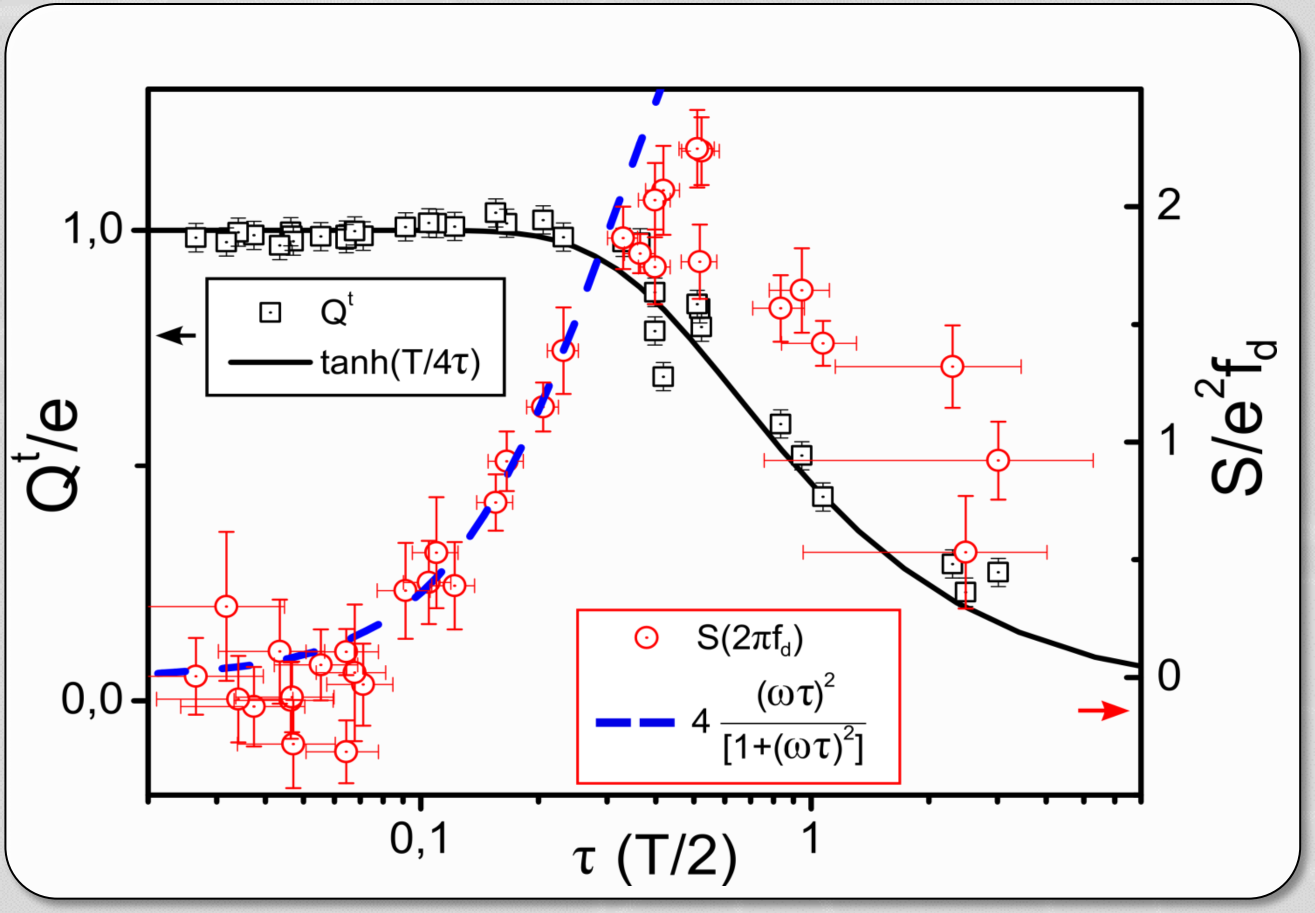
$$e^2 f \approx 4.10^{-29} A^2.Hz^{-1}$$

$$T_N \approx 100\,\mu K$$

$$\Delta f = 1.2 - 1.8\, GHz$$

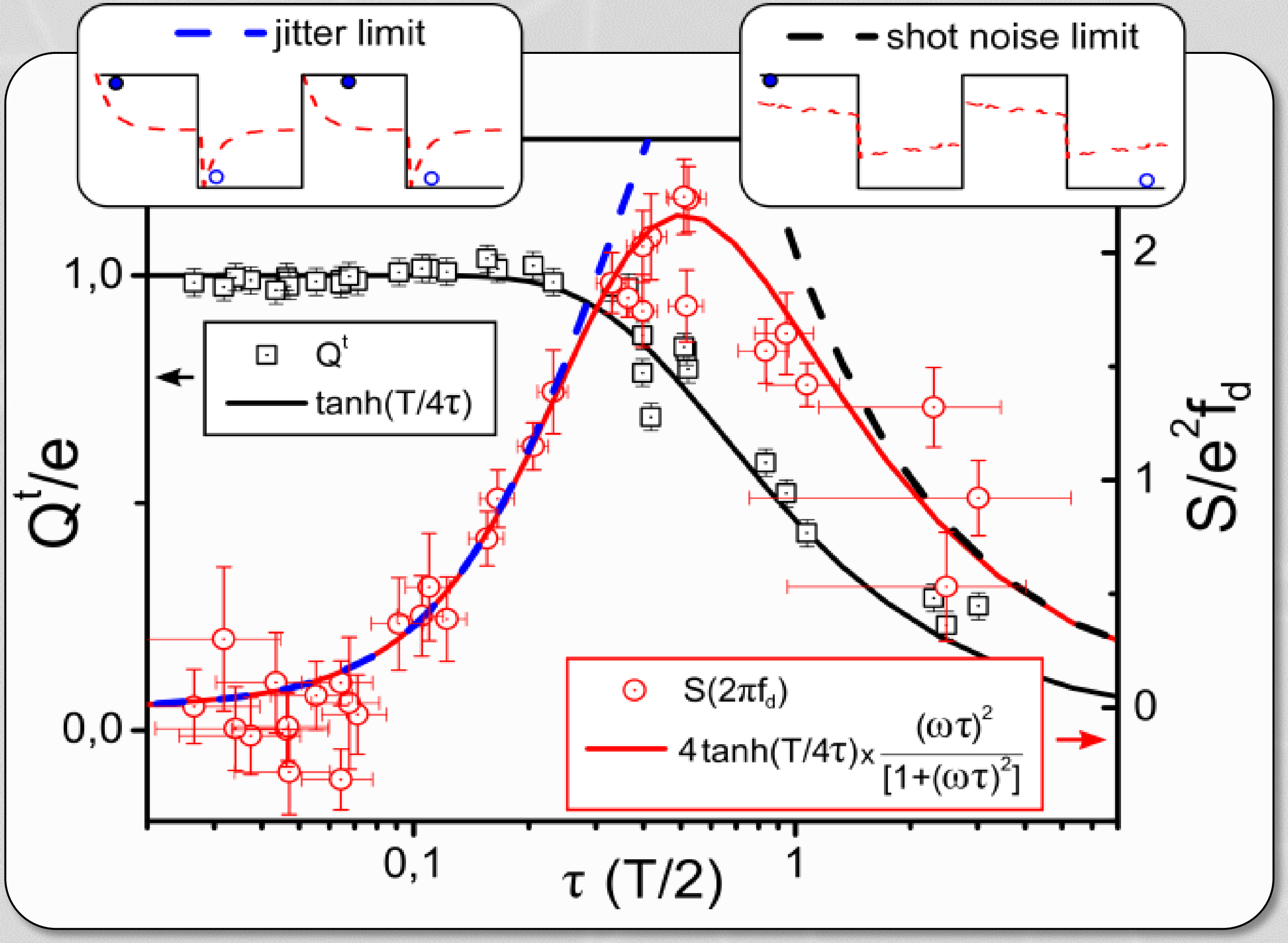
F.D. Parmentier et al., Rev. Sci. Instrum. **82,** 013904 (2010).

