A Kosterlitz–Thouless phase transition in the ground state of an antiferromagnetic spin-$\frac{1}{2}$ Heisenberg chain with nearest and next-nearest-neighbor interactions is re-investigated from a new perspective: A mapping of the components of the scalar product onto a set of loops is found. One can classify these loops according to whether any two of them can be transformed into each other in a continuous way (i.e., whether they have the same winding number). A finite size scaling of the fidelity susceptibility and geometrical phase calculated within each class of above mentioned loops leads to the accurate critical coupling constant value and enables one to find that the critical exponent $\nu = 2.000 \pm 0.001$. 