We study a two-dimensional (2D) effective orbital $t$-$J$ model [1] derived for strongly correlated $e_g$ electrons in doped manganites. At half doping, ferromagnetic (FM) and antiferromagnetic (AF) correlations closely compete, and a relatively small additional AF for the $t_{2g}$ core spins can trigger either antiferromagnetism or ferromagnetism. In the ferromagnet, alternating orbital order develops, while predominantly, but not exclusively, the orbitals within the plane are occupied in the AF case. Because the FM and AF states have very similar energy, small changes in the concerning parameters can have a large impact and can switch the system from one scenario to the other. For the same reason, spin order is destroyed easily with rising temperature. Upon doping, the kinetic energy drives a transition to ferromagnetism in the AF case and enhances occupation of the more mobile in-plane orbitals for the FM plane. We investigate the 2D model for parameters relevant to monolayer manganites and also treat the case of two coupled layers, i.e., bilayer compounds. Finally, we discuss the cooperative character of the changes in magnetic correlations and orbital reordering found recently in monolayer manganites.