Noise measurements



Mahé et al. Phys. Rev. B **82**, 201309 (2010).

Noise measurements



Mahé et al. Phys. Rev. B **82**, 201309 (2010).

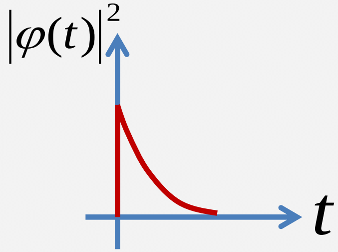
Albert et al., Phys. Rev. B **82,** 041407 (2010)

Talk of Mathias Albert yesterday

Information on electronic wavepackets

- average ac current + noise :

a single electron is emitted on demand



$$\langle I(t)\rangle = e\langle \psi^+(t)\psi(t)\rangle = e|\varphi(t)|^2$$

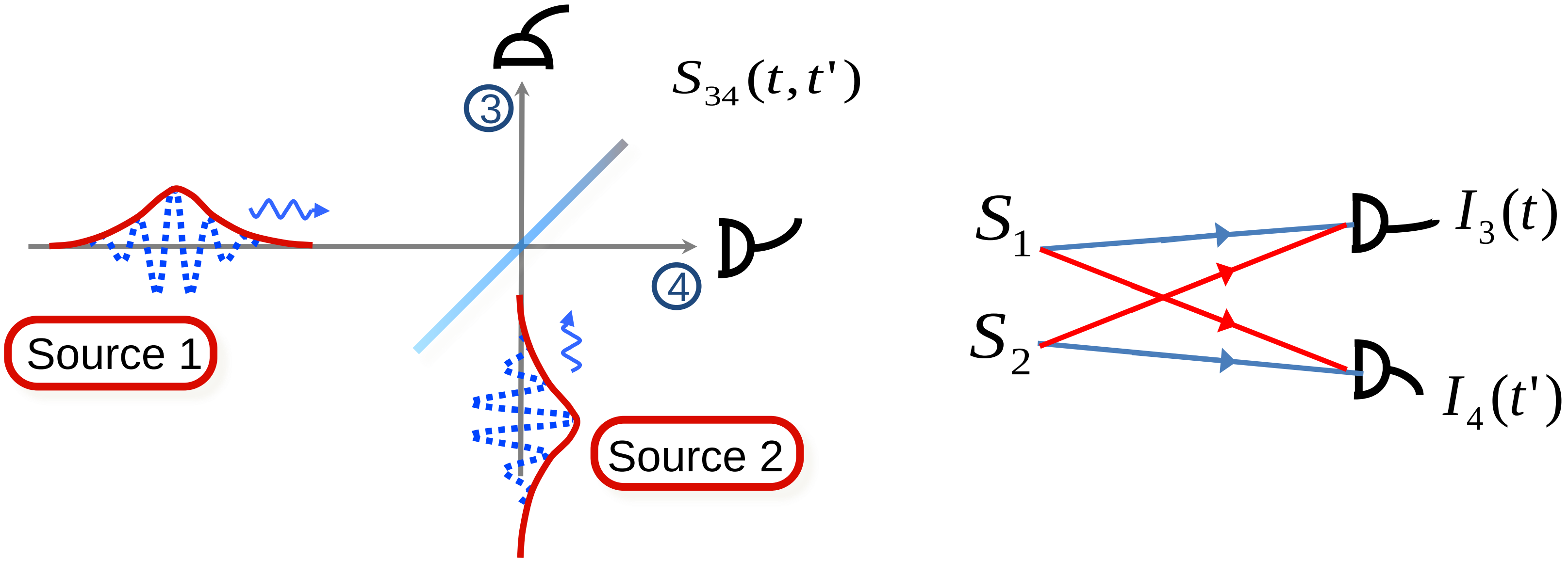
$$\langle I(t)I(t+\tau)\rangle = e^2|\varphi(t)|^2\delta(\tau)$$

- Probes the wavepacket envelopes, not the wave packet coherence

$$\rho(t,t') = \varphi(t)\varphi^*(t') \longrightarrow \varphi(t)\varphi^*(t')D(t-t')$$

$$D(0)=1 \quad D(\infty)=0$$

The electronic Hanbury Brown and Twiss experiment



$$S_{34}(t,t') = RT\left[S_{11} + S_{22} - Q\right]$$

$$I_1(t) \propto \psi_1^+(t)\psi_1(t)$$

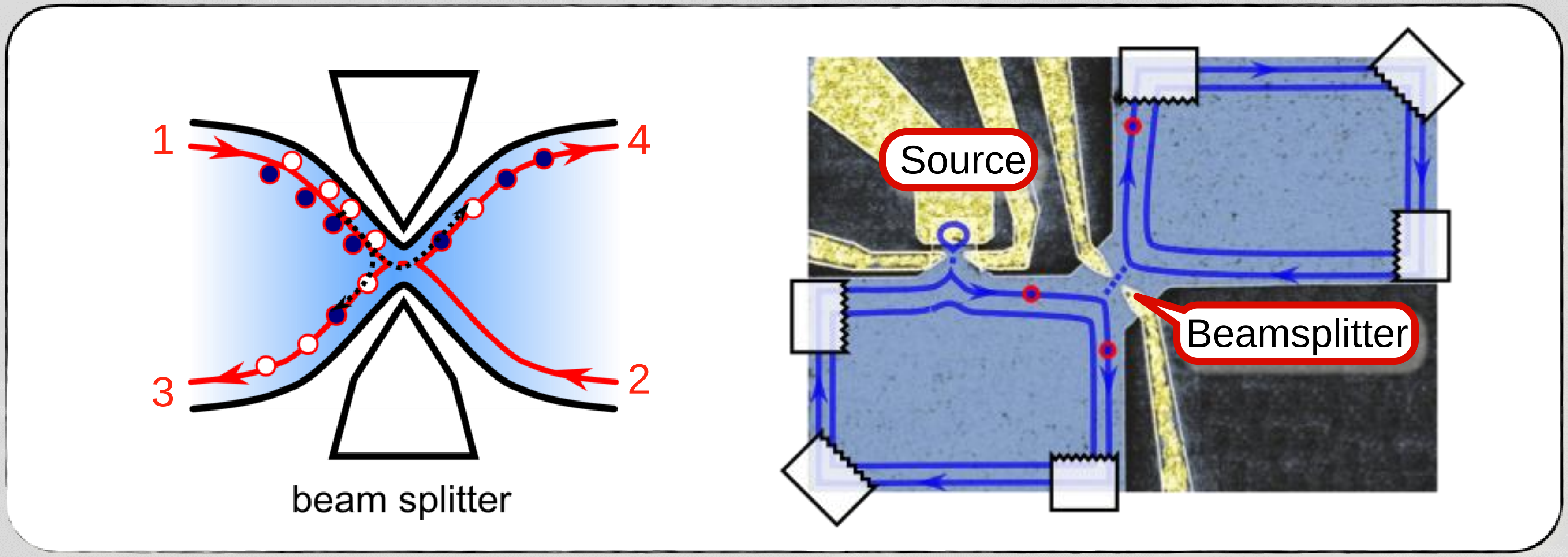
Diagonal term

$$Q(t,t') = \langle \psi_1^+(t')\psi_1(t)\rangle\langle \psi_2(t)\psi_2^+(t')\rangle + \boxed{\langle \psi_2^+(t')\psi_2(t)\rangle\langle \psi_1(t)\psi_1^+(t')\rangle}$$

$$S_{34}(\omega = 0) \propto \frac{1}{T_{meas}} \int dt\,dt'\,Q(t,t')$$

Overlap between
off diagonal terms

Single source partitioning



$$Q_1 = N_e - N_h \qquad \left\langle \delta Q_1^{\,2} \right\rangle = 0 \qquad \underline{S_{11}(\omega = 0) = 0}$$

