# *Unconventional superconductivity in double quantum dots*

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# Outline

**Introduction**

- Symmetry of pair amplitude
- Classification of superconductivity
- Unconventional SC in quantum dots

**Model**

**Results**

- Pair amplitudes
- Order parameters
- Transport signatures

**Summary**

# Introduction

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# Symmetry of pair amplitude

- Consider pair amplitude:

$$F_{\alpha\beta}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = -i\langle T\psi_\alpha(\mathbf{r}_1, t_1)\psi_\beta(\mathbf{r}_2, t_2)\rangle$$

- Pauli principle:

$$F_{\alpha\beta}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = -F_{\beta\alpha}(\mathbf{r}_2, t_2; \mathbf{r}_1, t_1)$$

- Different ways to realize antisymmetry

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# Classification of superconductivity

| Frequency | Spin | Orbital | |
|---|---|---|---|
| + (even) | - (singlet) | + (even) | even-singlet |
| + (even) | + (triplet) | - (odd) | even-triplet |
| - (odd) | + (triplet) | + (even) | odd-triplet |
| - (odd) | - (singlet) | - (odd) | odd-singlet |

- Even-singlet: $s$-wave BCS superconductivity
- Even-triplet: Superfluid $^3$He, $Sr_2RuO_4$, Majorana nanowires
- Odd-triplet: SFS heterostructures
- Odd-singlet: ???

# Unconventional SC in quantum dots

- Can unconventional SC be induced in quantum dots?
- Interplay between superconductivity, strong Coulomb interaction and nonequilibrium
- Quantum dots offer easy tunability of their properties
- Quantum dot as tunable source of even-singlet, even-triplet, odd-triplet and odd-singlet correlations?
- Signatures in transport properties?

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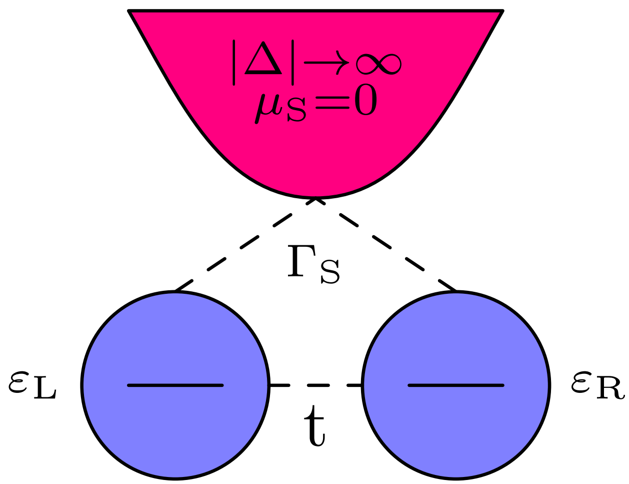
# Model

# Model: Double dot




Effective Dot Hamiltonian for $\Delta \to \infty$

Quantum dots $i =$ L,R

$$H_i = \varepsilon_i \sum_\sigma c^\dagger_{i\sigma} c_{i\sigma} + \mathbf{B}_i \cdot \hat{\mathbf{S}}_i$$

Intra- and interdot Coulomb interaction

$$H_{\text{inter}} = \sum_i U_i n_{i\uparrow} n_{i\downarrow} + U \sum_{\sigma\sigma'} n_{\text{L}\sigma} n_{\text{R}\sigma'}$$

Local and nonlocal proximity effect

$$H_{\text{prox}} = -\sum_i \frac{\Gamma_{\text{S}i}}{2} \left( c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} + \text{H.c.} \right) - \frac{\Gamma_{\text{S}}}{2} \left( c^\dagger_{\text{R}\uparrow} c^\dagger_{\text{L}\downarrow} - c^\dagger_{\text{R}\downarrow} c^\dagger_{\text{L}\uparrow} + \text{H.c.} \right)$$

Interdot tunneling

$$H_{\text{tun}} = t \sum_\sigma (c^\dagger_{\text{L}\sigma} c_{\text{R}\sigma} + \text{H.c.})$$