- electron and holes independently transmitted (or reflected) with probability T (or 1-T)

$$\underline{\left\langle \delta Q_3 \delta Q_4 \right\rangle = -e^2 T(1-T)\left(\left\langle N_e \right\rangle + \left\langle N_h \right\rangle\right)}$$

$$\underline{S_{I_3 I_4}(\omega = 0) = -4e^2 f\, T(1-T)\left\langle N_{e/h} \right\rangle}$$

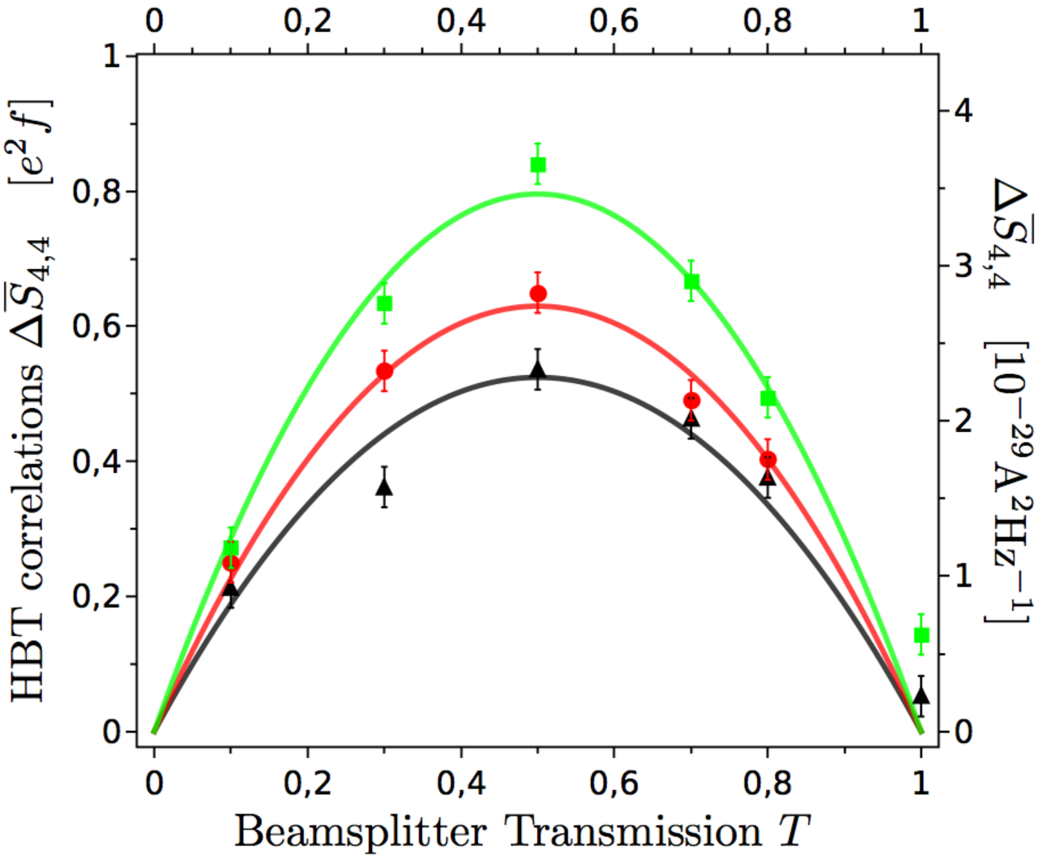
M.H. Pedersen *et al.*, PRB **58**, 12993 (1998)

G.B. Lesovik, JETP Lett. **70**, 208 (1999)

L.-H Reydellet *et al.*, PRL **90**, 176803 (2003)

Low frequency HBT correlations

$$\Delta\overline{S}_{4,4} = -\overline{S}_{3,4} = 4e^2 f \times T(1-T)\delta N_{HBT}$$



E. Bocquillon et al., PRL **108**, 196803 (2012)

— Square, $D = 0.4$

$Q^t = 1$

$\delta N_{HBT} = 0.80$

— Sine, $D = 0.3$

$Q^t = 0.93$

$\delta N_{HBT} = 0.63$

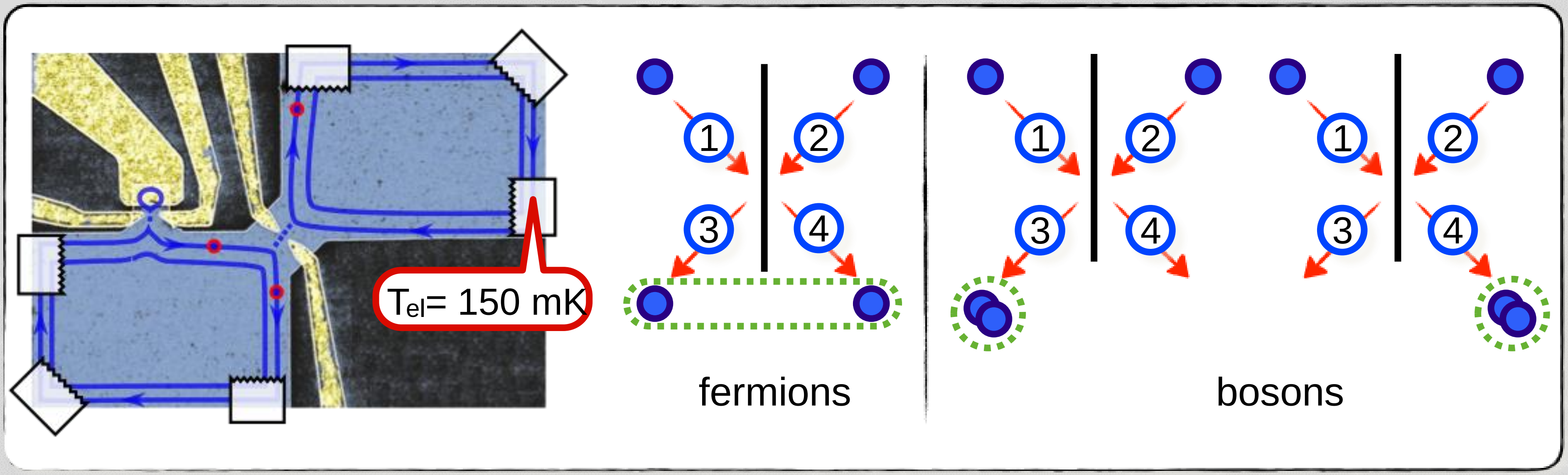
— Sine, $D = 1$

$Q^t = 1.27$

$\delta N_{HBT} = 0.51$

HBT correlations: the quantum version

- Input 2 : **Fermi sea at T_{el}= 150 mK** (calibrated) ≠ vacuum !



- HBT signal :

$$\delta N_{HBT} = \frac{\langle N_e\rangle + \langle N_h\rangle}{2} - \int_0^\infty \boxed{d\epsilon(\delta n_e(\epsilon) + \delta n_h(\epsilon))f(\epsilon)}$$

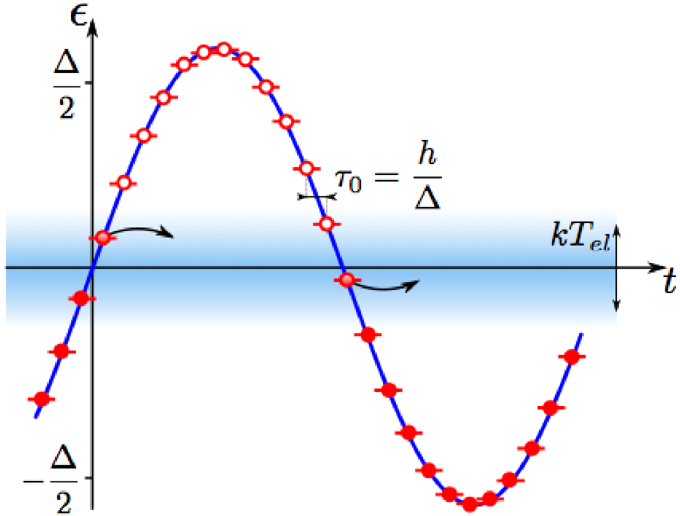
$$\langle N_{e/h}\rangle = \int_0^\infty d\epsilon\ \delta n_{e/h}(\epsilon)$$

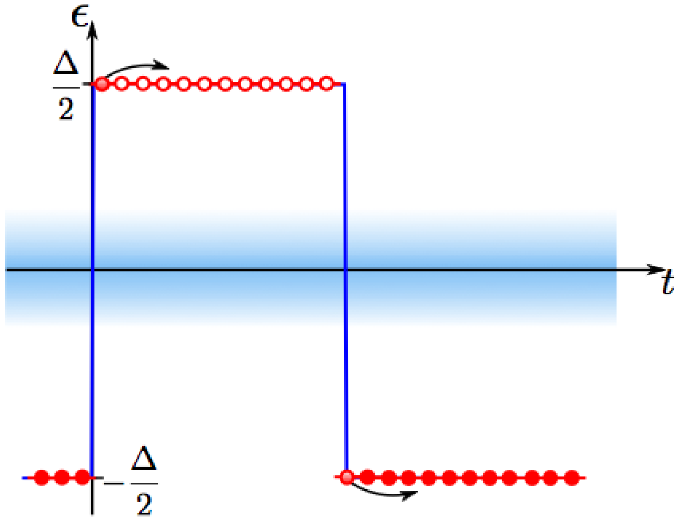
Classical contribution

Minus sign: fermions

antibunching with thermal excitations

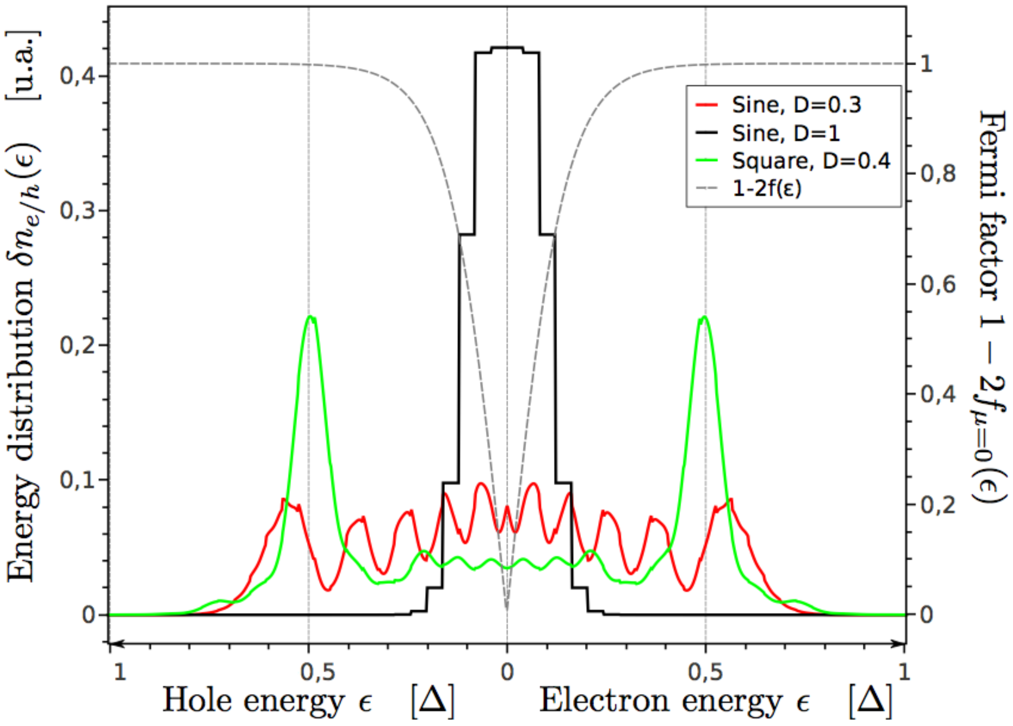
HBT correlations as a probe of energy distribution





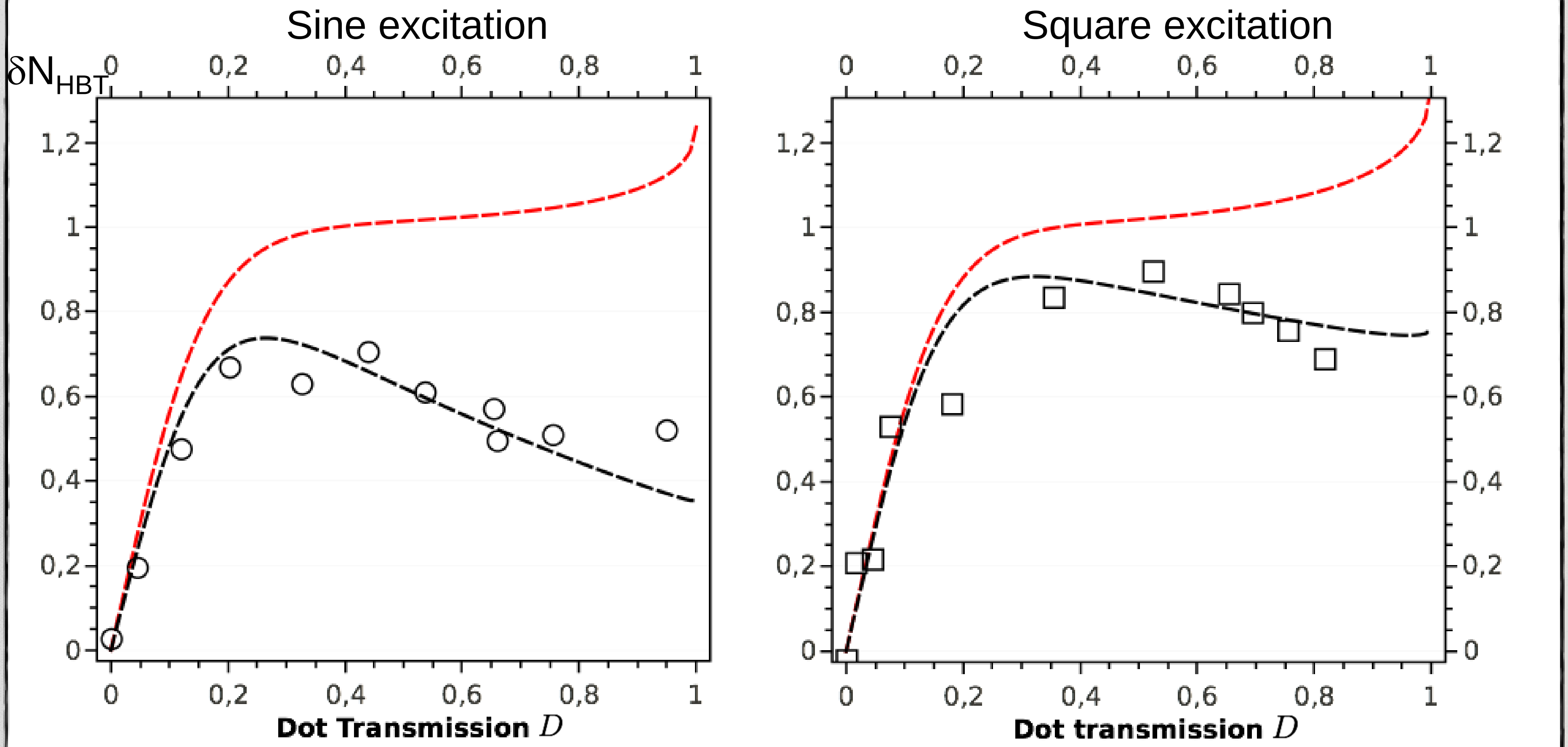
Engineering of the wavepacket through :