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# Dynamics in Hilbert space

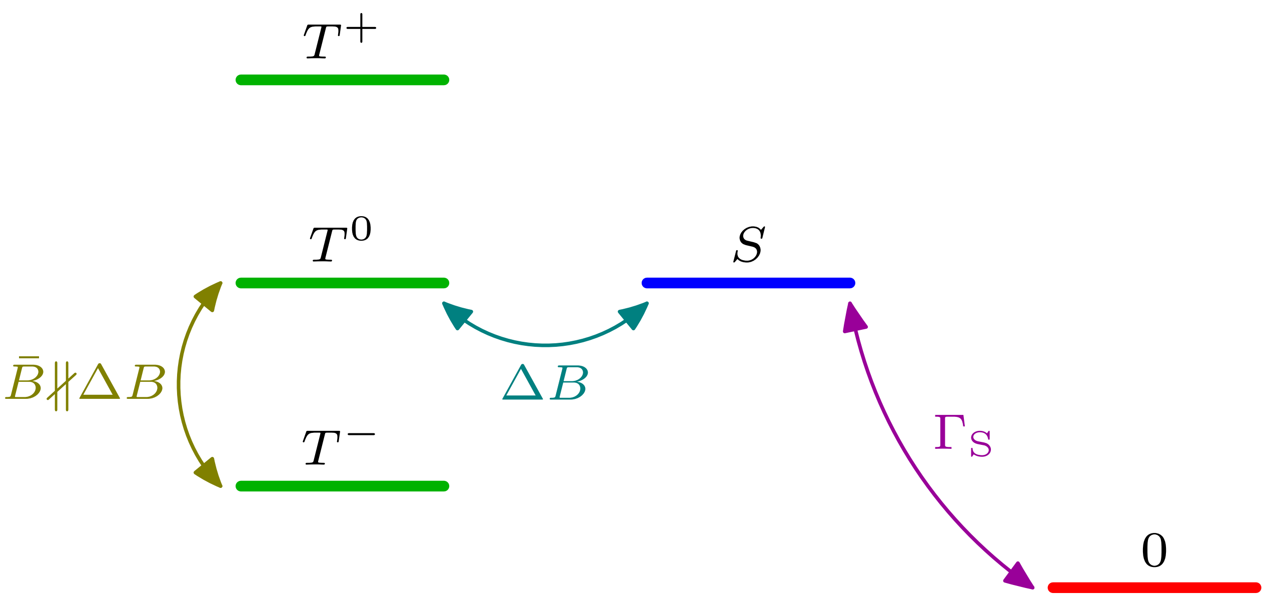
Relevant dot states with even occupation for $U_i \to \infty$

Empty dot $|0\rangle$, nonlocal singlet $|S\rangle$, 3 triplets $|T^\sigma\rangle$

Time evolution of density matrix elements $P^{\chi_1}_{\chi_2} = \langle\chi_1|\rho_{\text{dot}}|\chi_2\rangle$

$$\frac{d}{dt}P^{\chi_1}_{\chi_2}(t) + i\sum_{\chi}\left(h_{\chi_1\chi}P^{\chi}_{\chi_2} - h_{\chi\chi_2}P^{\chi_1}_{\chi}\right)(t) = 0$$

Matrix elements $h_{\chi_1\chi_2} = \langle\chi_1|H_{\text{ddot}}|\chi_2\rangle$

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# Results

# Pair amplitudes

Pair amplitude $F_{i\sigma i'\sigma'}(t) = \langle \mathrm{T}\, c_{i\sigma}(t) c_{i'\sigma'}(0) \rangle$

- Even/Odd-singlet pair amplitude:

$$F^{S}_{\text{e/o}} = (F_{\mathsf{L}\downarrow\mathsf{R}\uparrow} - F_{\mathsf{L}\uparrow\mathsf{R}\downarrow} \mp F_{\mathsf{R}\uparrow\mathsf{L}\downarrow} \pm F_{\mathsf{R}\downarrow\mathsf{L}\uparrow})/(2\sqrt{2})$$

- Even/Odd-triplet pair amplitudes:

$$F^{T^{+}}_{\text{e/o}} = (F_{\mathsf{L}\uparrow\mathsf{R}\uparrow} \mp F_{\mathsf{R}\uparrow\mathsf{L}\uparrow})/2$$

$$F^{T^{0}}_{\text{e/o}} = (F_{\mathsf{L}\downarrow\mathsf{R}\uparrow} + F_{\mathsf{L}\uparrow\mathsf{R}\downarrow} \mp F_{\mathsf{R}\uparrow\mathsf{L}\downarrow} \mp F_{\mathsf{R}\downarrow\mathsf{L}\uparrow})/(2\sqrt{2})$$

$$F^{T^{-}}_{\text{e/o}} = (F_{\mathsf{L}\downarrow\mathsf{R}\downarrow} \mp F_{\mathsf{R}\downarrow\mathsf{L}\downarrow})/2$$