- excitation : sine, square
- dot transmission



M. Moskalets *et al.*, PRB **66**, 205320 (2002)
F. D. Parmentier *et al.*, PRB **85**, 165438 (2012)

Comparison with experiments



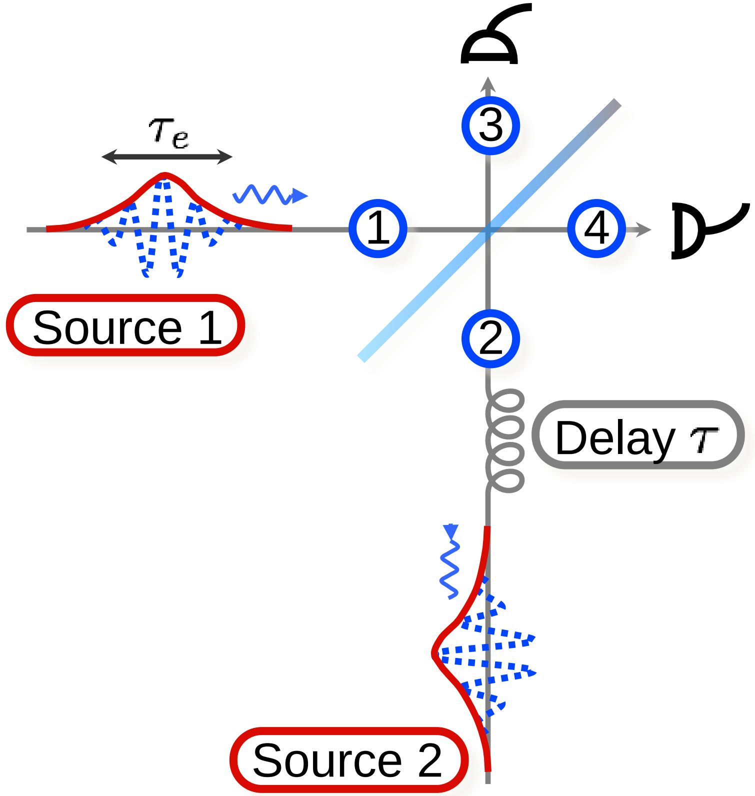
○□ Experimental data

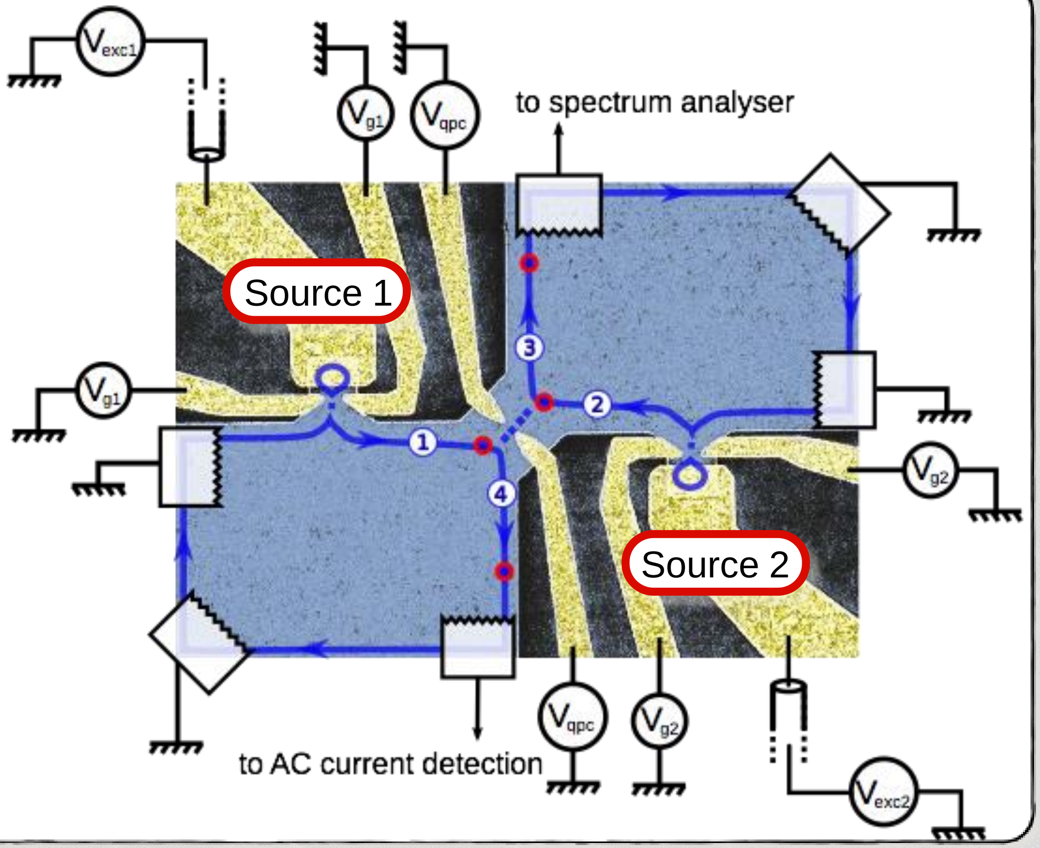
······ Simulation of $\delta N_{HBT}, T_{el} = 0\,mK$

······ Simulation of $\delta N_{HBT}, T_{el} = 150\,mK$

E. Bocquillon et al., PRL **108**, 196803 (2012)

Two electron interferences with two sources



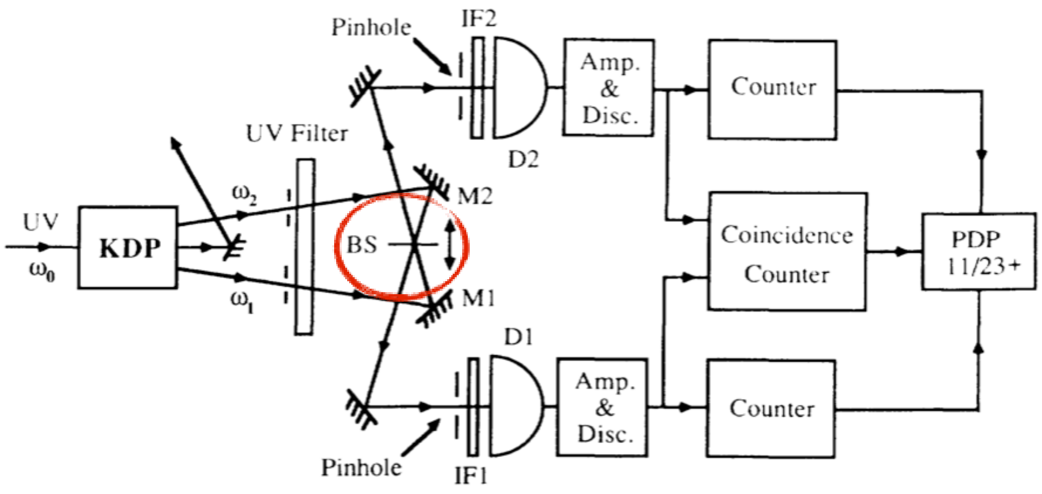


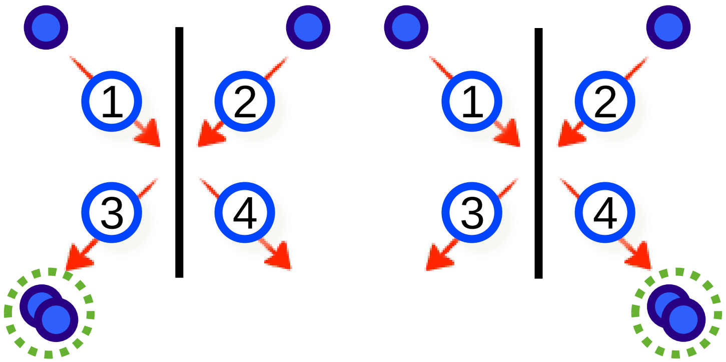
Two sources:

- independently tuned parameters
- synchronized excitations, with tunable delay τ (within a ±7 ps error)

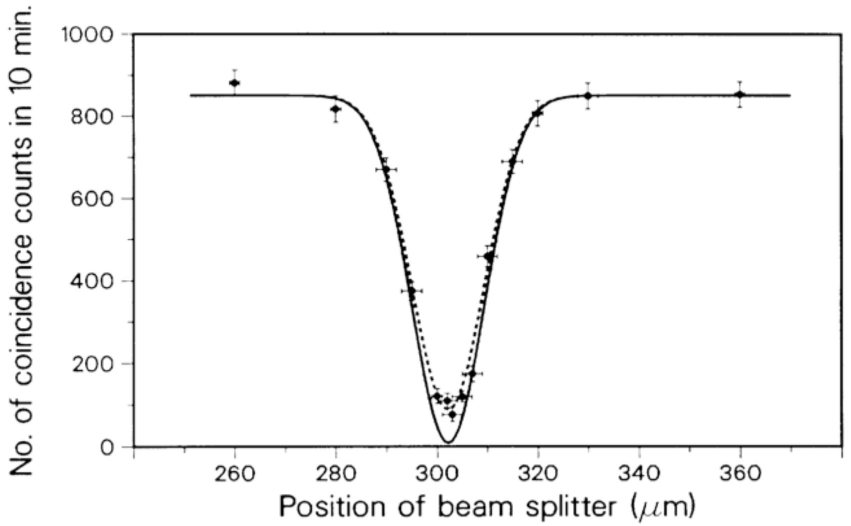
S. Ol’khovskaya *et al.,* PRL **101**, 166802 (2008)
G. Fève *et al.*, PRB **77**, 035308 (2008)
T. Jonckheere *et al.*, PRB **86**, 125425 (2012)

Seminal Hong-Ou-Mandel experiment





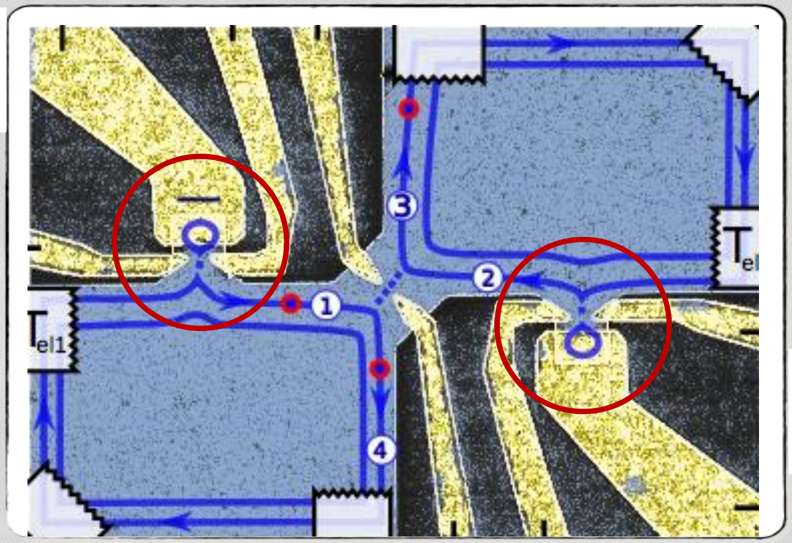
Undistinguishable photons



Photons pairs :
C. Hong *et al.,* PRL **59**(18), 2044 (1987)
Independent emitters :
J. Beugnon *et al.*, Nature **440**, 779 (2006)
P. Maunz *et al.*, Nature Physics **3**, 538 (2007)
E. B. Flagg *et al.*, PRL **104,** 137401 (2010)
C. Lang *et al.*, Nature Physics **9**, 345 (2013)

Two electron interferences with two sources

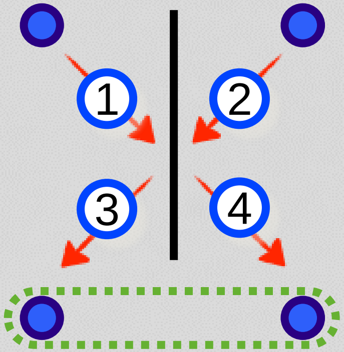
Single electron emitter



Single electron emitter

Two particle interferences

$\phi_1^e(x)$ $\phi_2^e(x)$



$$P(1,1) = \frac{1}{2}\left[1 + |\langle \phi_1^e | \phi_2^e \rangle|^2\right]$$

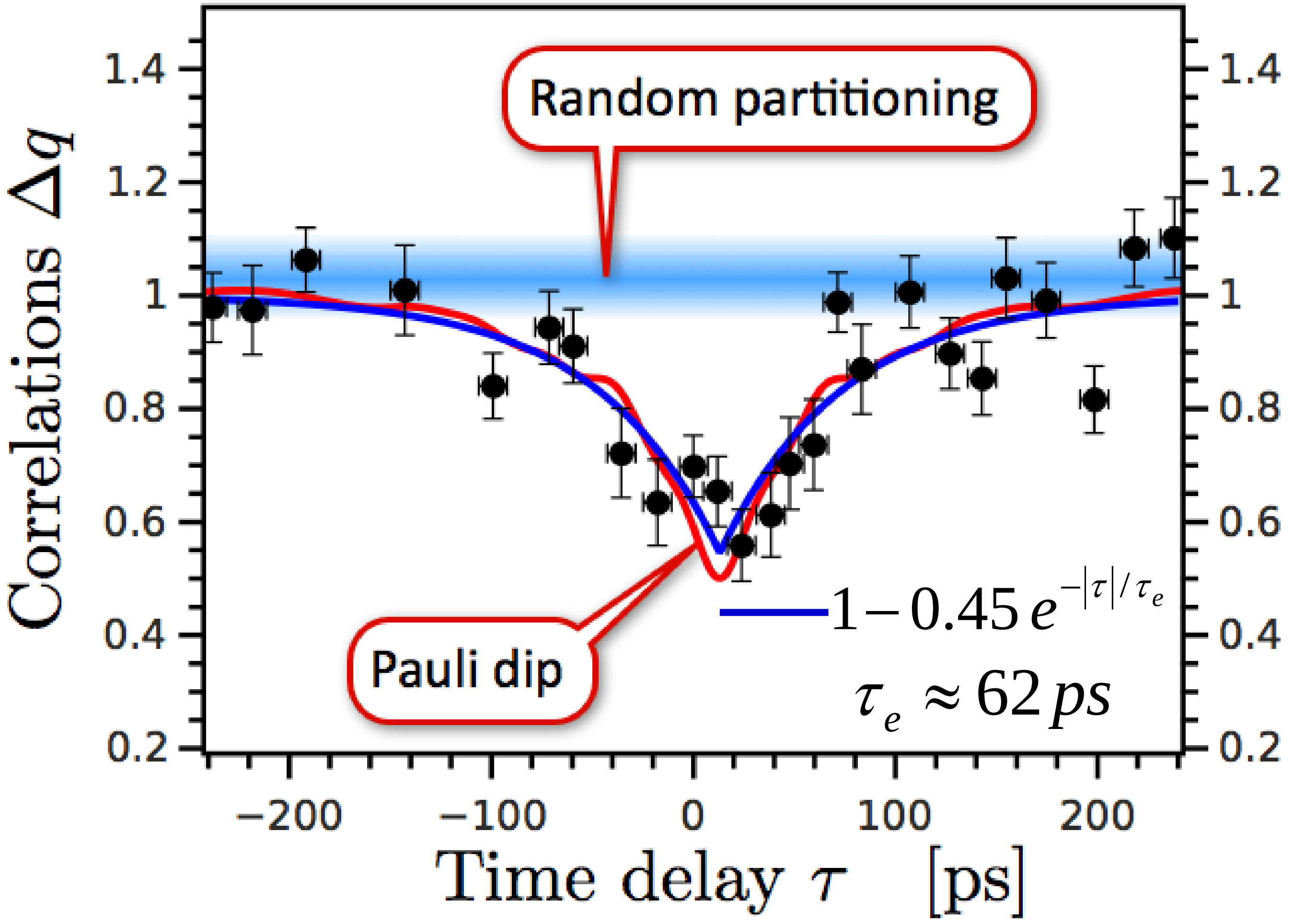
$$\frac{S_{HOM}}{S_{HBT}} = 1 - \left| \int dt \, \varphi_1(t+\tau)\varphi_2^*(t) \right|^2$$

$$= 1 - e^{-|\tau|/\tau_e}$$

Electronic Hong-Ou-Mandel dip

f=2.1 GHz

$$D_1 = D_2 \approx 0.4 \qquad \tau_e \approx 58\,ps$$



E. Bocquillon et al., Science DOI:10.1126/science.1232572 (2013).

Possible suppression of HOM dip

- Differences in emission energies of the dot (controlled by static potential)