# Even-frequency order parameters

Order parameter: Pair amplitude at equal times $\Delta_e = F_e(0)$

For $U_i \to \infty$:

$$\Delta_e^S = P_0^S$$

$$\Delta_e^{T\alpha} = P_0^{T\alpha}$$

- Coupling to superconductor $\Gamma_{\mathrm{S}}$: Even-singlet
- Inhomogenous magnetic field $\Delta B_\alpha$: Even-triplet along the inhomogeneity
- Noncollinear magnetic field $\Delta\mathbf{B} \nparallel \bar{\mathbf{B}}$: Even-triplet perpendicular to inhomogeneity

In agreement with findings for SFS heterostructures Bergeret, Volkov, Efetov, RMP 2005

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# Odd-frequency order parameters

- Pair amplitude at equal times vanishes $F_o(0) = 0$
- Consider time-derivative of pair amplitude at equal times
  $\Delta_o = \left.\frac{dF_o(t)}{dt}\right|_{t=0}$
- Express odd-frequency order parameter in terms of even-frequency order parameters and local and nonlocal expectation value of charge and spin

$$\mathbf{\Delta}_{\mathsf{o}}^{T} = -\frac{i}{2}\Delta\varepsilon\mathbf{\Delta}_{\mathsf{e}}^{T} + \frac{i}{2}\bar{\mathbf{B}}\Delta_{\mathsf{e}}^{S} + \frac{1}{4}\Delta\mathbf{B}\times\mathbf{\Delta}_{\mathsf{e}}^{T} + \frac{i}{2\sqrt{2}}\left(\Gamma_{\mathsf{S}}\mathbf{S} - \Gamma_{\mathsf{SL}}\mathbf{S}_{\mathsf{R}}^{\mathsf{L}} - \Gamma_{\mathsf{SR}}\mathbf{S}_{\mathsf{L}}^{\mathsf{R}}\right)$$

$$\Delta_{\mathsf{o}}^{S} = -\frac{i}{2}\Delta\varepsilon\Delta_{\mathsf{e}}^{S} + \frac{i}{2}\bar{\mathbf{B}}\cdot\mathbf{\Delta}_{\mathsf{e}}^{T} - \frac{i}{4\sqrt{2}}\left(\Gamma_{\mathsf{S}}\Delta N + \Gamma_{\mathsf{SL}}N_{\mathsf{R}}^{\mathsf{L}} - \Gamma_{\mathsf{SR}}N_{\mathsf{L}}^{\mathsf{R}}\right)$$

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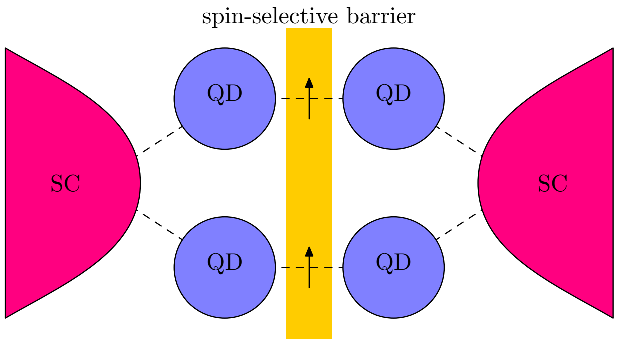
# Transport signatures

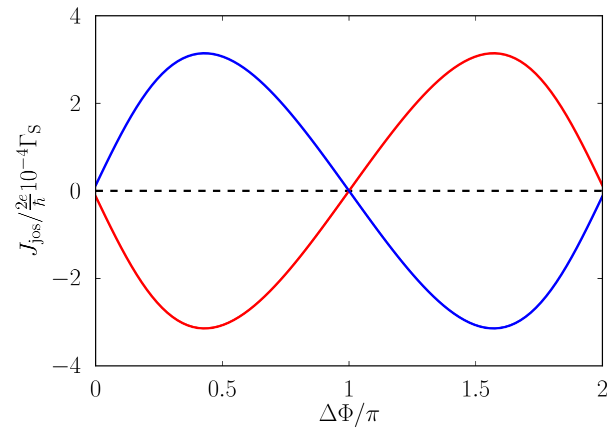
How to probe unconventional correlations in transport?