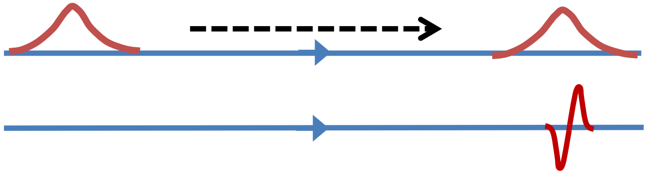
$$\varphi_1(t) = e^{i\varepsilon t/\hbar} e^{-\Gamma t/2}$$

$$\varphi_2(t) = e^{i\varepsilon t/\hbar} e^{i\delta\varepsilon t/\hbar} e^{-\Gamma t/2}$$

$$\boxed{C(\delta\varepsilon) = \frac{1}{1+(\delta\varepsilon/\hbar\Gamma)^2}}$$

$1/\Gamma = 60\, ps$ $C = 0.5$ for $\delta\varepsilon = \Delta/10$

- Decoherence along propagation



$\nu = 2$

C. Wahl et al., arXiv:1307.5257 (2013)

$$\frac{S_{HOM}}{S_{HBT}} = 1 - \int dt\, dt' \varphi_1(t+\tau_D)\varphi_1^*(t'+\tau_D)D_1(t-t')\varphi_2^*(t)\varphi_2(t')D_2(t-t')$$

$$D_1 = D_2 = e^{-|t-t'|/\tau_c}$$

$$\boxed{C(\tau_c) = \frac{\tau_c\Gamma/2}{1+\tau_c\Gamma/2}}$$

$\tau_c \approx 100\, ps$

Conclusion

- Strong electron/photon analogies in quantum conductors

- short time current correlations prove single charge emission

- Hanbury-Brown and Twiss interferometry provide a counting of excitations

- HOM: partial coherence and indistinguishability

- Fundamental differences remain
  - statistics, presence of the Fermi sea, Coulomb interaction

- Perspectives

  -Single electron wavefunction reconstruction : HOM/MZ interferometry, energy/time domain

  C. Grenier *et al.*, NJP **13**(9), 093007 (2011)
  M. Moskalets *et al.*, PRB **83**, 035316 (2011)
  G. Haack *et al.*, PRB **84**, 081303(R) (2011)

  -Decoherence by Coulomb interaction

People involved

Mesoscopic Physics group, LPA ENS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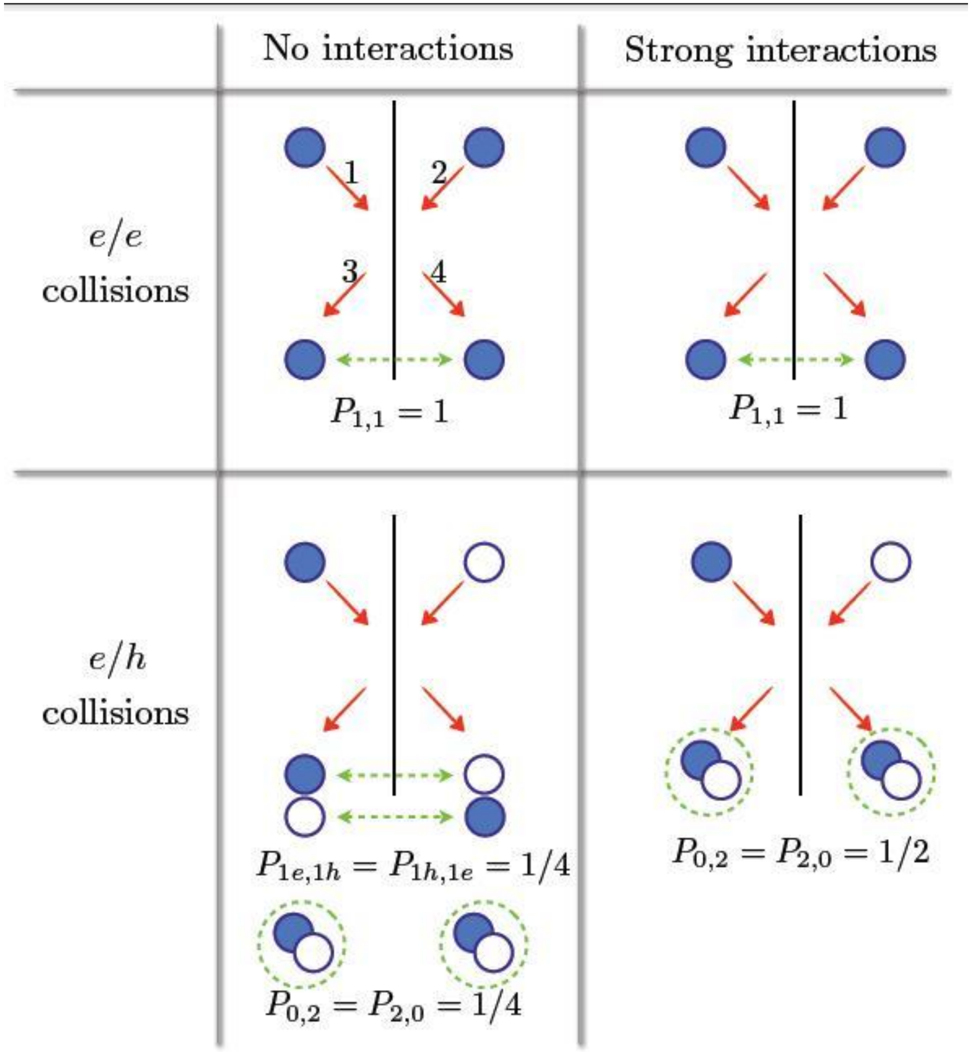
Y. Jin	C. Grenier	T. Martin	M. Büttiker
	D. Ferraro	J. Rech	C. Flindt
	E. Thibierge	C. Wahl	G. Haack

Coulomb interactions

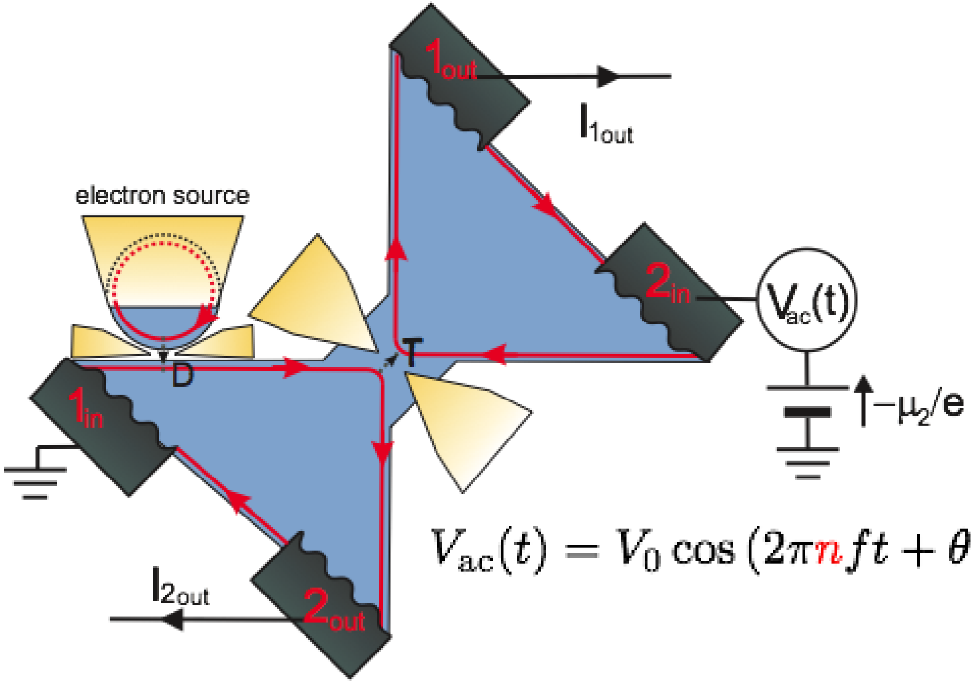


$$S_{HOM} = S_{\text{int}} = 0$$

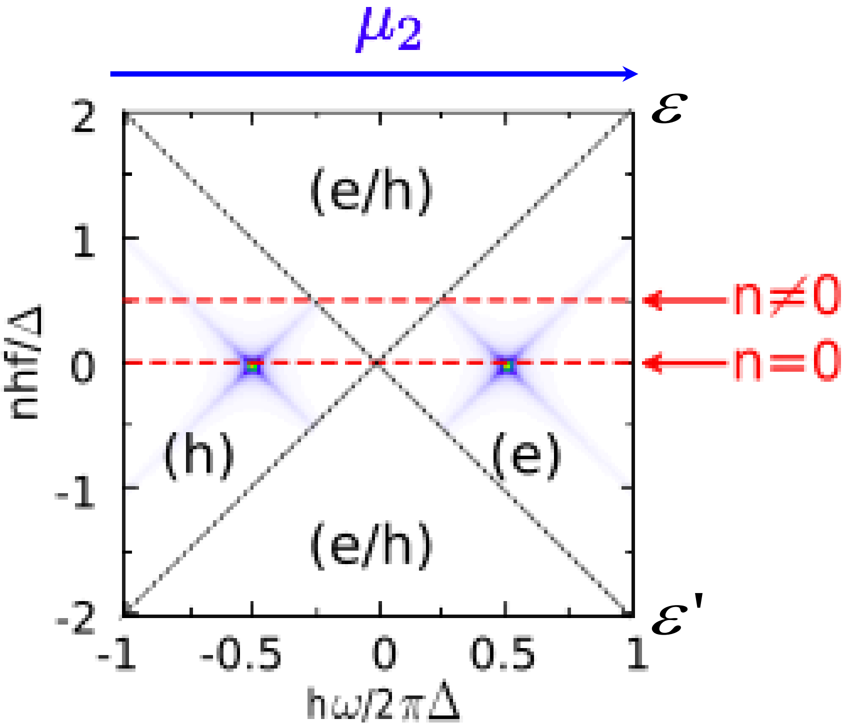
$$S_{HOM} = 2e^2 f \qquad S_{\text{int}} = 0$$

No effects of Coulomb interactions between electrons and holes

Tomography of a single electron emitter



$$V_{\mathrm{ac}}(t) = V_0 \cos\left(2\pi n f t + \theta\right)$$



Current noise in terms of the DC bias

$$\Delta\mathcal{G}^{(e)}_{n=0}(\omega)$$

Current noise response to the AC drive

$$\Delta\mathcal{G}^{(e)}_{n\neq 0}(\omega)$$

Future prospects

- Imaging a single electron wave function, tomography of single electrons states

Time domain, Mach-Zehnder interferometer

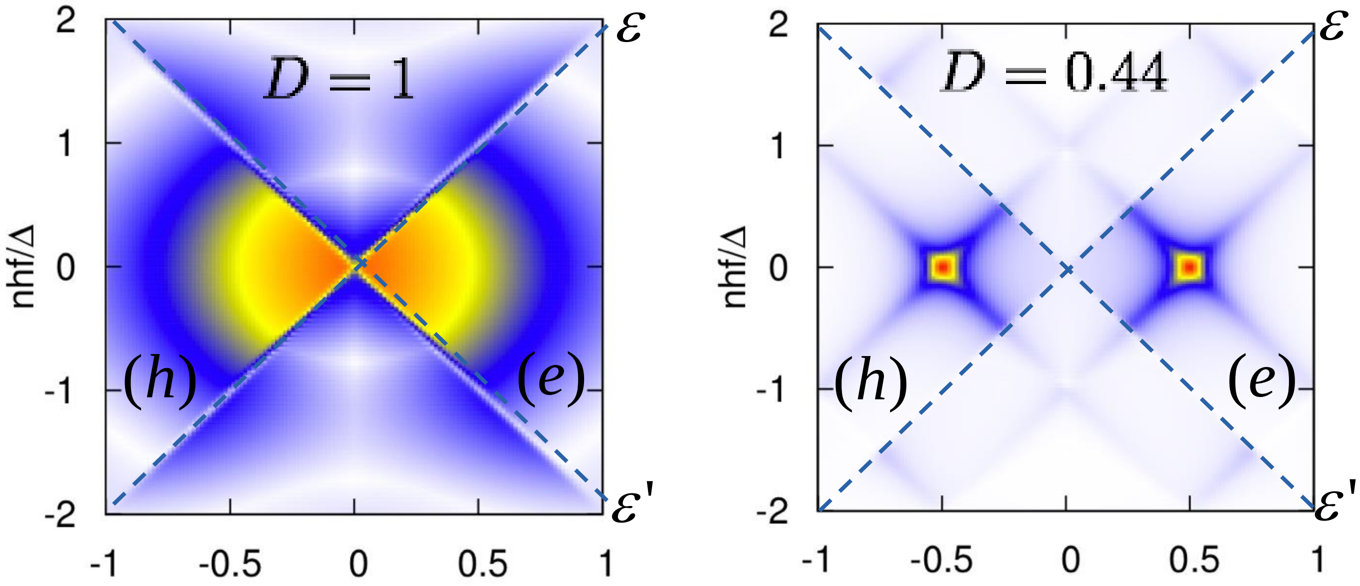
G. Haack et al., PRB. **84**, 081303 (R) (2011)

$$\Delta G^{(1)}(t,t') = \left\langle \psi^{+}(t')\psi(t) \right\rangle = \varphi(t)\varphi^{*}(t')$$

Energy domain, HBT interferometer

C. Grenier et al., New J. Phys. **13**, 093007 (2011)

$$\Delta G^{(1)}(\varepsilon,\varepsilon') = \tilde{\varphi}(\varepsilon)\tilde{\varphi}^{*}(\varepsilon')$$



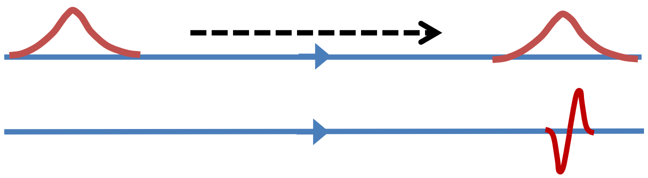
Future prospects

- Imaging a single electron wave function, tomography of single electrons states

Time domain, Mach-Zehnder interferometer $$\Delta G^{(1)}(t,t') = \varphi(t)\varphi^*(t')$$

Energy domain, HBT interferometer $$\Delta G^{(1)}(\varepsilon,\varepsilon') = \tilde{\varphi}(\varepsilon)\tilde{\varphi}^*(\varepsilon')$$

- Life and death of a quasiparticle, Coulomb interaction (between channels)



$$\nu = 2$$