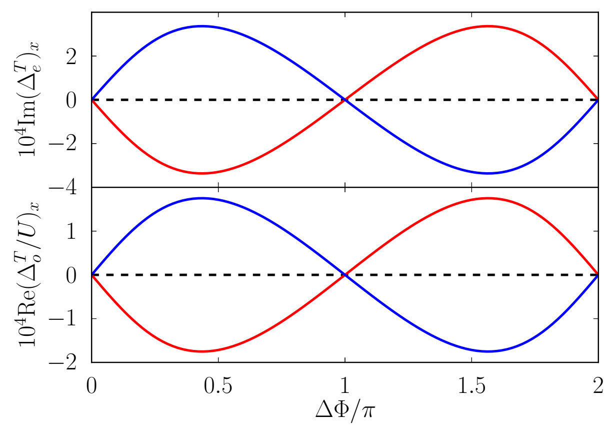
- Conventional SC only probes even-singlet pairing
- No unconvential electrodes available
- Indirect signatures needed!
- Contribution from even-singlet dominates transport
- Find situation where current due to even-singlet vanishes

Here: Two examples for detecting triplet correlations in Josephson current

# Triplet Josephson current




- Double dots connected via spin-selective barrier
- No Josephson current via spin singlet
- No magnetic field: Only even-singlet pairing: No Josephson current
- Inhomogenous magnetic field: Even- and odd-triplet pairing: Finite Josephson current
- $\pi$-junction possible

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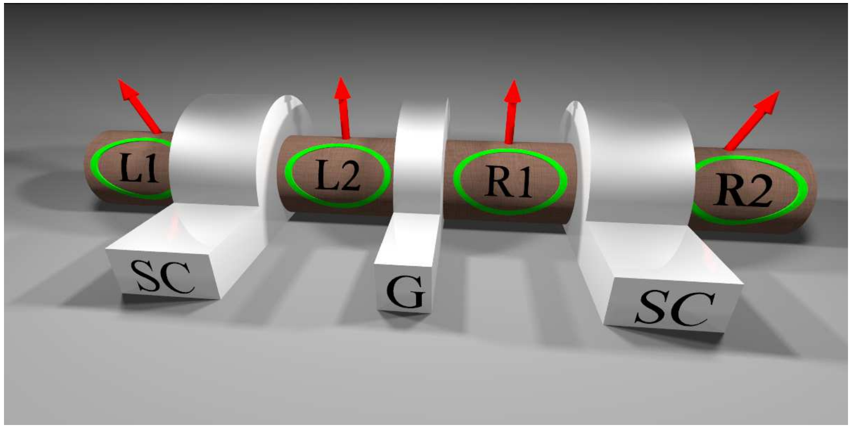
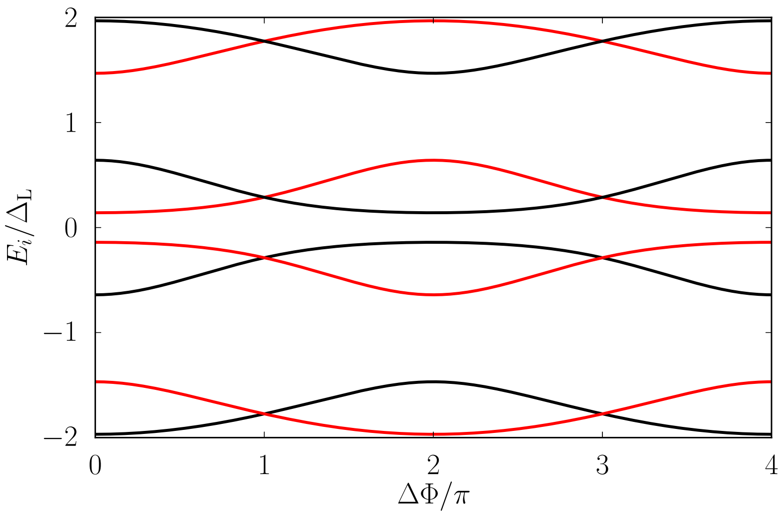
# Fractional Josephson effect

Sothmann, Li, Büttiker, NJP 2013




- Quadruple quantum dot with inhomogenous magnetic field
- Dots can be tuned to host Majorana fermions
- Fractional Josephson effect possible

# Summary

# Summary & Outlook

- Quantum dots can host unconventional SC
- All types of unconventional SC can be generated
- Tune system between different types of unconventional SC
- Signatures of unconventional SC in transport

- Can one find clear signatures of odd-singlet SC in this system?